Abstract

La ricerca è stata condotta al fine di studiare l'interdipendenza tra volatilità del mercato azionario e prezzi dell'oro. I dati sono stati raccolti con frequenza giornaliera, settimanale e mensile per il mandato su diversi orizzonti temporali di un paio di anni a partire dal 1 gennaio 2018 al 31 dicembre 2019, quinquennale dal 2014 al 2019 e per dieci anni dal 2009 al 2019. Gli strumenti utilizzati per analizzare ai set di dati sono: analisi di decomposizione wavelet, trasformata continua wavelet e analisi di correlazione incrociata tra dati aggregati e ciascun livello di decomposizione. I risultati indagano gli orizzonti temporali per gli investimenti in oro e il riequilibrio dei portafogli degli investitori al fine di difendere gli asset in periodi di elevata volatilità e imprevedibilità del mercato azionario. Sulla base dei risultati si è concluso che gli investitori tendono principalmente a considerare l'oro come un'attività difensiva contro la volatilità su orizzonti temporali di medio termine entro 2 - 2,5 anni.

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Introduction

Gold is a metal that throughout the history of mankind has played the role of a special commodity - money. For thousands of years, gold has successfully fulfilled the function of the universal equivalent, until it was replaced by inferior money. The combination of its physical properties and aesthetic qualities provided gold with absolute value. Therefore, even now, when the gold standard has become history, gold remains one of the main defensive assets.

Protective assets are anti-crisis instruments, means of hedging investment risks. In a situation of economic crisis, when stock markets collapse, and the currencies of most countries are devaluing, it is difficult to predict which companies and national economies will grow in the post-crisis period, and which ones will be especially negatively affected by the recession. Market hypothesis makes an assumption that gold will remain a stable value.

The fact is that the central banks of economically developed countries also consider gold as a protective asset that can strengthen the position of the national currency in a situation of global crisis. The maximum gold reserve is in the US gold and foreign exchange vaults, it is about 8 thousand tons.

For the national currency, gold is a solid security, a guarantee of reliability and stability. In addition, gold is a highly liquid asset, a traditional international payment and settlement instrument. Gold reserves serve as an indicator of the economic independence of the state, its competitiveness in the world market. Therefore, the trend of increasing gold reserves remains unchanged, and prices for this asset in the long term only increase (with minor adjustments during periods of stability).

A fall in stock markets is usually marked by an increase in the price of gold. Investors, getting rid of falling stock instruments, buy gold in order to reduce the risks of their investment portfolio. Large crashes can also have the opposite, albeit short-lived, effect: to cover their losses on stocks, investors sell gold from their own reserves, which increases its supply in the market and temporarily reduces the price. But then this asset rises again.

The "Great Depression" that erupted in 1929 and the abolition of the US gold standard in 1933 led to a devaluation of the dollar and an increase in gold prices. Then the price of gold for several

decades remained fixed at $ 35 per ounce, so the economic crisis of 1957 did not affect it at all. But with the abolition of the gold exchange standard in 1971 (and with the final departure from the gold backing of the dollar), gold prices rose sharply, and the 1973 economic crisis intensified this trend. After 1980, the price of gold began to decline, but the Black Monday of 1987 made its own adjustments, and gold rose in price for a while. A new upward trend was established after the tragic events of 2001. Gold reacted to the 2008 crisis with a sharp rise and a slight correction that followed. After a peak in 2011, there was a decrease in demand for gold as an asset, but the crisis in 2020 and the collapse in stock markets again led to an increase in prices for this precious metal.

There are several ways to invest in gold through the exchange market:

- By buying shares of gold mining companies;
- Purchasing gold futures;
- Investing in ETFs - index funds for gold;

We will consider futures contracts as the instrument closest to gold in terms of value, since there is an interest rate for fund management included in the ETF, and shares of gold producers cannot accurately reflect the movement of gold prices, since they are influenced by the company's management and reputation.

Chapter 1

1.1 Importance of risk assessment and the role of defensive strategies

Evaluating risks is a key concept in modern finance theory. Financial risk management is a process of defining the price for taking certain risks and controlling equal compensation for taking it using a range of financial instruments.

They are associated with the likelihood of losing a certain amount of money or not receiving it. Make sense to note that speculative risks are parts of financial risks for which both positive and negative results are possible. They are risky by their nature and the main feature is the likelihood of damage as a result of such operations.

There are the main financial risks:

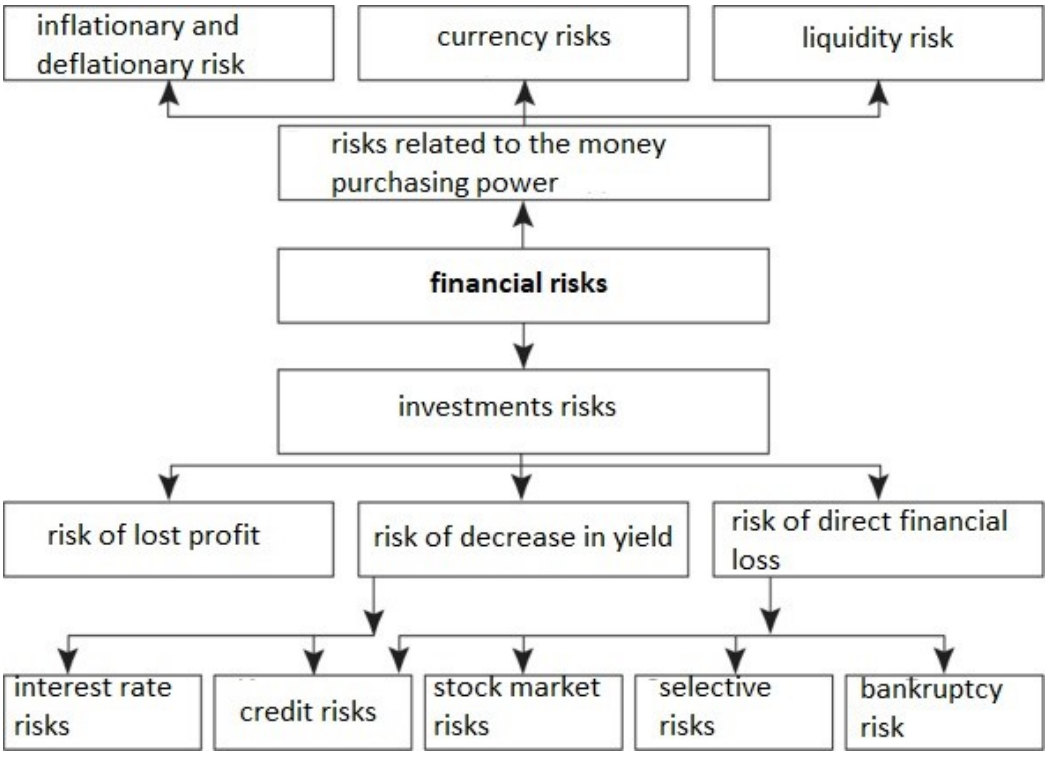


Figure 1.1 Financial risks

There are two main groups of financial risks:

1. Risks related to the money purchasing power: inflation or deflation, appreciation or depreciation of the local currency, liquidity risks.
2. Investments risks: risk of lost profit (interest rate risks, credit risks, stock market risks), risk of decrease in yield, risk of direct financial loss.

Financial assets flows through different elements of financial system form companies that have surplus in their balance to companies that have financial deficit. Some flows of assets are directed via financial intermediary in the market, e.g. banks or brokers. Similar to financial assets, risks tend to flow as well. There are some intermediaries in that market such as insurance companies that specialize on transfering and buffering risks. In the most cases financial assets and risks are connected to each other and are flowed together.

Financial risk, having different possibilities of occurrence, has a mathematically expressed probability of losses that might occur if the considering situation has a few mutually exclusive outcomes with a known probability of distribution. If the distribution is unknown, then the corresponding situation is considered as uncertainty. There is usually no distinction between risk and uncertainty in economic practice, especially in financial practice. Uncertainty related to some possible losses caused by the set of random adverse events is often understood as a risk. The loss can be objective, e.g. be determined by external influences as loss of purchasing power of money (inflation risk), however, losses often arise due to the decisions taken relate to financial assets management and here the owner has to choose the optimal solution and try to evaluate the probability of an adverse event occurring and choose an optimal strategy to defend his assets. The higher this probability, the greater the risk.

In general the main aim for any financial operations, initial and final conditions of which can be evaluated pretty accurately, is the profit maximizing which is the difference between the initial condition of a financial transaction and the final one. Most of financial transactions are conducted under conditions of uncertainty and therefore their outcome cannot be predicted in advance, consequently all financial transactions have inherited degree of risks. An investor who invests money in a bank, in a financial instrument, securities etc., counts on gaining profit and in case of getting less than expected, bears losses. Since the initial transaction had several possible outcomes, that were not equal for the investor and in terms of gaining profit, this transaction is considered as risky.

The bright side here is that an investor has a possibility to influence on the extent and magnitude of the risk through financial mechanisms. The impact is carried out using financial management techniques and special defensive strategies. Taken together these tools form a risk management mechanism, thus risk management is aт inalienable part of financial management.

Risk management is based on purposeful research and organization of work to assess, avoid, retain, transfer and reduce the extent of risk. The ultimate goal of risk management is to obtain the greatest profit with the optimal ratio of profit and risk acceptable to every investor.

Having various possible alternatives, the investor evaluates and compares them while it is assumed that for each conceivable mode of action, the predicted consequences may not actually happen, due to the influence of uncontrollable factors. The scatter of possible values relative to the expected value depends on the randomness of these mismatches and also on the amplitude characteristics. Therefore, each alternative should be weighted at least by two criteria: one gives a predictive estimate of the option (for example, the average value of the possible option), the second, as a measure of possible discrepancy, evaluates the degree of risk, meanwhile the riskiness of the option increases with the growth of the expected performance and profitability. Which alternative will be taken depends on the investor attitude to risk, how seriously he considers assets diversification as a risk management instrument and the proportions in which he is ready to accept the ratio of risk and reward.

Financial models are suited to analyze and predict the behavior in the financial markets and its individual parts, including markets in which money, bank loans and securities serve as goods. In accordance with the type of goods, the financial markets are divided into money and capital markets, which consist of credit and stock markets. The main purpose of the financial market in a market economy is to serve the production system, to promote the products of production that became commodities to consumers.

Risk management is an important part of the stock trading but often ignored by the market participants, a trader who has gained significant profits over the course of his work can lose it all in bad trades if proper risk management is not applied, which includes using predictive financial models.

There are certain strategies that can be used to reduce risk in the stock market:

1. Following the market trend

This is one of the proven ways to minimize risk in the stock market, however, the problem might be the market trend itself as it tends to change rapidly. A market trend can last for one day, a month, or there is a situation when short-term trends operate within long-term trends.

2. Portfolio diversification

It is a strategy in terms of which an investor allocates the risks depends on the trading strategy. Usually, he diversifies his investments in several industries, companies and types of assets. There is a possibility that while the market value of a certain investment might decrease, the value of another one might increase instead. One of probable solutions to this is mutual funds, that is also one of an option of risk diversification.

3. Buy and sell planning

The most successful traders and investors have trading strategies no matter what is happening in the market at the moment, moreover, often following their strategies allow them to make the most circumspect decisions when there is a high degree of volatility in the market. Besides, having trading strategy allow the participants to exclude the emotional part from the making decisions process.

4. Stop Loss and Take Profit points

Stop Loss and Take Profit points represent two main paths a trader takes into account while elaborating the trading strategy. The most successful traders usually know the price they are willing to pay for a certain asset and the price they are ready to sell at, if the probability of a fall in securities and possible consequent loss of investments is high enough versus the probability of possible profit if they continue holding the securities. In this case they execute the security transaction even if it will result in financial loss.

The strategies were elaborated in order to defend new investors in the market as they tend to hold securities in case of loss and hope that it will grow later, even though there might not be any indicators for that movement in the future. The opposite situation is also possible: investors tend to hold the position when they have gained a certain profit, hope it will grow more and simply

not pay attention to market trends and microclimate which might result in loss instead of expected greater profit eventually.

One of indicators of profitability which should be taken into account is return on investments (ROI). It shows how effective they are and is also used to compare the effectiveness of several different types of investments. It is calculated as the ratio of the profit to the investment's expenses. If there is a positive ROI but at the same time there are other opportunities in the market with a potentially higher ROI, then it is impractical to pursue the first option.

5. Forecasting models

Forecasting models cannot be considered as a defensive strategy itself, but they play an important role in overall risk assessment. A forecasting model is a formalized way of describing the process under study (forecasting object), which is the basis for obtaining its future values, which is based on the use of forecasting methods. The set of methods in each model is different and determines its belonging to one or another class of forecasting models.

Forecasting models in financial markets belong to the "super-complex" category, which means that in their description it is necessary to take into account the relationships between a large number of factors and indicators. There are multiple functions that describe these connections and financial forecasting models themselves are also complex due to differences in understanding of modeled objects.

One of the most popular forecasting models is candlestick formations, man of candlestick patterns match the shape of chart patterns. Together they form a set of geometrical objects that became an integral part of existing technical analysis.

The second step of forecasting models is an attempt to put market fluctuations into a mathematical model. During this a large number of mathematical objectives arise and the main purpose of mathematical modeling, in this case, is to calculate a logical numerical sequence of values to which the entire market can be offset.

The next step is based on the collected statistic data and statistical methods to search for patterns that might explain the behavior of model variables without significant reference to the economic theory. Among the most popular existing statistical models we can distinguish time series analysis, regression models and technical analysis.

Time series analysis is based on historical data and is supposed to follow the assumption that a sequence of values in a series of data repeats at regular intervals. Time series analysis can be combined with regression models. The construction of this type models must be approached systematically: the process must include an experimental plan, an algorithm for choosing a model structure an assessment of the model coefficients.

As for technical analysis, first of all it studies the price changes themselves and based on construction of charts, technical indicators, studying of open positions in the market and the volume of trading and assumes that prices contain all the market information available. However, there are a range of problems arise with making forecasts. Institutional factors cannot be descripted statistically, thus there is no opportunity to construct corresponding time series.

Efficient Market Hypothesis

Efficient market hypothesis states that all the new information is instantly reflected in market prices, therefore, nobody can predict the market prices dynamic in the efficient market and all the market prices are considered as fair for the assets. There are three form of efficient market: weak, semi-strong and strong.

Weak form supposes that the price of any assets fully reflects the previous information related to this asset, first of all – dynamic of the market value and trading volume.

Semi-strong form reflects not only full past information about an asset, but also public information available (e.g. companies' financial reports, official meetings, dividends amount etc.).

Strong form of efficiency reflects, apart from the above mentioned factors, the impact from insider information which is known by only few people. Analytics tend to consider the strong form as the most relevant and suppose that technical analysis hardly can assist in future value assessment as all the factors that might have affected an asset, has already affected it.

According to the random walk theory, changes in assets' prices fluctuate randomly and practically completely unpredictable. Although, all these theories are controversial, usage of insider information, lack of information in general and manipulations in stock markets also make the building of forecasting models more complicated.

1.2 Risk assessment and defensive strategies investigated in other studies

Full technical analysis is practically based on identifying and following the current market trend, meanwhile identifying the duration of the trend depends on the strategy an investor is following. There is a sample of evidence that stock past returns can predict future returns, one of the most comprehensive studies in this field is Moskowitz et al. (2012), which investigates time-series momentum (TSM) strategy. Time series momentum (TSM) refers to the predictability of the past 12-month return on the next one-month return. There are two main sections in the Moskowitz study: the first one examines the predictability of TSM, the second one addresses the profitability of a strategy based on TSM signals. Moskowitz and his coauthors found strong positive predictability from a security's own past returns for almost five dozen diverse futures and forward contracts that include country equity indexes, currencies, commodities, and sovereign bonds over more than 25 years of data.

Although, their findings were argued in a new study by a group of researches Dashan Huang , Jiangyuan Li , Liyao Wang, Guofu Zhou in 2020. By investigating the given data and results from the initial study the authors suggest that the t-statistic in a pooled regression is large which is cannot be considered as a reliable indicator. Therefore, they agreed on the conclusion that the empirical results from the Moslowitz study on TSM are weak in both asset-by-asset time series regressions and a pooled regression accounting for size distortions. However, the authors do not claim fully that there is no predictability in the asset classes. The authors assume that the predictability, if it exists, is not as easy to define as a constant 12-month return rule.

The overall conclusion from the studies on TSM is that the TSM strategy is profitable but its performance is practically the same as any other strategy based on historical sample mean and does not require predictability.

Portfolio diversification

A lot of investors, especially new in the market, tend to underestimate the strategy of portfolio diversification. Indeed, a large literature documents that many individuals invest in a few stocks, rather than diversifying through equity funds, tend to invest in the same stock of the firm their work with, concentrating human capital risk and financial risk, and opt often fortheir firm’s

stock when allocating retirement saving (Dorn and Huberman, 2005). The main question is to which extent assets should be diversified and what percentage of allocation should be designated to each asset.

A wisely diversified investment portfolio should include 10 shares. According to the law of profitability, approximately three quarters of the total (in this case it is 6-8 shares) will bring profit, and only 24 shares will gain losses. Having established the same level of fixing the loss and profit for each security in the portfolio, the risk of gaining overall loss tends to decrease. The total profit will probably be gained by 6-8 assets in the portfolio and the loss by only 2-4 (maximum 6) assets. According to experts, a well-formed investor's portfolio should be divided into 4-7 assets. This range is considered as the most optimal for a competent diversification and at the same time tries to exclude the risk of 'loss profit' in case when the portfolio is over-diversified. The main types of diversification that should be taken into account by any investor are: Assets diversification and Currency diversification.

One more important point of the diversification strategy is the portfolio rebalancing. If an investor splits the portfolio into, for example, gold and stocks at the ratio 50:50, and stocks rises over time, but gold falls, the proportions of the portfolio will become 40:60 and the initial balance will be distorted. In order to return back to the initial allocation, the investor should sell part of shares and buy fold with the money earned. Technically, the assets that have risen in price may also fall later and vice versa, if we do not apply rebalancing on the regular basis, profits will be lost. That is why assets prices should be monitored and rebalancing should be done if need as the main goal of the diversification strategy is to minimize risks and increase the rate of return.

Buy and sell planning

Buy and sell planning cannot be considered specifically as a defensive strategy but as a trading strategy in general. First, buy and sell planning is based on moving averages, that are elaborated from price history. Moving averages show fluctuations over time, smoothing occasional price picks to illustrate general direction of a stock over time. Traders tend to track two moving averages: one for short duration and the second for longer duration. The method of two moving averages consists of usage of, for example, 50-day and 200-day moving averages. When the 50-day average crosses above the 200-day one, it generates a buy signal. When the opposite happens

– it generates a sell signal. Moving averages help traders to set up signals for buying or selling at the right point in the current trend.

Buy and sell planning also takes into account cycles from fear to greed and back over time. Times of maximum fear is the best time to buy assets, times of maximum greed is the best to sell, even though none can define the picks certainly but can be fairly close to them. Such tendencies happen a couple of times every decade following cyclicality of business and economy. When economy is in a recession, fear in the markets dominates, when economy is on the rise – prices go up as well. Studying and monitoring such cycles might be useful not only for traders but also for investors following the long-term investing strategies for indicating the best time for buying expensive but promising on the long run assets.

Stop Loss and Take Profit points

The most popular algorithmic strategies is developed in Wang et al. (2009) study. In their study decision on choosing the most suitable trading algorithm depending on the investor's objectives is investigated.

Wang et al. (2009) introduced a process of trading strategy forming (Figure 1.2.1) separated in four steps:

1. Market data analysis and relevant external news
2. Software (e.g. spreadsheets or charts) in order to support and demonstrate the main outcomes from the analysis and generate the trade signals
3. Forming a trading model and decision making strategy
4. Execution of the chosen trading strategy which can be executed by a computer automatically

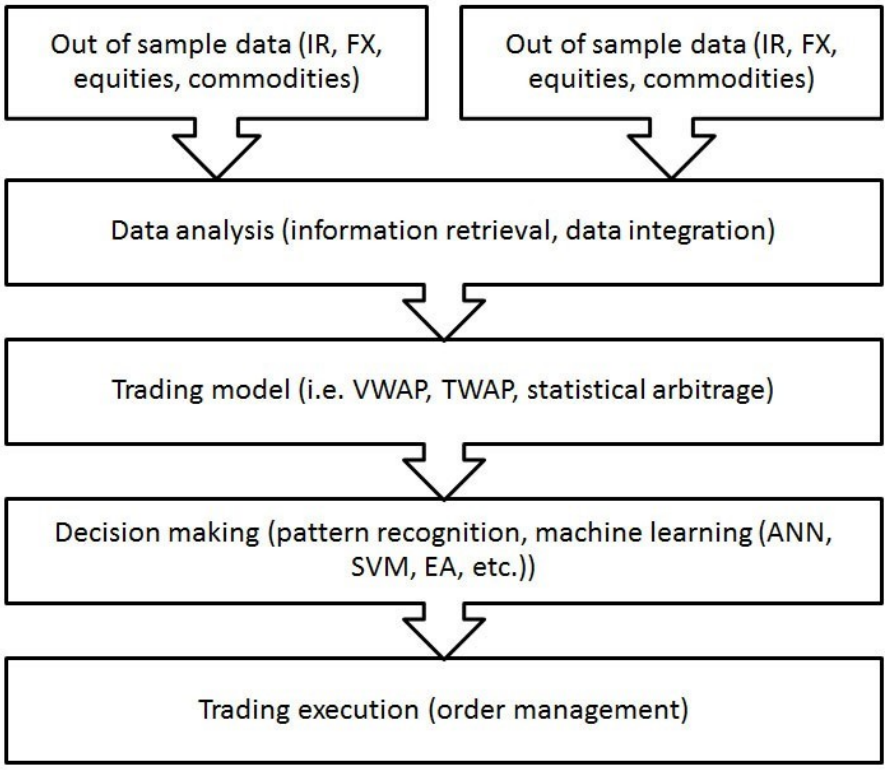


Figure 1.2.1 Algorithmic trading process (Wang et al., 2009)

Algorithm trading models provide traders with tools to reduce transactional costs, enhance control on risks and use information and newest technologies for decision making.

Forecasting models

There is range of forecasting methods for predicting time series. One of the most famous forecasting models is the autoregressive integrated moving average model (ARIMA) which was proposed by Box and Jenkins in 1970 for the analysis and forecasting of time series. Some studies using this model were conducted aiming to predict stock market returns (Adebiyi and Oluinka, 2014; Mondal et al., 2014), however many other studies showed that ARIMA models do not tend to predict financial time series precisely enough (Zhang, 2003; Khandelwal et al., 2015). To account for the nonlinearities resulting from regime change in the economy, some researchers have used the Markov mode switching and Threshold Autoregression (TAR) models, which involve nonlinear stationary processes to predict stock prices (Hamilton, 1989; Tong, 1990). Tasy (1989) proposed a simple but widely used model construction procedure for autoregression threshold models, as well as a threshold non-linearity test. Gooijer (1998) examined mode switching in a moving average (MA) model and used validation criteria to select

a self-excited threshold autoregression (SETAR) model. Several empirical studies comparing different methods with SETAR have shown that this method provides better results than linear models (e.g. Boero, 2003; Firat, 2017).

Later at the end of the twentieth century, artificial intelligence (AI) was involved into modeling, these models were represented mostly by neural network models. The main difference of AI models is that they are data-driven, nonlinear and adaptable. These factors made the models the most attractive and promising for forecasting purposes. Many studies have shown already that AI models yield more accurae forecasts compared to the others (Mostafa, 2010; Aras and Kocakoc, 2016).

Even though nowadays investors have various of different techniques to their disposal for forecasting stock market returns, none of them can be considered as a multipurpose one. Researches tend to agree that a suitable method should be chosen by each investor depending on the class of assets and the world stock markets they operate – developed, emerging or frontier.

Recently, frequency-domain models, such as spectral analysis and wavelets have been proposed for increasing the degree of accuracy of financial time series.

1.3 Why gold became a defensive instrument

It is generally accepted that gold acts as a protective asset, which tends to grow especially strongly during periods of crisis. The last rally in gold began back in November 2018, when gold gained about 20% for the year. But there is no crisis, stock markets continue to rise and renew highs. Therefore, the question arises: has gold always been a protective asset?

In 1944, the Bretton Woods Agreement was adopted. A gold exchange standard was introduced, based on gold and two currencies - the US dollar and the British pound sterling, which put an end to the monopoly of the gold coin standard. Under the new rules, the dollar became the only currency directly pegged to gold. The US Treasury pledged to exchange dollars for gold to foreign government agencies and central banks at a rate of $ 35 per troy ounce. In fact, gold went from being a primary currency to being a reserve currency.

The basic principles of the Bretton Wood system were the following:

- the US dollar is recognized as an international settlement currency that can be exchanged for gold at the rate of $ 35 per troy ounce;
- other world currencies, such as the British pound sterling, the German mark, etc., are pegged to the dollar;
- central banks ensure a stable exchange rate of their currencies using foreign exchange interventions;
- if necessary, the participating countries can devalue or revalue the currency;

For mutual assistance, the countries also agreed to establish a number of international financial organizations - the World Bank, the International Monetary Fund. The International Monetary Fund was formed to manage the system, which initially consisted of 44 countries. Each member of the organization determined the gold content of their currency and, on this basis, fixed the exchange rate in the currencies of other participating countries.

The American dollar, the only formally convertible into a metal national currency, became the international reserve instrument along with gold. The official gold price was $ 35 per troy ounce. The British pound sterling was declared the "second" reserve currency.

Gold, as any exchange commodity, has a number of economic factors that affect its pricing, this the functioning of this system could be carried out without any obstacles only as long as the US gold reserves could allow the free conversion of the dollar. As practice showed, this situation could not last long. The crisis of the Bretton Woods system was caused by a change in the balance of the main forces of the world economy and the fall in the US dollar. Since it was the key currency of the system, its fall caused the destruction of fixed rates and the entire world monetary system.

In August 1971 the US president R. Nixon canceled the dollar peg to gold. After that "market laws" ruled the gold price and contributed to the revaluation of gold. From 1971 to 1975, its rapid growth was observed: during this period, the price of gold increased 4.5 times - from $ 42 to $ 183. After that, until mid-1976, there was a downward correction to $ 114.During this period, the oil crisis occurred (from 1973 to 1975), which provoked a recession in the US economy. Gold actively started growing, acting as a defensive asset during the recession.

Later in 1976 the exchange of currency for precious metals was completely discontinued. As a result, the leading powers entered into a new international agreement the so-called Jamaican monetary system. Within this framework of the Jamaican monetary system, which is still in effect today, there was a complete decoupling of the US dollar from gold, which contributed to the continued growth of the price of the valued metal. Growth continued until 1980, reaching $ 748, setting the highest price in the 20th century.

Over 9 years (from 1971 to 1980), gold rose almost 18 times, and the risk of an expected correction or transition to a sideways trend only increased. And so it happened: from the end of 1980 to the middle of 1982, gold was continuously declining, and then turned into a sideways trend, from which the price managed to break up only in mid-2005. Thus, after the strongest growth in gold in history, quotes for 25 years could not update the maximum of $ 748.

In contrast to gold, since mid-1982, one of the longest-lasting uptrends in the American stock market began. For 18 years, there were several local crisis falls in the market, but none of them helped gold to get out of the “flat”.

The next point in the gold history starts at the beginning of 2001, a new wave of gold growth began, which continued until 2012. In the period 2007-2008 there was a global financial crisis, due to which gold corrected in mid-2008 by 13%. Though, despite this, it quickly recovered and continued its active growth. In 2013, the US stock index broke through the 2008 high, after which gold went into correction.

Figure 1.3.1 Gold prices 1971 – 2020

Figure 1.3.2 S&P 1971 – 2020

If we compare the charts of the American stock index S&P500 and gold, we can see that the turning points between them do not have specific dates, but are accompanied by long time zones, within which there is a change in investors' preferences and the flow of money from stocks to gold. Such an overflow, to some extent, gives gold the status of a defensive asset, since in the long-term period global trends are not co-directional: one is growing, the other is falling or is in the "sideways". At the same time, the presence of gold in a portfolio cannot always protect an investor at the time of global collapses in prices for all asset classes.

Besides, it should be noted that over the past year, gold and the S&P500 have risen together, which could mean a shift in preferences and a gradual flow of capital from the stock market to gold, but it is not known how long the transition will take.

Chapter 2

2.1 Time series and assessment of time series data

Aims of studying time series targets can be different. One can, for example, strive to predict the future based on the knowledge of the past, control the process that generates a series, try to figure out the mechanism underlying at the center of the process, to clear the time series of components that obscure its dynamics, or simply briefly describe the characteristic features of the series.

A time series is a sequence of observations, usually ordered in time. The main feature that distinguishes time series analysis from other types of statistical analysis is the importance of the order, in which observations are made.

There are two types of time series. Some quantities are measured continuously, at least in theory. In this case, observations can be recorded in the form of a graph. But even in the case when the studied values are recorded continuously, in practice, during their processing, only those values are used that correspond to a discrete set of points in time.

Consequently, if time is measured continuously, the time series is called continuous, but if time is fixed discretely (i.e., at a fixed time interval), then the time series is discrete.

Discrete time series are obtained in two ways:

- By sampling from continuous time series at regular intervals (for example, the capital of the firm, the volume of the money supply, the stock rate), - such time series are called moment series;
- Accumulation of a variable over a period of time (examples: the volume of production of a certain type of product, the amount of precipitation, the volume of imports) - in this case, the time series are called interval.

When analyzing economic time series, different types of dynamics are traditionally distinguished. These types of dynamics can, generally speaking, combine. Thus, the decomposition of the time series into components (components) is set, which, from an economic point of view, carry a different meaningful load. The most important are the following:

- A trend corresponds to a slow change in a certain direction, which persists for a significant period of time. A trend is also called a trend or long-term movement.

- Cyclical fluctuations are quasi-periodic dynamics faster than a trend, in which there is a phase of increase and a phase of decrease. Most often, the cycle is associated with fluctuations in economic activity.
- Seasonal fluctuations correspond to changes that occur regularly throughout the year, week or day. They are associated with the seasons and the rhythms of human activity
- Calendar effects are deviations associated with certain predictable calendar events such as holidays, number of working days per month, leap year, etc.
- Random fluctuations are random movements of relatively high frequency. They are generated by the influence of dissimilar events on the studied value (non-systematic or random effect).
- Outliers are abnormal movements of the time series associated with rare events that abruptly, but only very briefly, deviate the series from the general law by which it moves.
- Structural shifts are abnormal movements of the time series associated with rarely occurring events that have an abrupt character and change the trend

Some economic series can be considered to represent certain other types of such movements are almost pure. But most of them are very complex. They may show, for example, both a general tendency to increase and some seasonal changes, for which random fluctuations are superimposed. Often for time series analysis it turns out to be useful to consider individual components in isolation.

In order to consider individual components of time series data of S&P500 and gold futures' prices we will use wavelet transform methodology to decompose time series on different levels and extract the main trends, and cross-correlation methodology to study interdependencies between the aggregate data and also between the levels of decomposition.

2.2 Description of wavelet transform methodology and its advantages over other methods

Graphical constructions of any degree of complexity (from commonplace templates like "Head and Shoulders" to the Gann fan and Andrews' villas) and technical indicators (including all indicator systems) are just tiny pieces of technique called "technical analysis of the financial market". Meanwhile, most investors and market players build their strategies on these graphic templates for buy and sell points.

The pinnacle of the modern "science of trading and investing" is the so-called spectral and neural network analysis, which constantly fall out of the field of investors' attention, and the base of the spectral analysis is the concept of cycle, spectrum and Fourier transform.

The word cycle (from the Greek κύκλος - circle) is customary to denote any repetition, periodicity. Every cycle develops in waves and moves in a sinusoid from its base to the top, then the wave unfolds and descends back to the base. At this point, the cycle ends and then repeats over and over again along the same trajectory. The height of the wave is called the amplitude. For an exchange, amplitude is the value of a security.

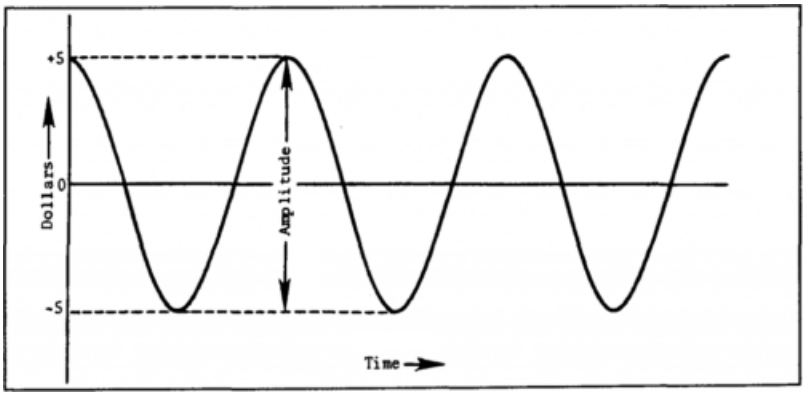


Figure 2.1.1 Amplitude

The second characteristic of the cycle is the period, that is, the time that separates the low points of the wave from each other.

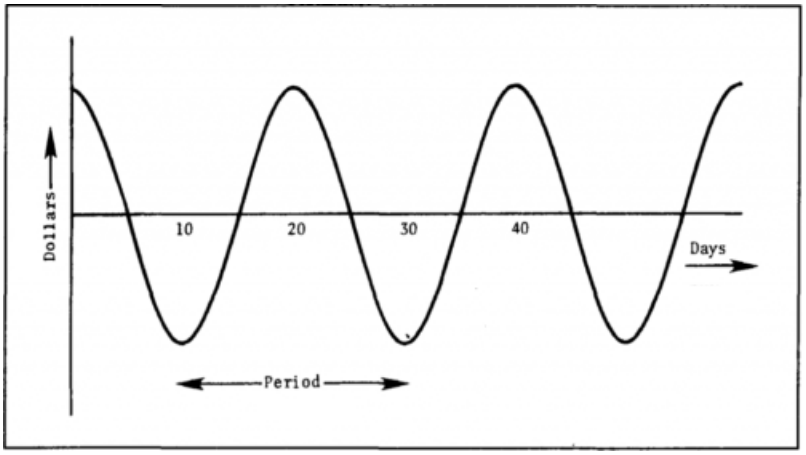


Figure 2.1.2 Period

The third characteristic of the cycle - phase - tells us where the bottom of the wave is located in time. The displacement of this base to the right (later) or to the left (earlier) along the X-axis is

called phase displacement. Two cycles can be completely identical in both period and amplitude, but differ in phase.

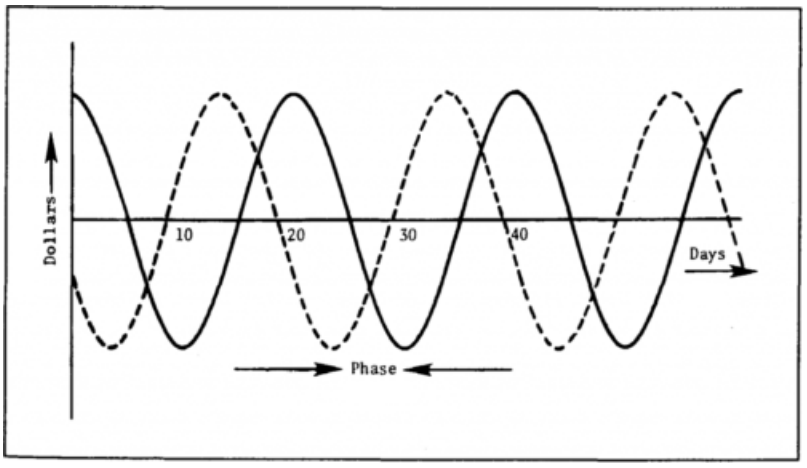


Figure 2.1.3 Phase

Having these pieces of information about amplitude, period and phase, we can easily build any cycle. After we can easily extrapolate cycle in future and use it for predictions, or, more accurately saying, for defining the direction of the least resistance of stock price changings.

There are four principles of cyclical changes in prices observed in the financial market.

The principle of summation, according to which each large price change is the sum of smaller changes. In other words, we can take any wave and decompose it into other waves that are built into it.

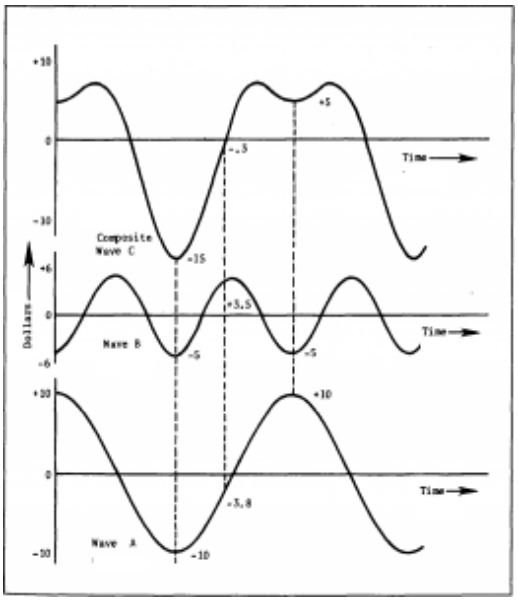


Figure 2.1.4 The principle of summation

The principle of harmony describes the relationship of waves embedded in each other. So, in the financial market, harmony is expressed in the fact that between the cycles of different periods, ratios are established that can be conveyed by simple small numbers, for example, 2, 4, 8, 16.

The principle of synchronicity is manifested in the fact that cycles, regardless of their periods, develop in time in such a way that they reach their foundations simultaneously, or almost simultaneously.

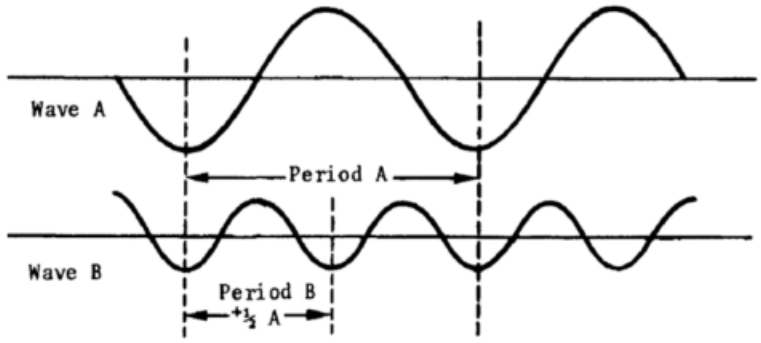


Figure 2.1.5 The principle of harmony and the principle of synchronicity

The principle of proportionality establishes a correspondence between the period and the amplitude and sounds like this: "The greater the period of the cycle, the greater its amplitude."

Apart from the above mentioned principles, cycles can be divided into two categories: external and internal.

External cycles are the so-called correlated cycles, that is, associated with some kind of external influence factors. The most famous external cycles are seasonal. They are associated with a particular month or season: for example, the “January rally” in the American market, the “summer dividend period” in the Russian market, etc.

Internal cycles are temporal patterns that are revealed directly from the signal itself. For example, the cycles that can be extracted from data reflecting the change in the price of an exchange instrument are internal.

The classical mathematical-statistical method used for cycles’ analysis in stock markets is the Fourier transform. The Fourier transform is an algorithm that allows to decompose any signal (sound, electromagnetic or light radiation, price changes in the market, etc.) into its constituent cycles, represented in the form of sinusoids, each of which has its own period, amplitude and phase. This decomposition is called spectral analysis.

The practical benefits of spectral analysis are enormous, for instance this technique allows to clear the signal from interference. The main use of the Fourier transform in the stock market is to decompose a price series into many of its constituent cycles. These cycles are located on the X-axis as their periods increase (for example, from 1 day to 5 years). The resulting cycles will differ in amplitude, the height of each cycle will be determined by the correlation coefficient: the higher the sinusoid, the stronger this cycle corresponds to the price behavior. After the performed analysis, it is necessary to clear the cycle from the "noise", keeping only the cycles that demonstrate the highest possible correlation. Having cleared the price chart from "noise" (e.g. other cycles with weak correlation), we combine the five obtained sinusoids into one cumulative wave. The cumulative wave, in turn, will help predict further price behavior in the price.

Despite the fact that the Fourier transform is a classic algorithm in spectral analysis, its use in the financial market raises disadvantages:

1. The Fourier transform uses a sum of periodic functions (sine and cosine) that go on to $+\infty$ and $-\infty$ on both ends of the real line. It does not say anything about the time coordinates of the identified cycles. It decomposes the signal into spectrums, and at the same time does provide with any information about cycles which worked out and are not valid now, which of them are decreasing, and which, on the contrary, are gaining strength.
2. A local characteristic of the function affects all the coefficients of the transformation (or of the series). In other words, it assumes stationery over the entire history of the signal, but this characteristic is not applicable to prices' cycles on stock markets. They constantly change their characteristics (amplitude, phase, period), and due to that they are still wrongly extrapolated to the future. For example, a cycle that shows a very high correlation in the past can fail at any moment, because its predicative qualities do not depend on the level of correlation.

Having these disadvantages, the trader needs to identify not cycles with high price correlation in the past, but cycles that are effectively projected into the future, that is, those that are endowed with high predictive ability. These problems of temporal relevance of cycles are solved by the so-called Wavelet transform.

Wavelet is a "small wave". When applied to a signal, "small" means "local", and according to two criteria at once - both in frequency and in time. The localized "small wave" is a snapshot of the signal. If we then arrange these snapshots one after another, we get a picture not of the cycle

as such, but of the cycle in its reference to time. In the context of stock markets, this technique allows us to determine which cycle was dominant a month ago, which was the week, and which is in effect today. In other words, the wavelet transform helps us to separate the "living" (still active) cycles from the "dead" ones (those that have already played and remained in the past). The window [X-ω] has variable and not fixed dimension: it gets smaller as frequency increases, and larger as frequency reduces.

Wavelet methods, therefore, are most suitable for the analysis of non-stationary financial and economic time-series due to its capability of breaking down the information into different layers of resolution and its time-scale localization properties. Moreover, wavelets are very handy in spotting the exact location in time of regime shifts, discontinuities, and isolated shocks to the dynamical system

The effectiveness of a wavelet based cross-correlation technique in analyzing the relation between two markets at different levels of time-frequency resolution is also demonstrated. This approach of multi-scale decomposition of a time-series using wavelet methodology allows us to detect changes in stock market behavior from a time-scale perspective where the data can be analyzed at different time horizons and frequencies simultaneously.

Wavelet transform methodology

Wavelet analysis begins with the consideration of a function known as the mother wavelet, which is given by,

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

(1)

Where a≠ 0 and b are real constants. The parameter a is known as the scaling parameter which determines window widths, whereas the parameter b known as the translation parameter determine the position of the window. Unlike Fourier analysis, a number of mother wavelets can be chosen depending upon the problem at hand. Just like in Fourier analysis, where a function is expressed as a combination of sines and cosines to transform the function into the spectral domain, wavelets can be used to project a function onto the time-frequency domain.

The scaling parameter a is typically taken to be a power of two so that a = 2j for some integer j. The compressed wavelet captures the finer scale resolution (high frequency components are

captured and well localized in time) of a given signal while the dilated wavelet (broad time window widths) captures low-frequency components of a signal by having a broad range in time. Thus scaling and translation, which allow adjustment of time window widths and their locations, are the most fundamental operations which enable go for higher and higher refinements in terms of time and frequency resolutions. Thus, the scaling and translation operations facilitate a given signal or function to be represented as a basis function which in turn allow for higher and higher refinement in the time resolution of a signal.

Since the large scale structures of a given signal in time are captured with broad time-domain wavelets, in which case the window width is broad (spread out in the time axis), the time resolution of the signal is very poor and captures only the low frequency components. However, finer and finer time resolution of the signal along with its high frequency components can be obtained by successive rescaling of the dilation parameter in time. The information at low and high scales is all preserved so that a complete picture of the time-frequency domain can be constructed. Ultimately, the only limit in this process is the number of scaling levels to be considered.

In the case of spectral analysis, a signal is taken and projected into the space of sines and cosines. Similarly a function can be represented in terms of the wavelet basis. The wavelet basis can be accessed via the integral transform of the form

$$\int_t K(t,\omega) f(t) dt$$

(2)

where K (t, ω) is the kernel of the transform and f(t)is a time domain signal. The above equation represents any generic transform where a modification in the kernel gives the required transform. In the case of Fourier transform the kernel K (t , ω) exp(-iωt) represents the periodic oscillations. The key idea now is to define a transform which incorporates the mother wavelet as the kernel. Thus we define the continuous wavelet transform (CWT) as:

$$W_{\psi}[f](a,b) = (f, \psi_{a,b}) = \int_{-\infty}^{\infty} f(t)\,\overline{\psi}(t)\,dt$$

(3)

Where Wψ [f](a, b) is the CWT which is a function of the dilation parameter a and the translation parameter b,f and ψa,b are as defined in equations (1) and (2). The CWT given in equation (3) should satisfy the following admissibility condition:

$$C_{\psi} = \int_{-\infty}^{\infty} \frac{|\widehat{\psi(\omega)}|}{|\omega|} d\omega < \infty$$

(4)

where ψ ω , the Fourier transform of the wavelet is defined as:

$$\widehat{\psi} = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} e^{-i\omega t} \psi\left(\frac{t-b}{a}\right) dt = \frac{1}{\sqrt{|a|}} e^{-ib\omega}\, \widehat{\psi}(a\omega)$$

(5)

The wavelet transform given in equation (3) is well defined subject to satisfying the admissibility condition given in equation (4). An important property of the wavelet transform is its ability to construct new wavelet bases

The working principle of wavelet transform is quite simple. A bunch of smaller signals are extracted from the main signal by translating (shifting time window location) the wavelet with parameter b over the entire time domain of the signal. Further, the scaling process is carried out where the same signal is processed at different frequency bands, or resolution, by scaling the wavelet window with the parameter a. This combination of translation and scaling allows for

processing of signals at different times and frequencies which in turn allows reading the signal at different scales of time and frequency resolutions. The above process of analyzing a given signal or function at different scales of resolution is termed as multi resolution analysis.

2.3 Description of cross correlation methodology

Currencies and exchange-traded assets cannot exist without interconnections. They are linked by numerous causal relationships. Therefore, a change in the price of one instrument inevitably entails a chain of other changes. The relationship between changes in two or more quantities is commonly called correlation.

A correlation between two quantities is a statistical relationship in which a change in one of the quantities leads to a systematic change in the other.

The cross-correlation function measures the similarity between a time series and lagged versions of another time series as a function of the lag.

XCF starts with the estimation of the sample cross-covariance function. Consider the time series y1t and y2t and lags k = 0, ±1, ±2,…. For data pairs (y11,y21), (y12,y22),...,(y1T,y2T), an estimate of the lag k cross-covariance is

$$c_{y_1y_2}(k) = \begin{cases} \frac{1}{T}\sum_{t=1}^{T-k}(y_{1t}-\bar{y}_1)(y_{2,t+k}-\bar{y}_2);\ k=0,1,2,\ldots \\ \frac{1}{T}\sum_{t=1}^{T+k}(y_{2t}-\bar{y}_2)(y_{1,t-k}-\bar{y}_1);\ k=0,-1,-2,\ldots \end{cases},$$

(6)

where y1 and y2 are the sample means of the series.

The sample standard deviations of the series are:

$$s_{y_1} = \sqrt{c_{y_1y_1}(0)}\text{, where } c_{y_1y_1}(0) = Var(y_1).$$

$$s_{y_2} = \sqrt{c_{y_2y_2}(0)}\text{, where } c_{y_2y_2}(0) = Var(y_2).$$

(7)

An estimate of the cross-correlation is

$$r_{y_1y_2}(k) = \frac{c_{y_1y_2}(k)}{s_{y_1}s_{y_2}}; \; k = 0, \pm 1, \pm 2, \ldots .$$

(8)

The practical meaning of calculating the correlation between financial instruments is to obtain important fundamental data necessary for making trading decisions. The reaction of the markets to the release of important economic news is expressed in the fact that at first the prices of basic assets (gold, oil, futures for industrial indices), sometimes the yield of government bonds, come into motion. As a result, exchange rates and stock prices change. By tracking the relationship of individual instruments, as well as the causal relationship between price changes, you can quickly revise your trading and investment plans. In addition, correlation analysis is used in investment portfolio management as a mandatory part of risk management.

- Cross-correlation is used to track the similarities in the movement of two factors over time.
- Stock investors use it to determine the degree to which two stocks move in tandem.
- Portfolio diversification requires selecting stocks and other assets that move in opposite directions in order to hedge losses.

Cross-correlation is generally used when measuring information between two different time series. The possible range for the correlation coefficient of the time series data is from -1.0 to +1.0. The closer the cross-correlation value is to 1, the more closely the sets are identical. Investors and analysts employ cross-correlation to understand how the prices of two or more stocks—or other assets—perform against one another.

Above all, cross-correlation is used in portfolio management to measure the degree of diversification among the assets contained in a portfolio. Investors increase the diversification of their assets in order to reduce the risk of big losses.

2.4 Description of data

In this section, we focus on data description from the stock market. The data selected consists of S&P500 prices as a volatility indicator and Gold prices as a defensive asset. Sample period for

both indicators is 2 years: 2018 and 2019, as in this long run period different internal trends appeared in terms of mid-term and short-term run.

Standard and Poor Index

The S&P500 Index was created on March 4, 1957 by Standard & Poor's, a rating society formed in 1941 as a result of the merger of Standard Statistics Company and Poor's Publishing. Standard Statistics released the first stock market indicator in 1923, which included 233 companies. After the merger, this number was increased to 416 and then to 500 in 1957.

Since then, the S&P500 has overthrown the Dow Jones 30 as the most representative US stock market index. Indeed, it includes more securities (505 S&P500 values versus 30 Dow Jones values). In addition, its value takes into account the market capitalization of the companies that make it up, while the Dow Jones value is based only on stock market prices. That is, a change in the dollar rate per one S&P500 share of a large company will have a greater impact on the index than a share of a smaller company. This is why the S & P500 is a more informative indicator.

Nowadays the S&P index consists of 505 shares issued by 500 large companies listed on the US stock market. This index covers approximately 80% of the total volume of such companies. The S&P500 shares are 505 because the index includes 2 categories of shares for 5 of its companies. To be listed on the S&P500, a company must meet the following financial criteria:

1. Market capitalization - $ 5 billion.
2. The minimum trading volume is 250,000 shares per month.

Gold futures contracts

Gold futures is one of the types of futures contracts, which is one of the most attractive investment and earning tools. A futures contract for the purchase / sale of an asset is concluded on a specified (clearly defined) future date at the current market value. In the process of concluding a contract, bonds and stocks, currency and commodities can act as the underlying assets that are the subject of the contract.

Futures is an agreement that stipulates the process of buying or selling products at the right time and at the price fixed in the contract. Agreements concluded between a buyer and a seller for the

supply of goods, services or shares, which are subsequently supplied at the cost at the time of signing the document, are futures, or futures contracts. With their help, the parties agree on a deferral of payments for products, but when signing the document, the cost is indicated. Such a contract helps to avoid problems with setting prices in case of sharp fluctuations in quotations. Since it is possible to opt for a no-delivery contract, futures are often viewed as a tool for speculating in the stock market.

Types of futures contracts:

Estimated - delivery of goods is not expected in reality. After the expiration of the contract, the profit / loss is recalculated, funds are credited / debited.

Deliverable - involves the delivery of the underlying asset in reality. The seller agrees with the buyer to conclude a deal for the supply of gold at the current price in six months, for example.

The futures price is the current price of the contract. While this paper is in circulation, the cost may change.

Key benefits of futures trading:

A wide range of instruments for trading on various financial exchanges, opportunities for portfolio diversification.

High liquidity of futures, which makes it possible to use different strategies in work.

The level of commissions is lower.

Collateral - when buying a futures, a trader invests less money than buying a real product: from 2 to 10% of the price of the underlying asset.

The value of gold in the medium / long term was influenced by the quantitative easing program by the US Federal Reserve, which was launched after the economic crisis and was supposed to stimulate lending, reduce interest rates, and further buyout from the asset market on the balance sheet Fed.

Gold futures can be bought in the format of standardized, regulated exchange-traded contracts. They usually trade intraday (during the same trading session) and close positions before completion.

The volumes of metal futures, which are traded on the London and Chicago Mercantile Exchanges, as well as on the COMEX, are measured in troy ounces, prices are set in dollars and US cents per troy ounce of 995 gold.
Gold futures are considered a universal hedging instrument.

Volatility index VIX

Since the mid-90s, the Chicago Board Options Exchange has been calculating the volatility indicator - VIX, or the so-called "Fear Index".
This indicator reflects the expectations of traders on the S&P 500 broad market index for the next 30 days, or rather its implied volatility. The indicator is calculated based on bid and ask quotes for index option contracts.

VIX shows the state of the market, its direction and mood. The regularity of the indicator is such that when the market falls, the volatility index rises, and when the market rises, the volatility index decreases.
According to the basic theory, if the VIX value is above 40-45, then this indicates panic in the market and the flight of investors from risky assets. Such situations develop when prices are at their lows and it is time to think about long-term purchases. If the value falls to 20 or below, then there is an uptrend in the markets and it seems that this will be so for a long time. In the area of long-term lows, it is time to think about closing long positions.

However, the range of fluctuations of the "fear index" may change in different historical periods. Moreover, the indicator is able to stay in the region of minimums for a long time. In recent years, the VIX has generally been low, largely due to the actions of the world's central banks, pumping money into the financial system. Since 2012, the index has been below 20 most of the time, around 11 at lows. Upward surges are just as rare as corrections in the US stock market.

Gold Volatility Index GVZ

The Volatility Index of Gold Exchange Traded Funds (Golden VIX, ticker - GVZ) shows market expectations of 30-day volatility in gold prices and is based on the VIX methodology based on options on SPDR Gold Trust Like other volatility indicators, GVZ can operate in a wide range of strike prices.

This index, like the VIX, reflects the expected level of fluctuations in the value of an asset, on the basis of which it is calculated. The indicator expresses the assumptions of trading participants on the forthcoming volatility of a financial instrument. In other words, any volatility index is an indicator of investor sentiment and their forecasts regarding the volatility of the underlying asset quotes over a certain period of time.

The data were collected based on a few criteria: relatively short-term period data (2 years) with high level of fluctuations and different trends to extract trend patterns on different time horizon within one main trend, mid-term period data (5 years) in order to investigate if the extracted patterns are applicable to extended time horizons and long-term data (10 years) to study if the same patterns are also applicable.

Index of volatility VIX was also studied in the same time periods with the same applied frequencies. VIX index was taken in order to compare its main trends with S&P and find the opposite trends.

Gold was chosen as an indicator of a defensive strategy instrument, which tends to grow in crisis periods. Time horizons for researching are short-term (2 year), mid-term (5 years) and long-terms (10 years). Volatility index of gold futures was also taken to compare its main trends with gold futures prices.

This choice of indicators is based on data availability at a daily frequency.

Chapter 3 Empirical study

3.1 Wavelet analysis with weekly data of S&P 500 and gold futures' prices

The analysis of the data will be done using the following tools applying to datasets with different time spans:

1. The first step consists of running wavelet transform and cross correlation sequence on the main studying data set with length of the sample for 2 years from 01 January 2018 to 31 December 2019 and with daily frequency of data.

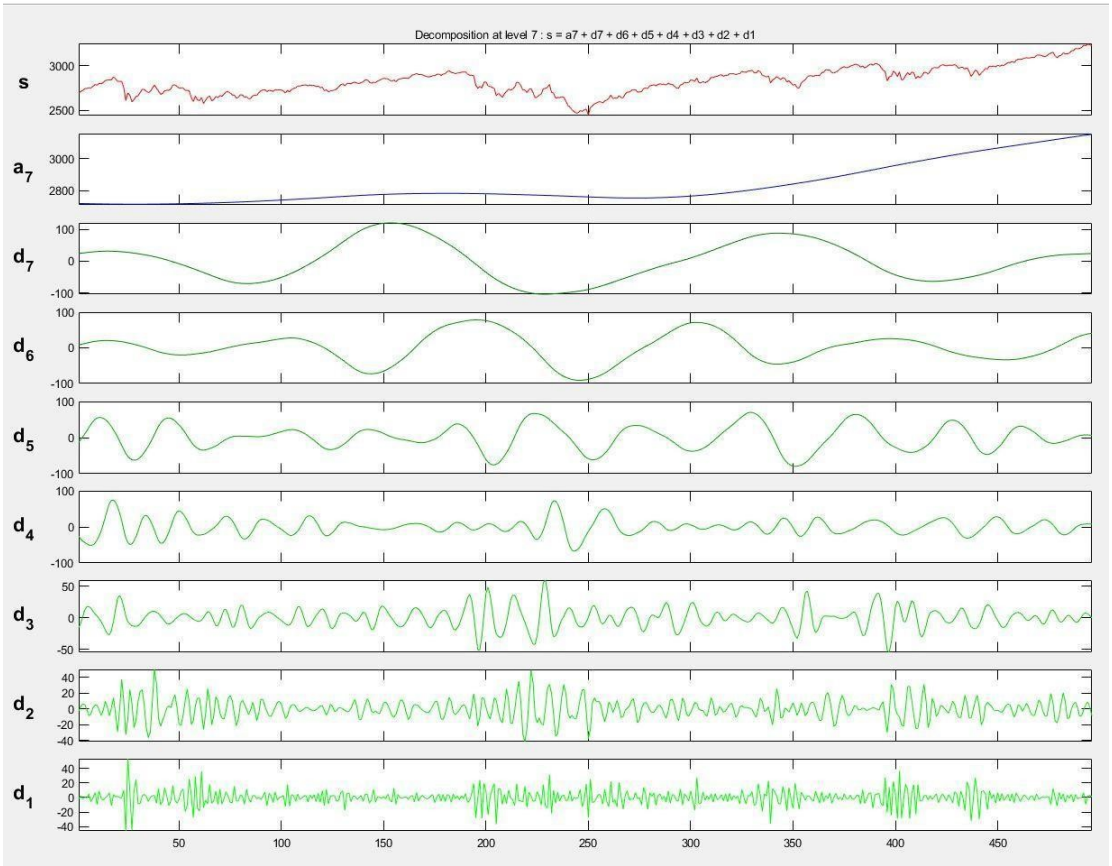
   The mentioned data sample was chosen based on a few main criteria: mid-term investment time horizon, relevance of the data close to present day and high levels of volatility within time span in order to monitor different movements and internal trends.
2. The second step consists of calculating wavelet power coherence in order to indicate possible correlations between the data samples, determine the level of correlation and specify the periods within the time span with the strongest and the weakest correlation.
3. The third consists of running wavelet transform and calculating wavelet power coherence on a larger time span of the data within 5 years from 2014 to 2019 and with weekly frequency range which is calculated as the average for each week. The range of data samples also will be extended and we will take into account changes in corresponded volatility indexes in order to see if the main hypothesis can be proved or refuted.
4. The fourth step consists of running cross-correlation analysis between widths of the data sets and also between each extracted component with different time horizons in order to see if the level of correlation changes along with changing time horizons.
5. The fifth step consists of repeating the same analysis as in step three, which is running wavelet transform and wavelet power coherence, but with data sets with larger time spans within 10 years and also with monthly frequency range.
6. The sixth step as well repeats step number four but with the large data set. Cross-correlation will be applied to each extracted decomposed level with different time periods as well.

The general graph of data sets will be indicated as S. All data sets will be decomposed in 7 levels from D1 to D7, and after the main trend will be presented and indicated as A7. Time horizon

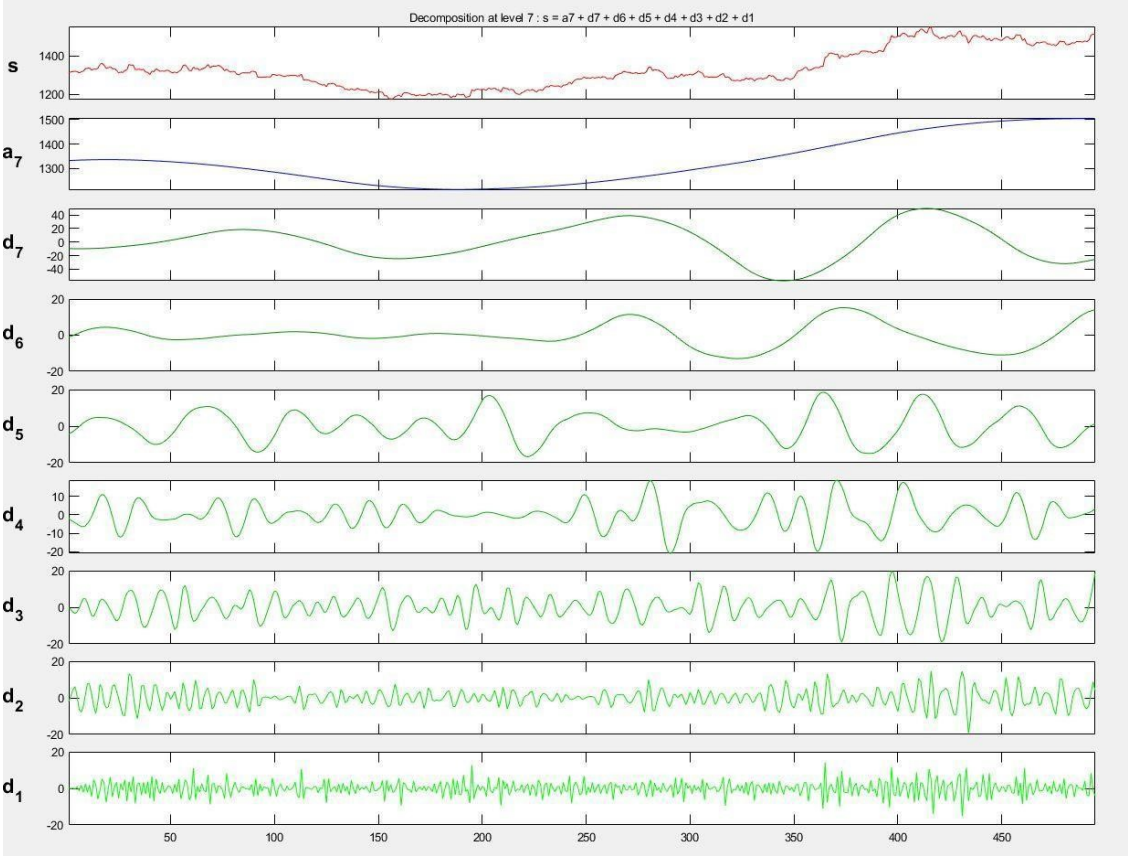
doubles with each level of decomposition and the actual time horizon depends on the frequency of the data set (days, weeks or months).

There are a few different types of wavelet transform that can be chosen for analysis. We decided to focus on the Symlet wavelet family which is also known as Daubechies' least-asymmetric wavelets. The symlets are more symmetric than the extreme phase wavelets. This type of wavelets was chosen based on the properties which allows to make the smoothed representation of the signal (i.e., processed by the scaling function) and its local features (obtained as a result of the wavelet transform) have two times redundancy. In other words, for a 2N wavelet (where N is the number of time periods), the result of signal transformation at each point is averaging of the previous signal and a set of details that distinguish the original signal from the averaged one. In this way the averaged signal is two times smoother than the original one and allows seeing the main trend of the original data set.

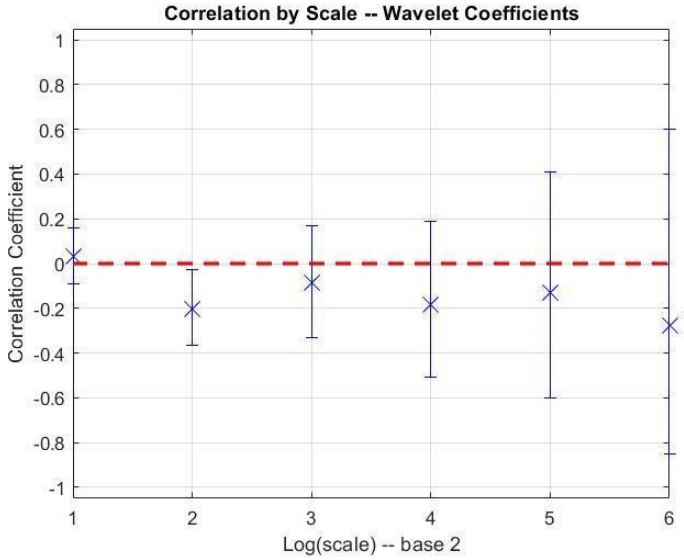
Analysis of the smoothed signal simplifies the identification of its characteristic properties, and the analysis of local features of the signal allows you to determine the nature and parameters of bursts and abrupt changes in the signal.



3.1.1 S&P daily from 01/01/1208 to 31/12/2019 wavelet decomposition



3.1.2 Gold daily from 01/01/1208 to 31/12/2019 wavelet decomposition



3.1.3 Correlation by scale S&P and Gold daily for two years

Comparing two trends from the Figures 3.1.1 and 3.1.2 of S&P and Gold data we can observe that the main difference in movements is at the beginning of the time span. Gold trend is in decline and reaches the lowest values closer to the middle of the time span when S&P trend acts differently and the trend demonstrates stability throughout the whole period with less significant fluctuations apart from a large decrease in the middle.

In the second half of the time span both trends behave similarly, they are on the rise till the end of the time span. The slops show almost identical dynamics, even though the gold trend is slightly steeper in the longer period which means the rate of change of this asset is faster than the rate of change of S&P. The observable increasing trend in the longer period after the middle of the time span is stable and constant, which indicates that the trend is not easily affected by changes significantly and the sensitivity remains relatively low.

Besides, from Figure 3.1.3 we can observe that S&P index and gold futures' prices are negatively correlated over the whole period. Positive correlation appears at longer scales at the end of the time span.

The second step is to run a continuous wavelet transform in order to get a wavelet power coherence to study relationships between gold and S&P. Wavelet scales are such that scale 1 is associated with 4 day dynamics, scale 2 with 8 day dynamics, scale 3 with 16 day dynamics, scale 4 with 32 day dynamics etc.

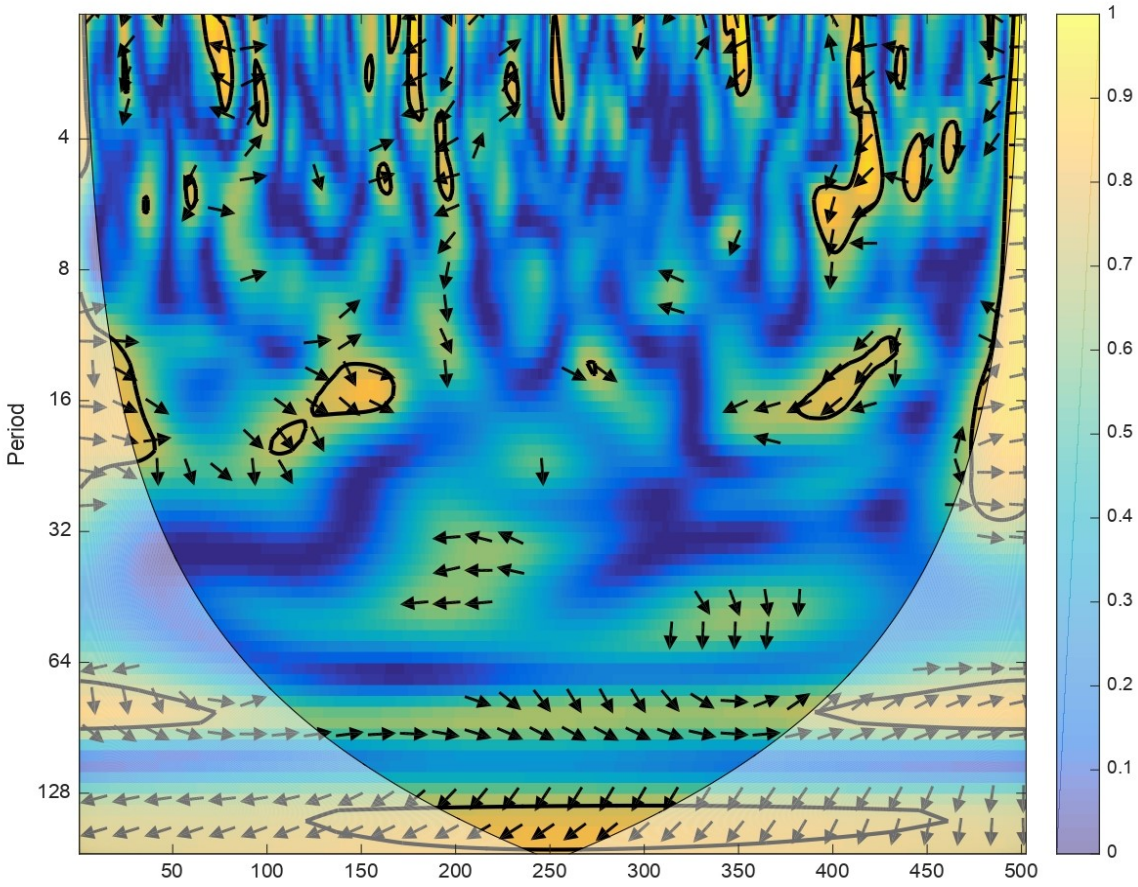


Figure 3.1.4 Wavelet power coherence of S&P and Gold daily observations for two years

The square of wavelet coherence is defined as the modulus of the transverse spectrum of the wavelet, normalized to the spectra of one wavelet, and measures the strength of the relationship between the two series in time and in frequency.

The thermo map of the wavelet power coherence can identify the regions in the time-frequency space where two phenomena have a strong local interaction. It is indicated by colour and its intensity that together highlight regions with different coherency and range from blue (low coherence) to red (high coherence).

A black contour line testing the wavelet power 5% significance level against a white noise null is displayed as is the cone of influence, represented by a shaded area corresponding to the region affected by edge effects at the beginning and the end of the time series.

From the spectrum map we can also extract information about wavelet phase difference. This difference is indicated by arrows and can be used to describe the phase of relationships between two time series as a function of frequency, e.g. phase of synchronization. The phase itself of x(t) wrt y(t) shows the relative phasing of the two time series and can be used to interpret lead or lag relationships.

In terms of phases, arrows can be interpreted as follows:

1. Right direction: two variables are in phase:
   1.1 right arrow points up y is leading (north-east)
   1.2 right arrow points down x is leading (south-east)
2. Left direction: two variables are in anti-phase:
   2.1 left arrow points up x is leading (north-west)
   2.2 left arrow points down y leading (south-west)

From the spectrum map we cannot observe red regions and can exclude strong correlation between gold and S&P.

Small positive correlation between trends is mostly observable on short and mid-term time periods (4-8 days and 8-32 days), this can refer to the same level of fluctuations on these time periods. One of the most significant positive correlations can be observed in the long time period which is 128 days, even though the relationships between the data sets on this period is positive but the arrows are indicating the lagging kind of the interconnection.

From the overall perspective positive correlation is mostly absent due to the difference in the trends in the first half of the time span. In the second half, in the longer time period, the relationships have positive but lagging tendencies.

Based on relatively short term data collected for two years and trends extracted, we can assume that gold does not work as a defensive asset against volatility in markets on short time horizons within 64 days, more likely investors tend to consider buying gold in terms of defensive strategy for mid-term time horizons form 64 to 128 days of volatility. In order to verify the validity of this assumption we take larger data samples with weekly and monthly frequency of data.

3.2 Wavelet analysis with weekly data of S&P 500 and gold futures' prices

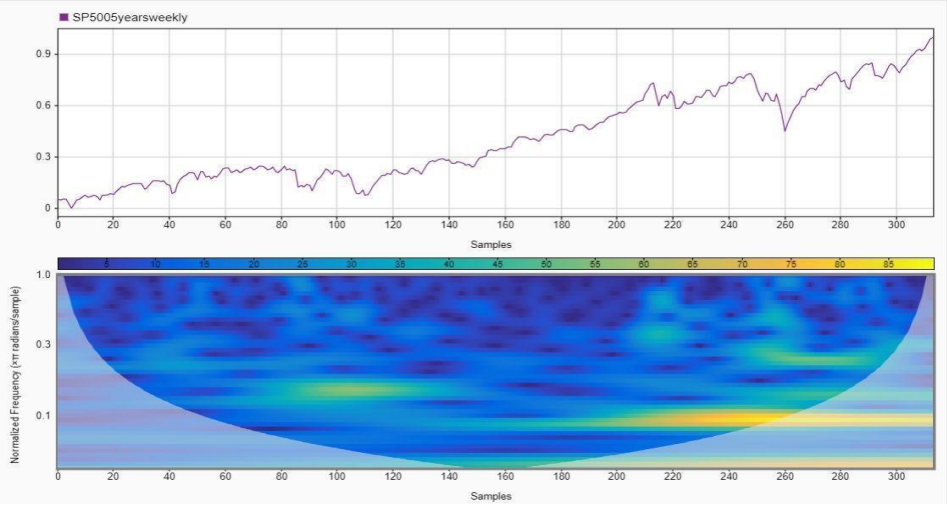


Figure 3.1.5 S&P weekly data scalogram (normalized)

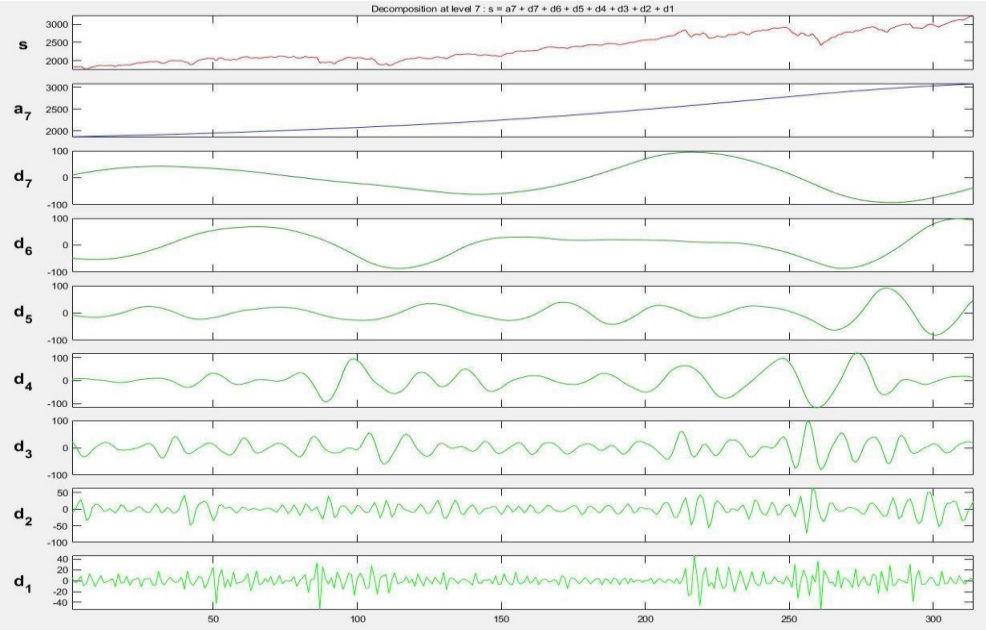


Figure 3.1.6 S&P weekly data decomposition

After decomposing the signal, the main growing trend can be seen very obviously. The biggest fluctuation periods could be seen at the beginning of the sample in 2015 and at the end around 2018. These fluctuations are also highlighted in yellow in the scalogram. Here we can assume

that the trend we extracted from the data for the two year period from 2018 to 2019 had been on the rise even before starting from 2014.

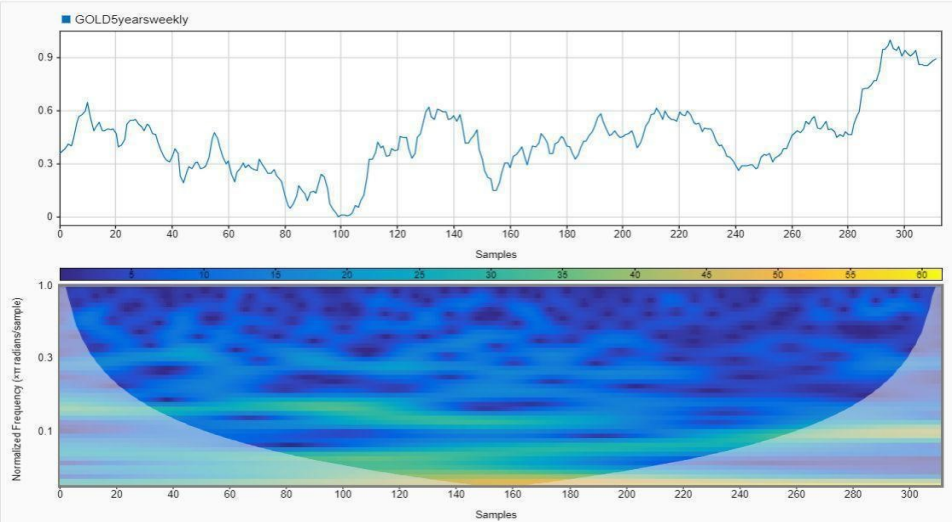


Figure 3.1.7 Gold weekly data scalogram (normalized)

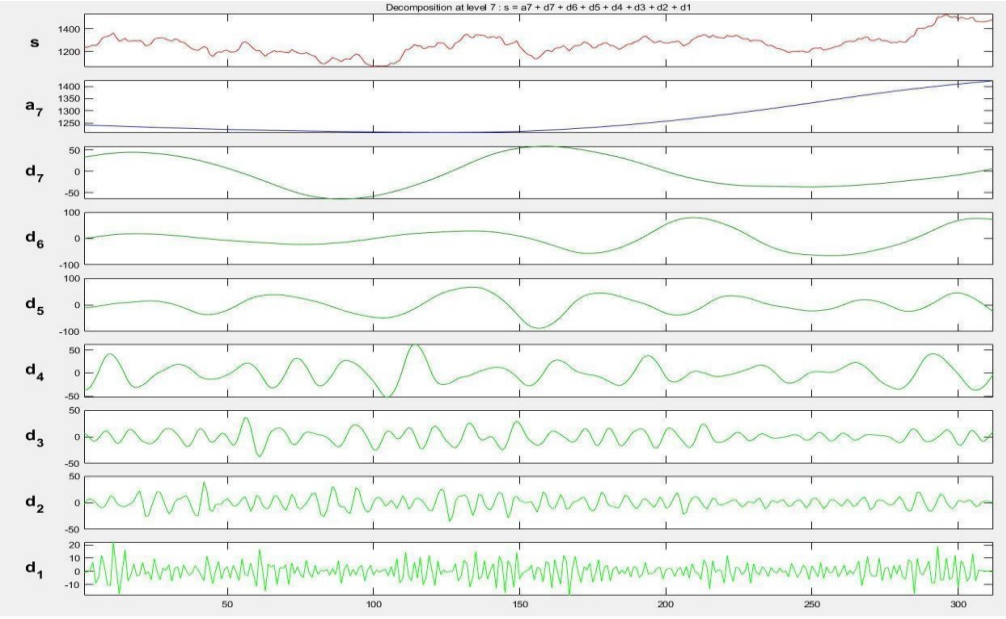


Figure 3.1.8 Gold weekly data decomposition

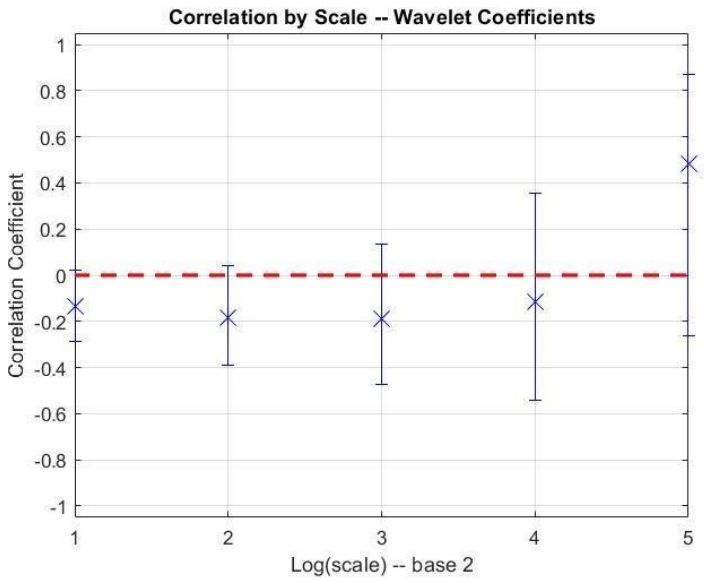


Figure 3.1.9 Wavelet coefficients S&P and gold five years weekly observations

Figure 3.1.9 shows that aggregate data are mostly negatively correlated over a period of five years, but the tendency of positive correlation at longer scales is stronger than over the time span of two years with daily observations.

After running decomposition of the gold data with weekly observations, we can see that the main trend is not as smooth as S&P500 in the same period of time, also the fluctuations are greater and occur with higher frequency, which for now can provide us with assumption that gold is more tradable asset than S&P500 index and investors do not tend to hold gold for years.

From the comparison of the normalized scalograms (Figures 3.1.5 and 3.1.7) and wavelet coefficients of gold and S&P we can observe that S&P trend has higher fluctuations but they happened only twice during the observable period, when the gold trend has higher number of fluctuations in general, but as the trend is not on the rise constantly, they are less visible and less sensitive to changes.

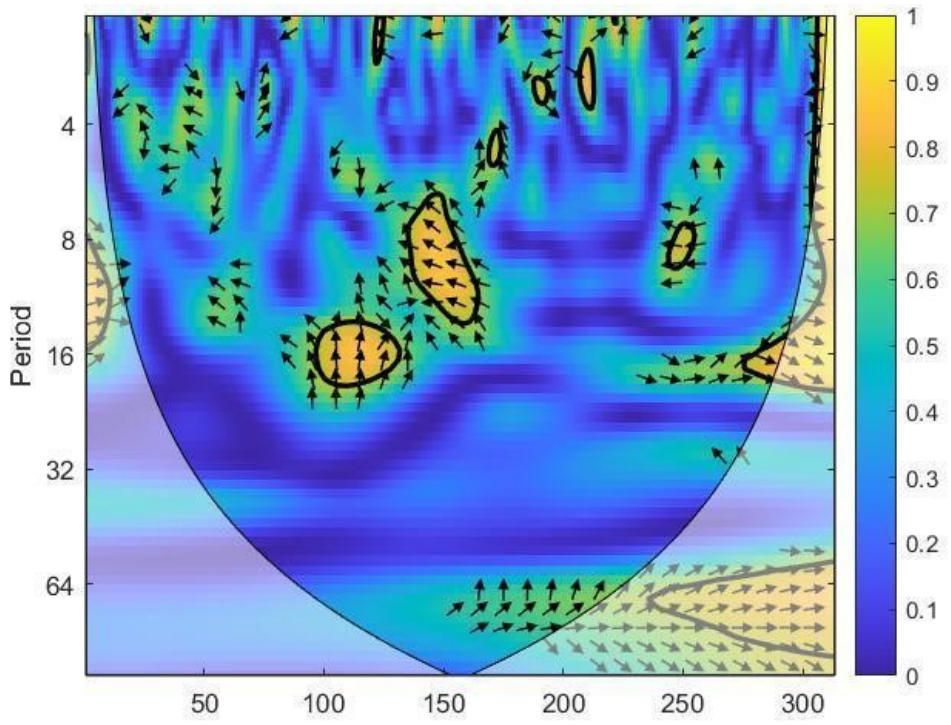
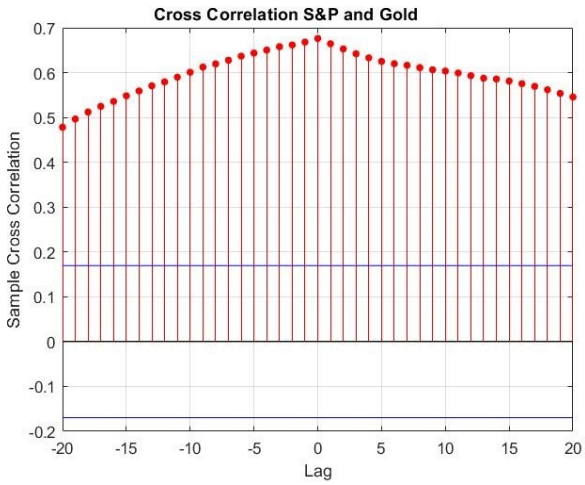
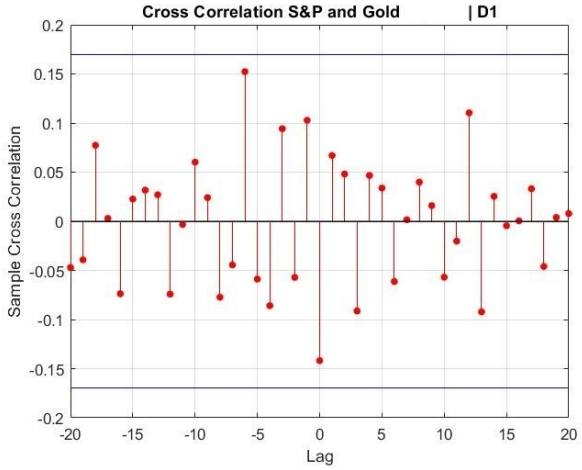


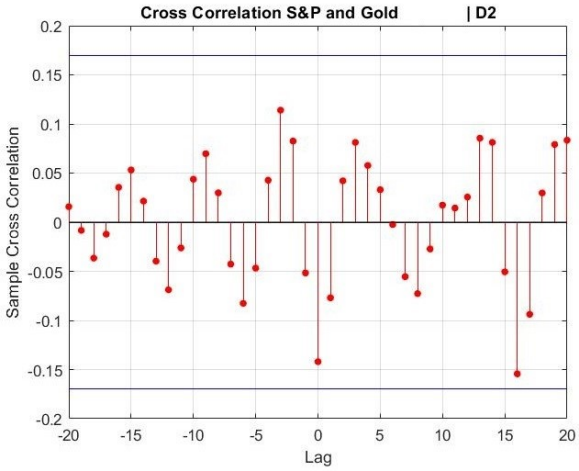
Figure 3.1.10 Wavelet power coherence of S&P and Gold weekly observations

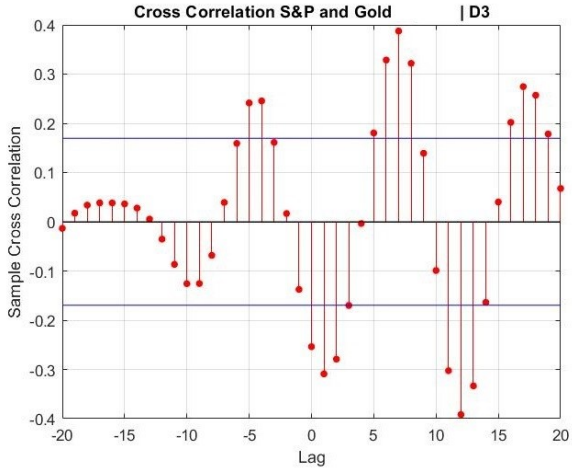
Referring to the figure 3.1.10 we can observe that the main correlation between the time series with weekly frequency of observations happens on the time horizon between 4th and 16th period clower to the middle of the considered time span, positive trend is mainly leading especially on the longer time horizon from 8th to 16th periods. A relatively small period of positively correlated relationships also can be seen at the end of the time span with 8th periods.

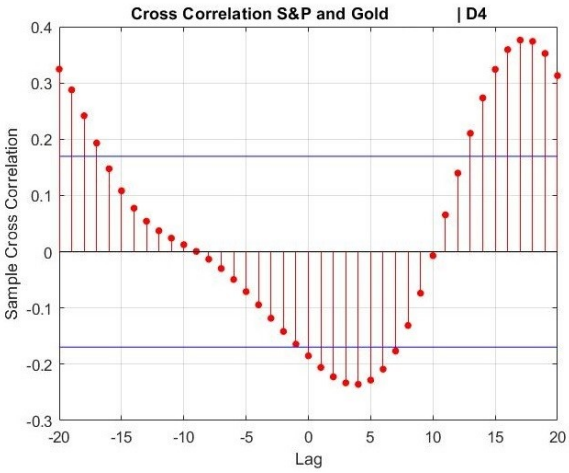
After applying multiresolution and extracting decomposed data, we study possible correlation between aggregate data and different levels of decomposition using cross-correlation analysis. Cross correlation analysis measures the extent of similarities between S&P index (vector X) and Gold future prices (vector Y) with a weekly shifted lag in time. The possible range for the correlation coefficient of the time series data is from -1.0 to +1.0. The closer the cross-correlation value is to 1, the more closely the sets are identical.


Cross Correlation S&P and Gold
Sample Cross Correlation
Lag


Cross Correlation S&P and Gold | D1
Sample Cross Correlation
Lag


Cross Correlation S&P and Gold | D2
Sample Cross Correlation
Lag


Cross Correlation S&P and Gold | D3
Sample Cross Correlation
Lag


Cross Correlation S&P and Gold | D4
Sample Cross Correlation
Lag

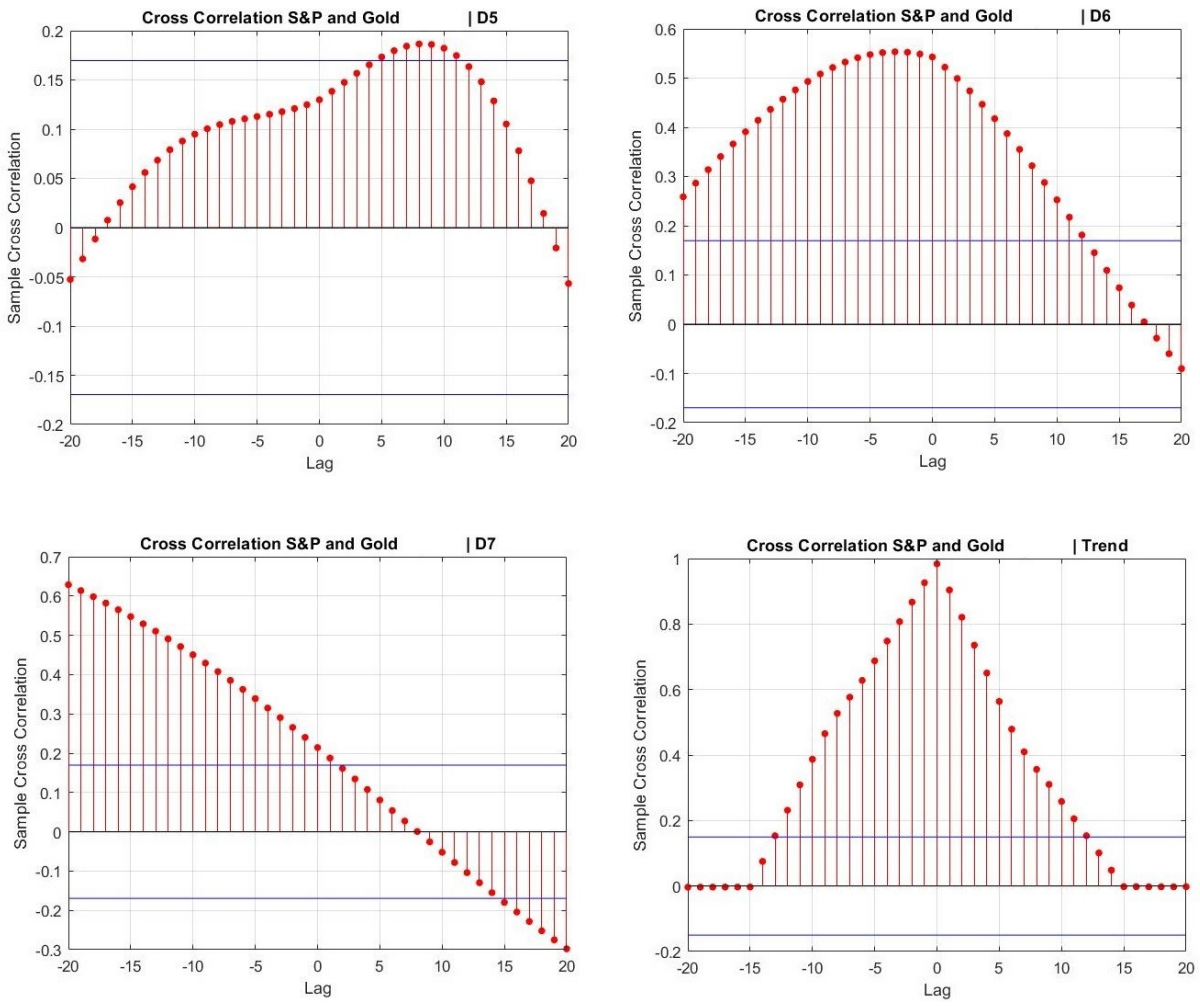


Figure 3.1.11 Cross-correlation analysis of S&P and Gold weekly observations for five years

As we can see from the graph on cross-correlation between the aggregate data, weekly lagged relationships between the time series are highly and positively correlated in the whole-time span of 5 years, most of calculated coefficients are above the 0.5 level and are outside the area delimited by confidence intervals.

However, the graphs of the decomposed level from D1 to D2 do not demonstrate any significant correlation between the decomposed data series as all of the calculated coefficients are inside the confidence levels, which means the data are practically uncorrelated. From the graphs we can see only weak negative correlation with peaks at 0.4 level.

Levels D3 and D4 demonstrate higher levels of correlation, but the correlation itself also becomes negative and exceeds confidence intervals in both cases: positive and negative. The

more the signals are decomposed, the more we can see that weekly changes in volatility affect gold future prices.

From level D 5 to D7 we can see that the higher the level of decomposition, the more positive the correlation becomes and barely exceeds the confidence interval which also can be explained by higher levels of fluctuations in gold futures' prices.

In the last step we can see the correlation of the main trends is strongly positive and lasts relatively long. It exceeds 0.4 which leads to a conclusion that the gold futures' prices were affected by weekly changes in S&P for around 2 years' time span.

Based on the wavelet power coherence and cross-correlation graphs we can conclude that the positive correlation between S&P and gold futures' prices is not obvious and mainly belongs to mid-term time horizons from 8 to 32 weeks. This correlation is affected by weekly fluctuations with one week lag. In other words, with effect from weekly frequency investors tend to consider gold futures as a defensive asset for mid-term investment time horizons from 8 to 32 weeks and these periods happened quite frequently and took 2 years out of 5 in total which is 40% of the total time span.

In order to receive a full picture on S&P and gold futures movements, we take into consideration specific indexes of volatility of these assets.

The Volatility index VIX shows the state of the market, its direction and mood. The regularity of the indicator is such that when the market falls, the volatility index rises, and when the market rises, the volatility index decreases.
According to the basic theory, if the VIX value is above 40-45, then this indicates panic in the market and the flight of investors from risky assets.

The Volatility Index of Gold Exchange Traded Funds (Golden VIX, ticker - GVZ) shows market expectations of 30-day volatility in gold prices and is based on the VIX methodology based on options on SPDR Gold Trust Like other volatility indicators, GVZ can operate in a wide range of strike prices.
This index, like the VIX, reflects the expected level of fluctuations in the value of an asset, on the basis of which it is calculated. The indicator expresses the assumptions of trading participants

on the forthcoming volatility of a financial instrument. In other words, any volatility index is an indicator of investor sentiment and their forecasts regarding the volatility of the underlying asset quotes over a certain period of time.

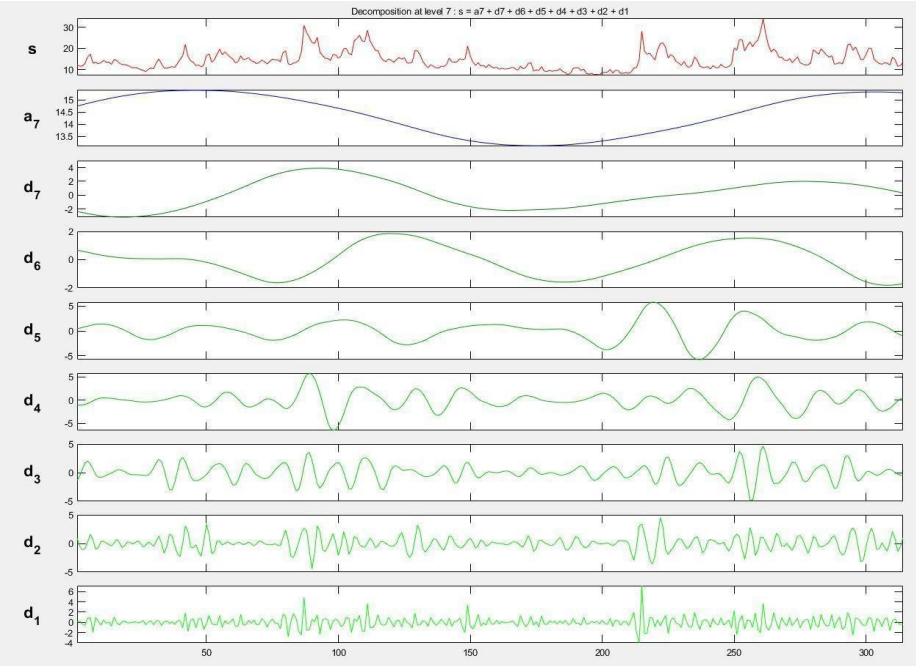


Figure 3.1.12 S&P volatility index VIX weekly data decomposition

As we can see from the figure 3.1.12, VIX index has higher fluctuations easily observable even in the main trend. First hypothesis from the observation is that VIX index is more sensitive to changes in markets compared to S&P500 and considered more as an instrument for speculative and even scalping strategies rather than for investment strategies. S&P500 in the same time span has smoother trend with much less fluctuations, it might happen due to bigger number of investors using S&P500 index as an instrument in their investment strategies. Moreover, careful investors who chose to follow long-term investment strategies tend to avoid high-risk positions and using speculative instruments like VIX but instead prefer to wait for volatility in markets to pass and continue following the chosen strategies.

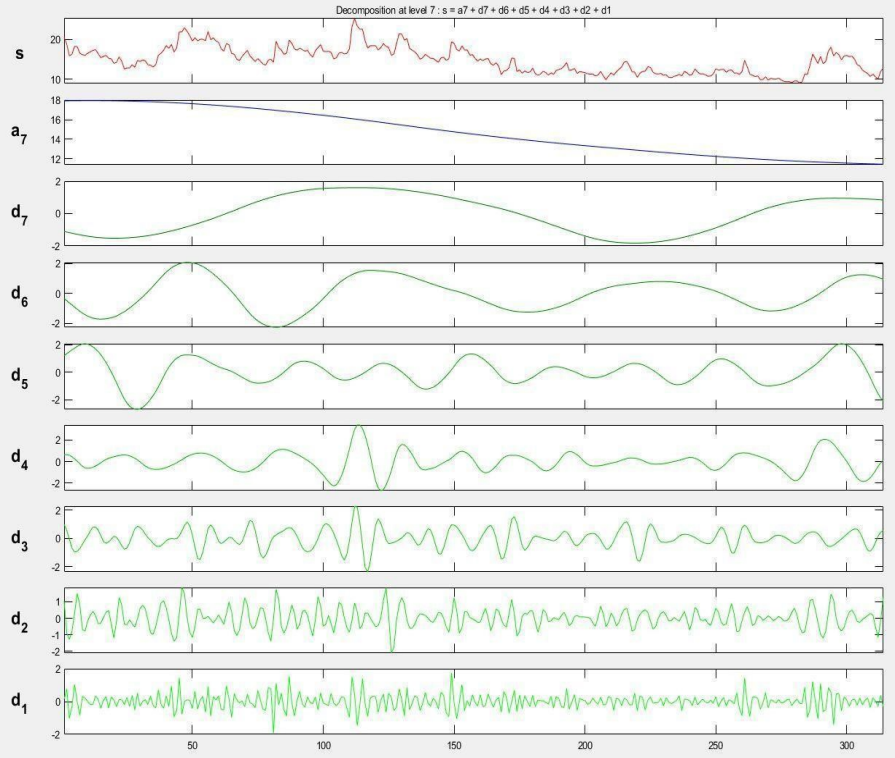


Figure 3.1.13 Gold volatility index GVZ weekly data decomposition

Volatility indexes in general should move the opposite direction to the asset. In the case of GVZ main trend, it behaves exactly as it is expected to behave: it demonstrates a completely opposite direction compared to the gold main trend for the same time period. Traders may use this index to hold positions against gold futures in markets when volatility periods come for relatively short period of time.

From the figures 3.1.12 and 3.1.13 we also can observe that general volatility trends of VIX and GVZ mainly go contrary to each other, though there is a fluctuation in the middle of the observable period of VIX when the directions of the main trends are broadly similar. That fact might occur exactly because the reasons mentioned above: that investors tend to hold S&P500 index in their portfolios as a long-term investment, which means that the final comparison between VIX trend and GVZ trend might be distorted.

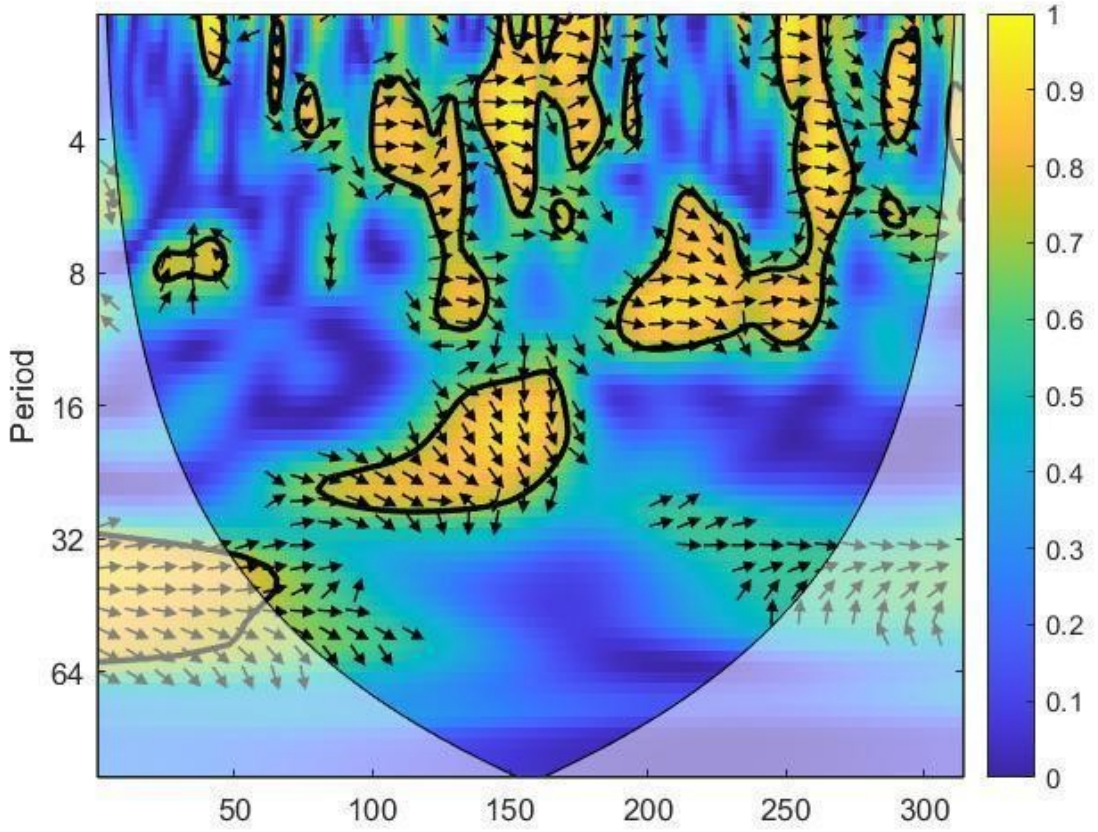
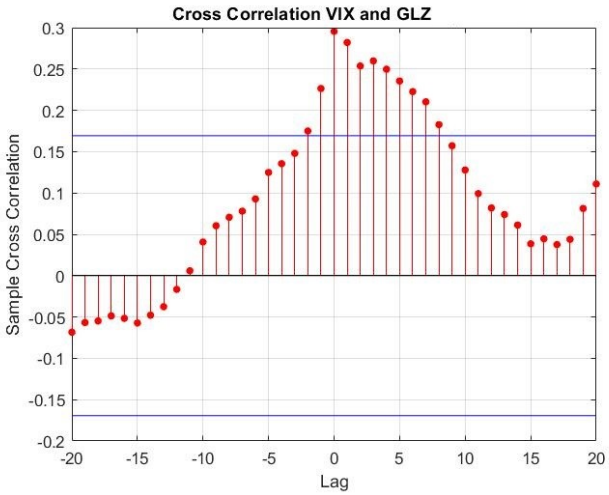
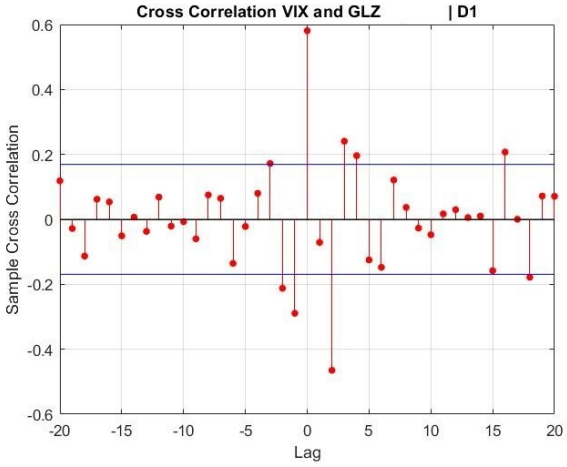


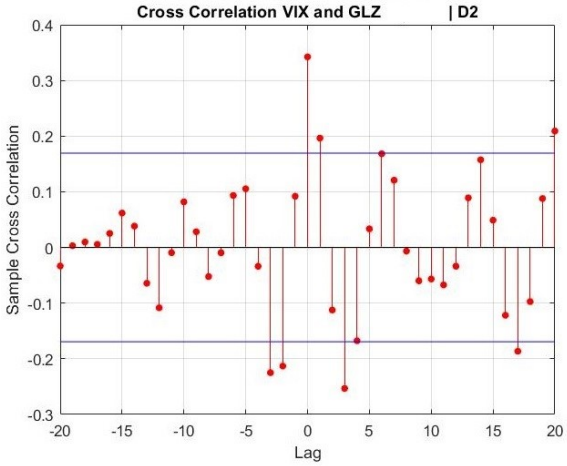
Figure 3.1.14 Wavelet power coherence of VIX and GVZ weekly observations for five years

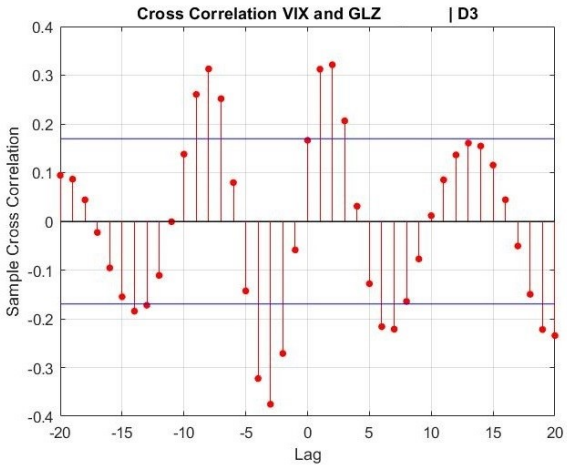
From the figure 3.1.14 we can observe strong positive correlation with the leading trend practically in most time horizons as volatility indexes are highly tradable instruments in the stock market and are mostly used for short time horizons to hold. They are not considered by investors for long-term investment strategies, but mostly as speculative instruments and as indicators of the decline trend in the market.

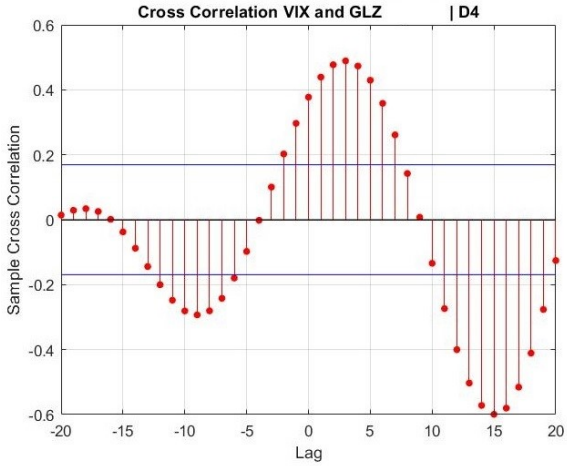
After applying wavelet decomposition and extracting decomposed data, we studied possible correlation between aggregate data and different levels of decomposition using cross-correlation analysis:


Cross Correlation VIX and GLZ
Sample Cross Correlation
Lag


Cross Correlation VIX and GLZ | D1
Sample Cross Correlation
Lag


Cross Correlation VIX and GLZ | D2
Sample Cross Correlation
Lag


Cross Correlation VIX and GLZ | D3
Sample Cross Correlation
Lag


Cross Correlation VIX and GLZ | D4
Sample Cross Correlation
Lag

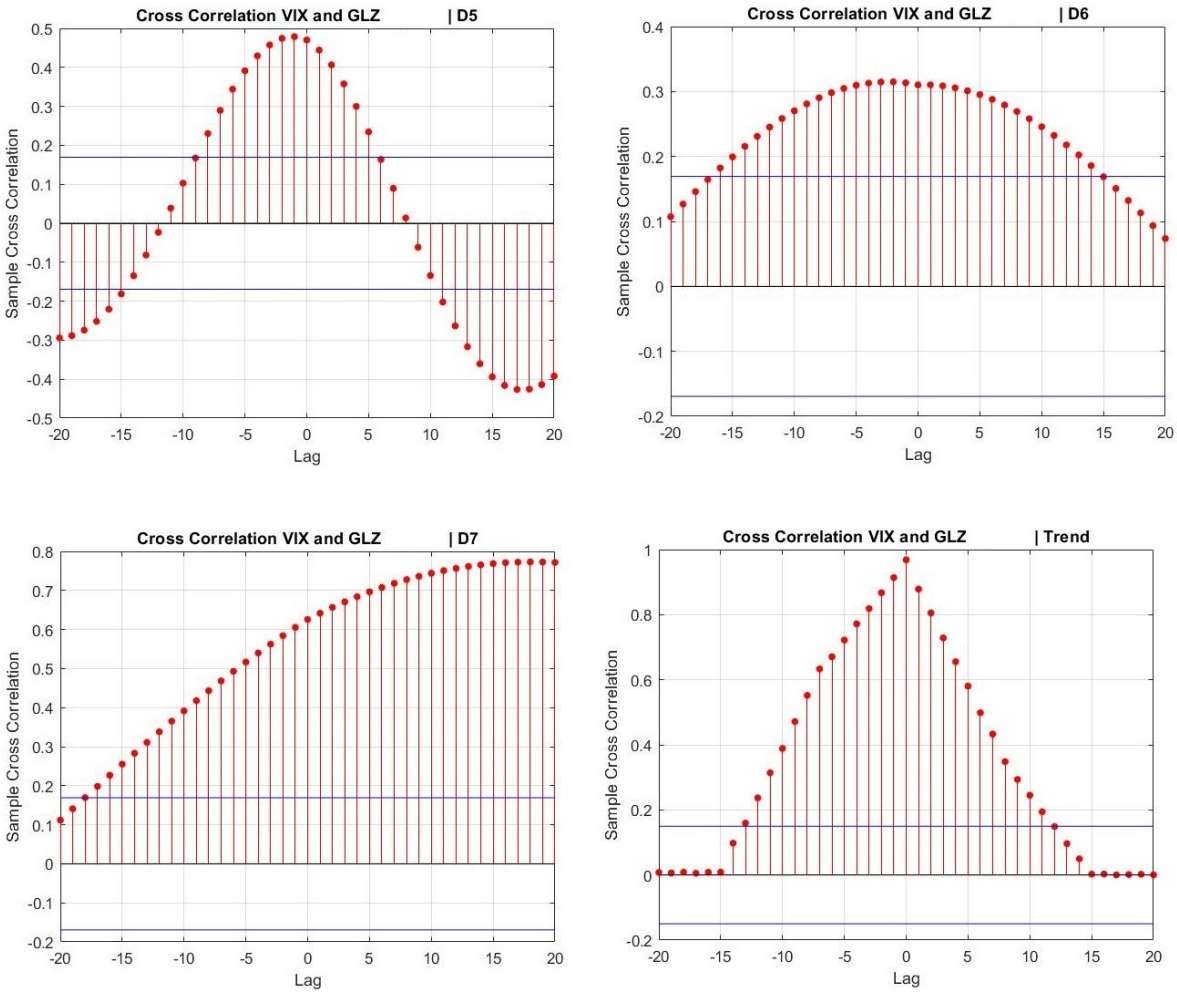


Figure 3.1.15 Cross-correlation analysis of VIX and GVZ weekly observations for five years

As we can see from the graph on cross-correlation between the aggregate data, the time series are mostly positively correlated in the whole time span of 5 years, however the calculated coefficients are inside the area delimited by the confidence intervals which means the correlation between aggregate data exists but it can be considered as negligible and the volatility index GVZ is not affected by weekly movements in VIX.

The decomposed level from D1 to D2 does not demonstrate any correlation between the decomposed data series and almost all the calculated coefficients are inside the confidence.

Decomposition from level D3 to D6 demonstrates changing dynamics in the positive direction, the more we decompose the initial signal, the more obvious the positive correlation becomes. Mostly positive peaks can be seen between lags -10 and 10. From that perspective we can

assume that weekly changes in VIX affect moves in GVZ in the positive direction, the level of correlation reaches 0.5 as the maximum.

From level D 5 to D7 we can see main trends of VIX and GVZ are positively correlated, the level of volatility in both indexes is high but the indexes tend to move in the same direction.

In the last step we can see that the dynamic of the correlation of the main trends is mostly the same as the dynamic between S&P index and gold future prices, weekly changes in volatility index VIX affect movements in GVZ to the same extent as S&P dynamic affects gold future prices. This fact derives from two main reasons: the first one is that VIX moves the opposite S&P direction and GVZ moves the opposite gold futures' direction. The second reason is that both instruments are mostly used for trading strategies and inherently have high levels of volatility, in other words if we compared any other volatility indexes (e.g., oil prices volatility index and VIX) the result would be broadly similar.

3.3 Wavelet analysis with monthly data of S&P 500 and gold futures' prices

In order to support the theory about using gold as a defensive asset mostly in short-term and mid-term time horizons and to investigate if the correlations between gold futures and S&P500 also takes place we studied longer periods of data within a 10 years' time span including volatility indexes for both assets.

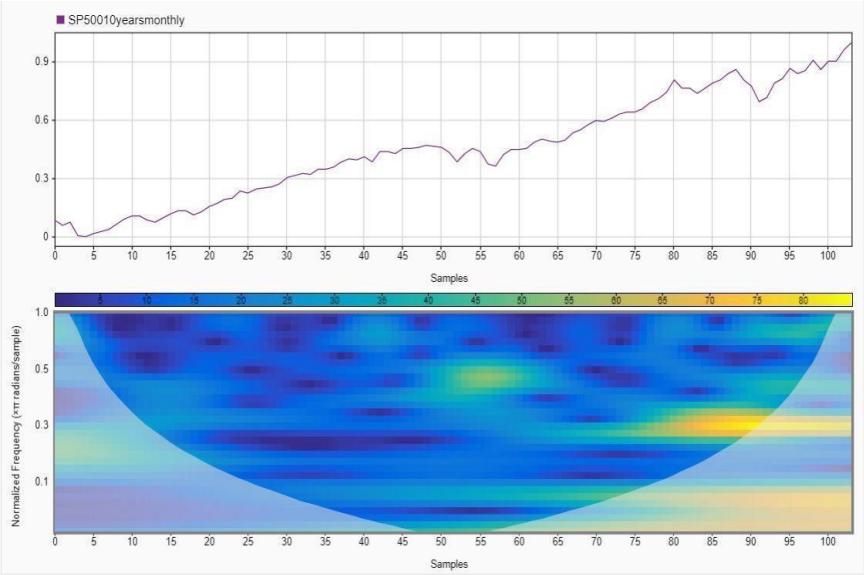


Figure 3.1.16 S&P 10 years monthly data scalogram (normalized)

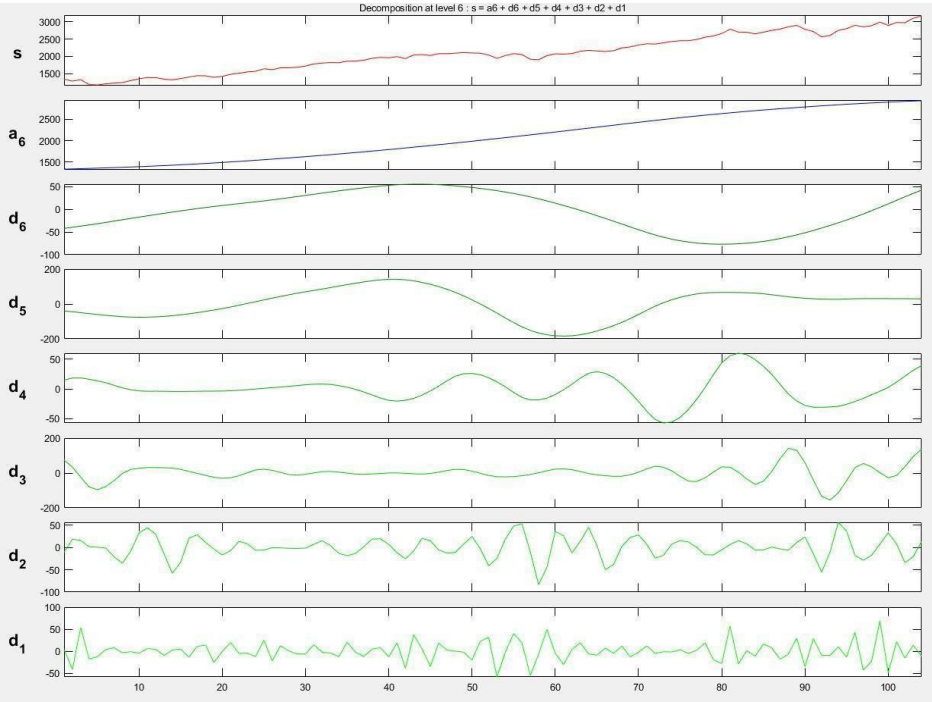


Figure 3.1.17 S&P 10 years monthly data decomposition

As we can see from Figure 3.1.17, the 10-year trend became smoother and with a more obvious positive tendency compared to the 5-year trend. Fluctuations are less abrupt and more negligible, this is also visible in the scalogram, where the fluctuations are highlighted in green and yellow, the most obvious and the longest fluctuations period happened practically at the end of the time span starting at the beginning of 2018.

The decomposition graph also shows a very smooth positive trend. It tends to grow as well as the 5-year trend and even reaches a steadily positive stage at the end of the time span.

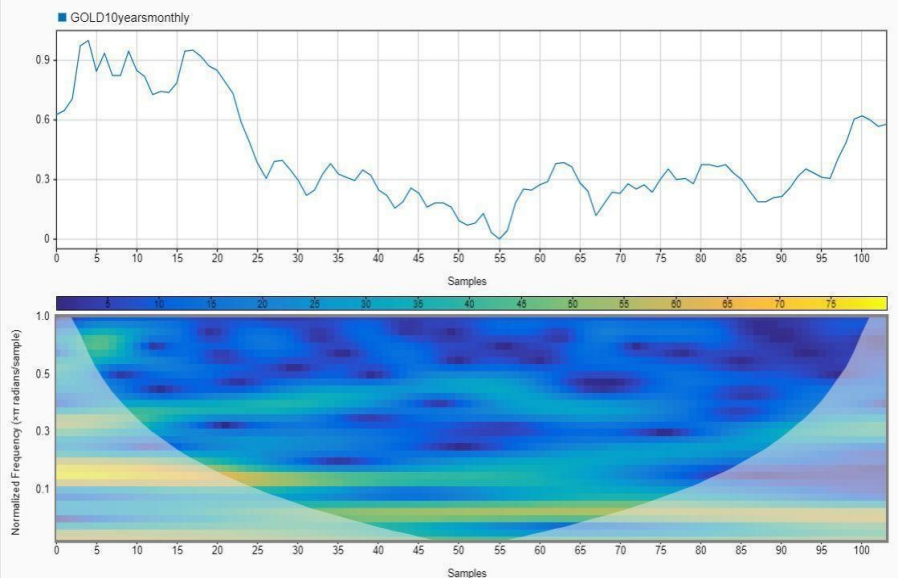


Figure 3.1.18 Gold 10 years monthly data scalogram (normalized)

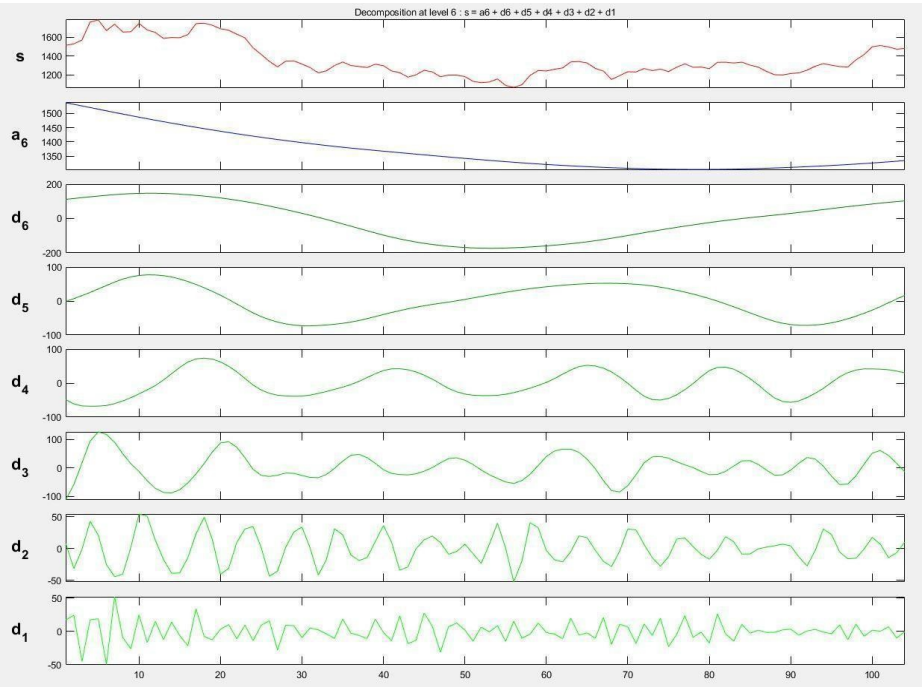


Figure 3.1.19 Gold 10 years monthly data decomposition

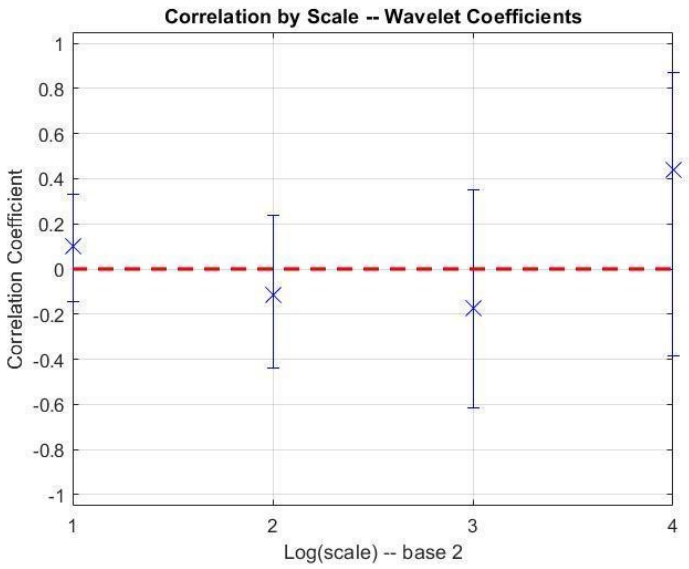


Figure 3.1.20 Wavelet coefficients S&P and gold 10 years monthly frequency of observations

Aggregated data for ten years' time span are also negatively correlated in most part of the time span, but the positive tendency is more obvious at the end of the time span compared to the shorter time spans of two and five years.

The 10-year trend and the scalogram of gold shows a negative trend for a long time even though the current prices still have not reached the level of prices 10 years ago at the beginning of the time span. Fluctuations are also present but look smoother, mostly because of the duration of the time span.

The main trend after applying wavelet decompositions also shows a negative trend on the long run compared to 2 year and 5-year periods, which partially confirms our theory about using gold as a defensive asset only in short-term and mid-term time horizons.

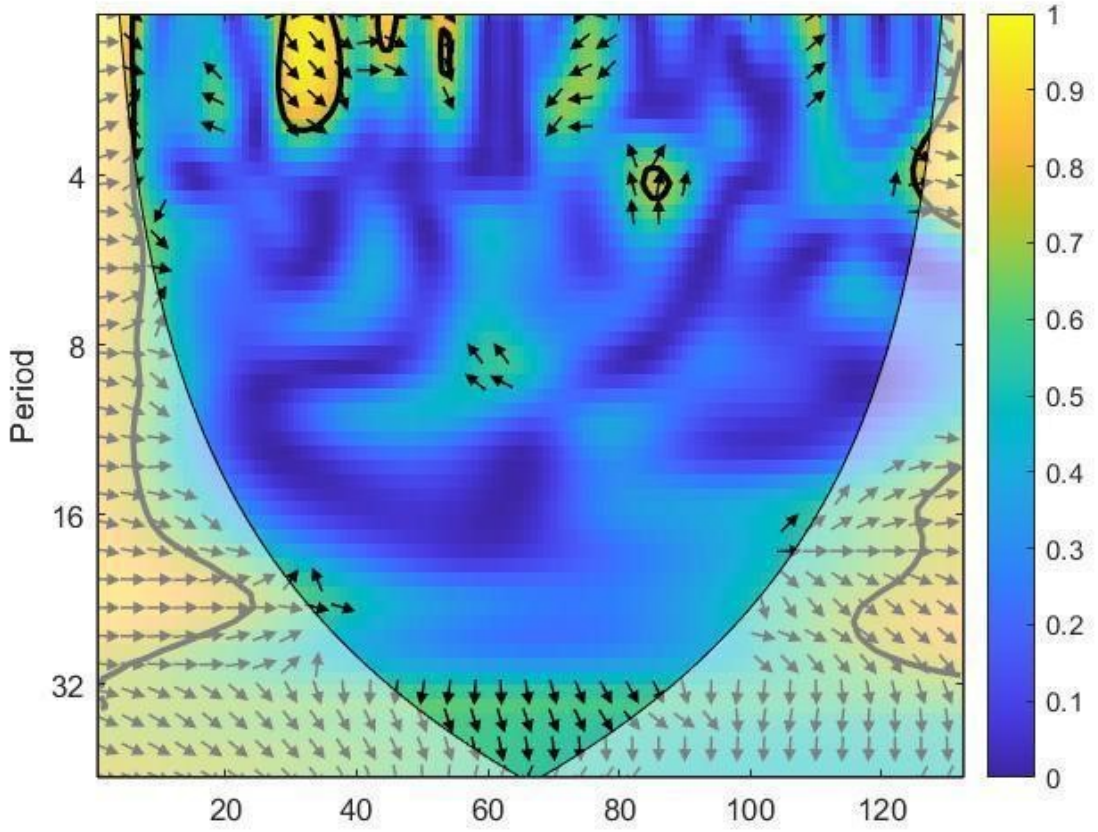
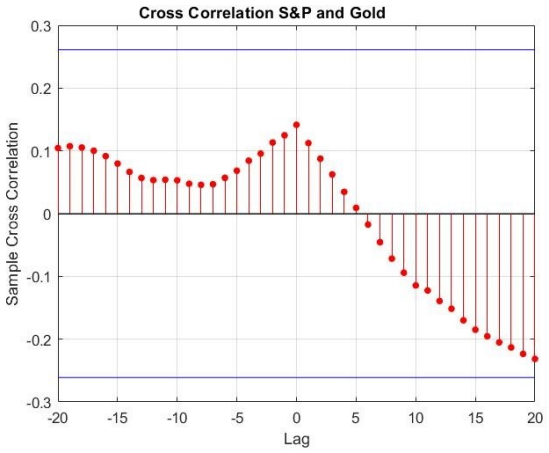
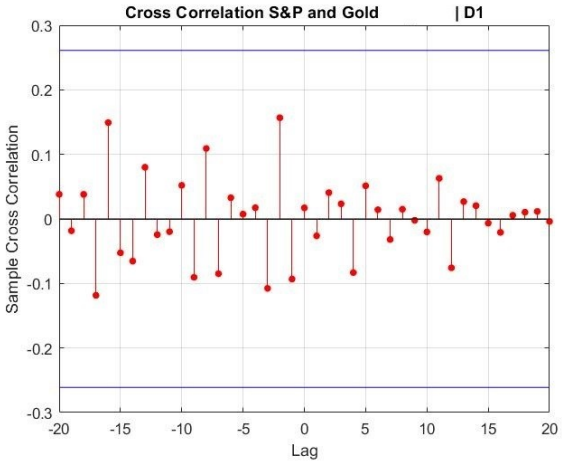


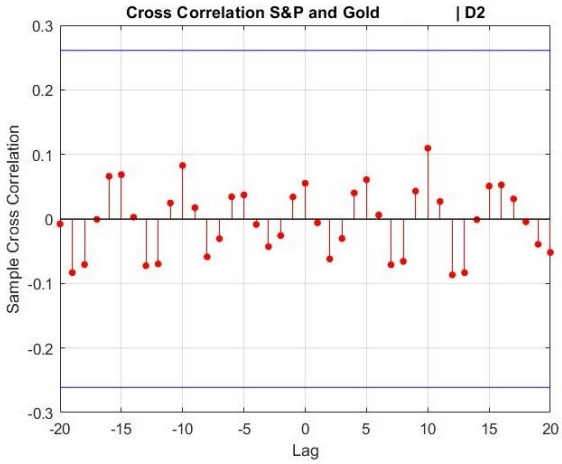
Figure 3.1.21 Wavelet power coherence of S&P and Gold weekly observations for ten years

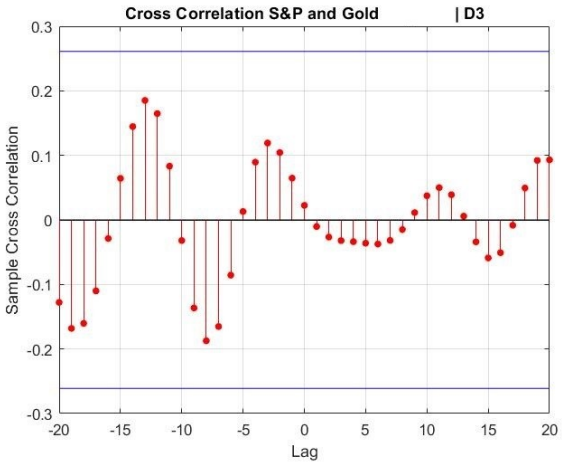
As we can observe from the figure 3.1.21 S&P and gold futures are barely correlated throughout the whole time span of 10 years. This is due to very different movements and trends in both assets: S&P tends to grow during the whole time span when gold futures' prices demonstrate a quite high level of fluctuations. A small spot of positive correlation can be seen only on a 4 period time horizon, but it did not last long and had a lagging relationship.

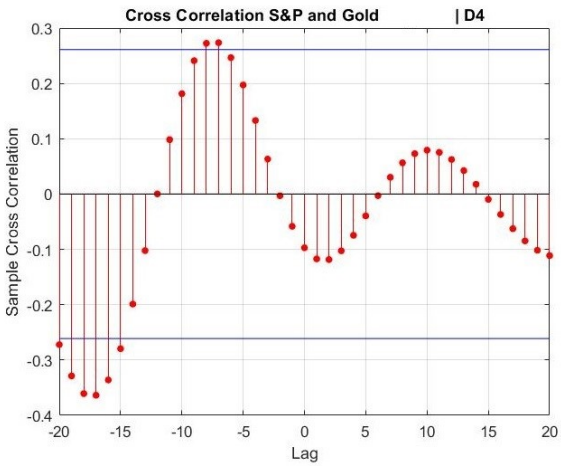
After applying wavelet decomposition and extracting decomposed data, we studied possible correlation between aggregate data and different levels of decomposition using cross-correlation analysis:


Cross Correlation S&P and Gold
Sample Cross Correlation
Lag


Cross Correlation S&P and Gold | D1
Sample Cross Correlation
Lag


Cross Correlation S&P and Gold | D2
Sample Cross Correlation
Lag


Cross Correlation S&P and Gold | D3
Sample Cross Correlation
Lag


Cross Correlation S&P and Gold | D4
Sample Cross Correlation
Lag

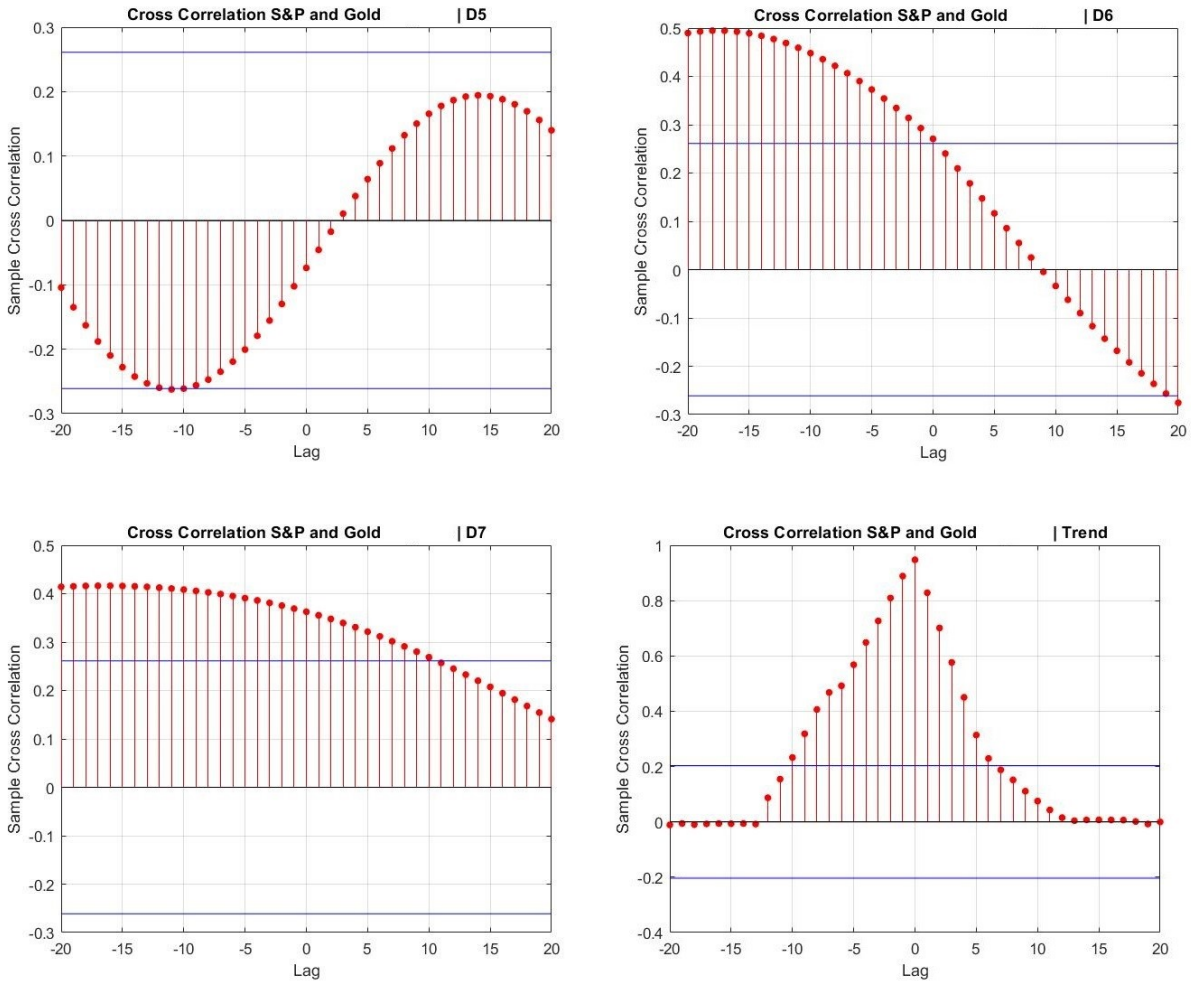


Figure 3.1.22 Cross-correlation analysis of S&P and Gold monthly observations

As we can see from the graph on cross-correlation between the aggregate data, the time series have very weak correlation that practically can be considered as insignificant. As the data sample has monthly frequency, we can make an assumption that the level of gold future prices is not affected by monthly movements in S&P.

The graphs of the decomposed level from D1 to D2 demonstrate the same results, the correlation itself is differently directed but in both cases it does not get close to the confidence intervals and main peaks barely exceed level of 0.1 in both positive and negative directions.

With higher levels of decomposition from D3 to D6 the correlation does not demonstrate any significant changes in both directions and level of the correlation itself, the effect of monthly changes in S&P on gold futures' prices is not strong.

In the last step we can see the correlation at the D7 and of the main trends becomes stronger and more positive. The dynamic of the effect of monthly changes in the main trend is similar to the dynamic of the effect of the weekly changes, even though the period of positive correlation is shorter in general and even more shorter when the level of the correlation exceeds at least 0.4.

In the time span of 10 years the correlation that might be considered significant and exceeds at least 0.4 lasts even less time than in the time span of 5 years. From this point of view, we can conclude that the effect of gold futures' prices from volatility in the stock markets made by weekly movements in S&P is less significant than the effect made by monthly movements.

In other words, having considered more aggregate data the investors' behaviour has the same tendency: they tend to consider gold futures as a defensive asset for mid-term investment time horizon from 4 to 8 periods (months) but that strategy was applied less frequently during the time span of 10 years compared to 5 years.

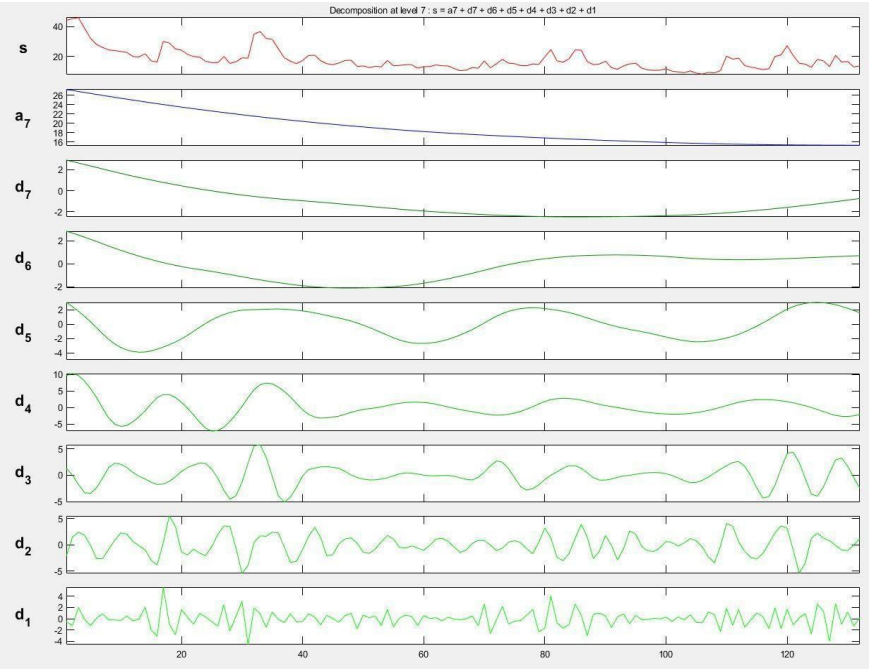


Figure 3.1.23 S&P volatility index VIX 10 years monthly data decomposition

The decomposition analysis in the figure 3.1.23 of VIX demonstrates that the main trend is decreasing, which is especially obvious in comparison with the main trend of the same index in the shorter time span over five years.

Compared to the five year VIX trend, the ten year trend demonstrates a decreasing tendency throughout the whole time span of data. Even though, mostly the trend goes the opposite S&P500 direction but higher level of fluctuations result in occasional positive correlations within the time span. It can be seen from the decompositon levels D3-D4.

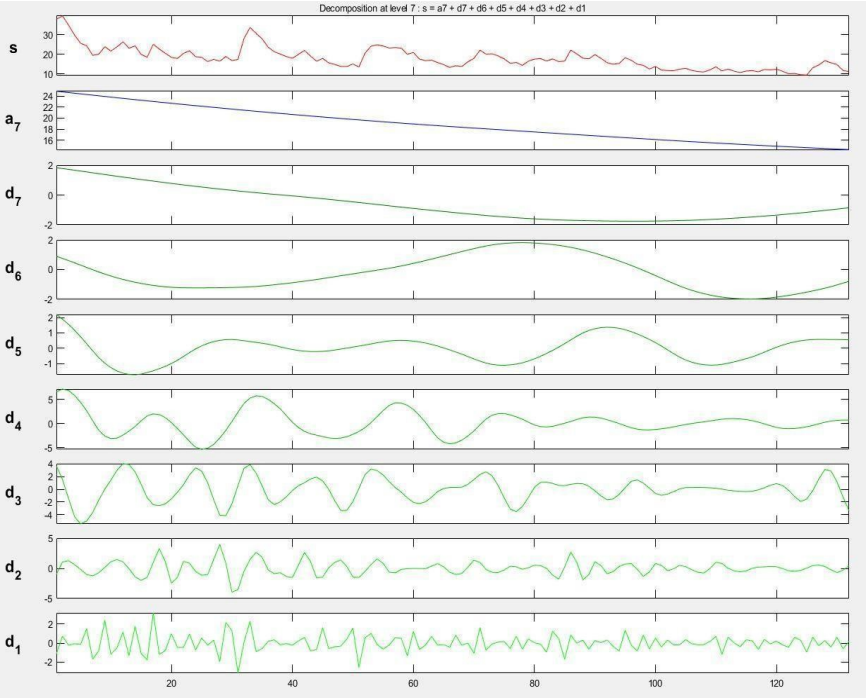


Figure 3.1.24 Gold volatility index GVZ monthly data decomposition

The main trend of GVZ over the longer period shows the same tendency as the five year trend. Besides, in the decomposition graph of the five year date we could spot a small fluctuation towards positive change at the beginning of the considerate period and do not observe the same in the ten years decomposition graph.

GVZ index does not demonstrate the expected tendency over a longer time span. Due to the fact that gold futures is an asset considered for short and mid-term time horizons, the level of fluctuation over long time span is high and the same is applicable to the index GVZ. Both trends have the same tendency, however, the GVZ trend is steeper than the gold futures.

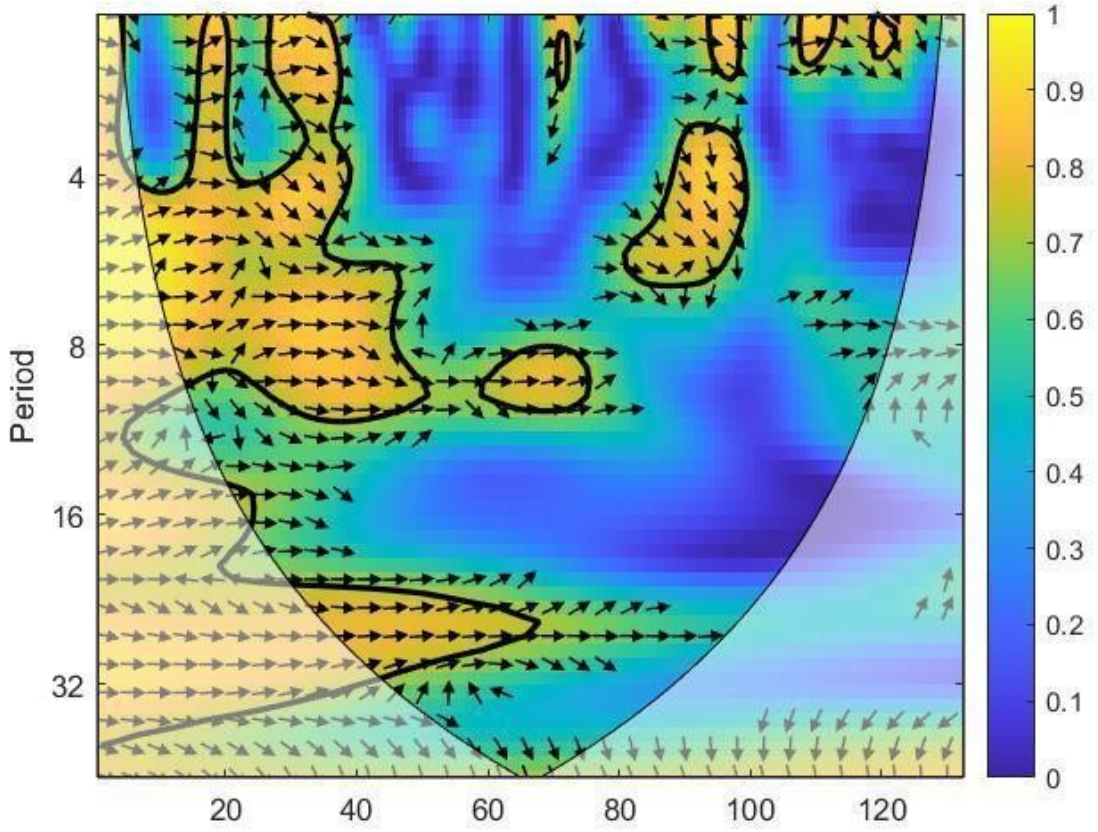
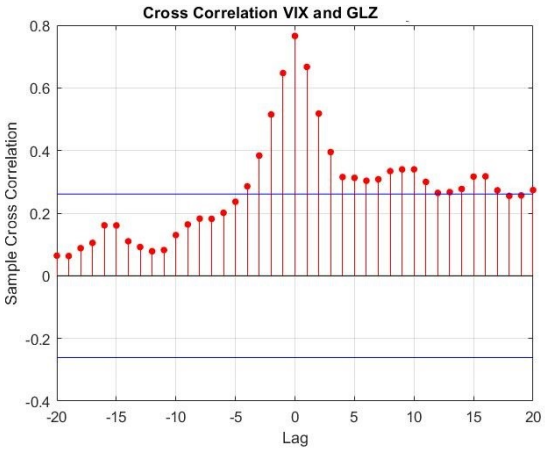
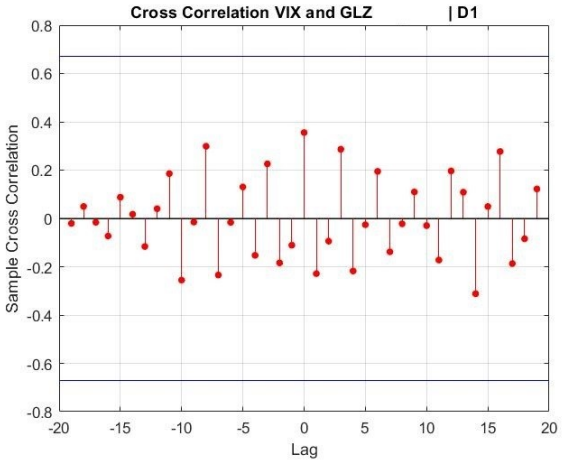


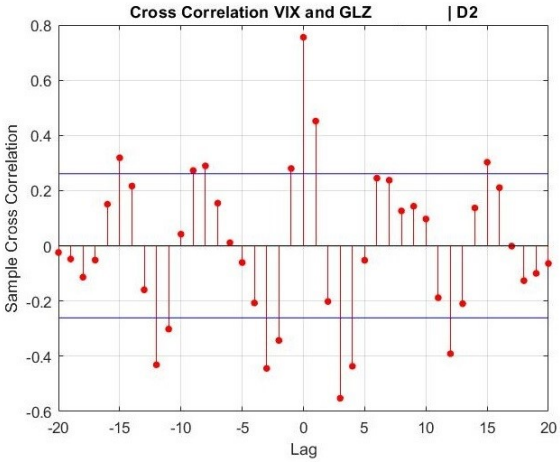
Figure 3.1.25 Wavelet power coherence of VIX and GVZ monthly observations

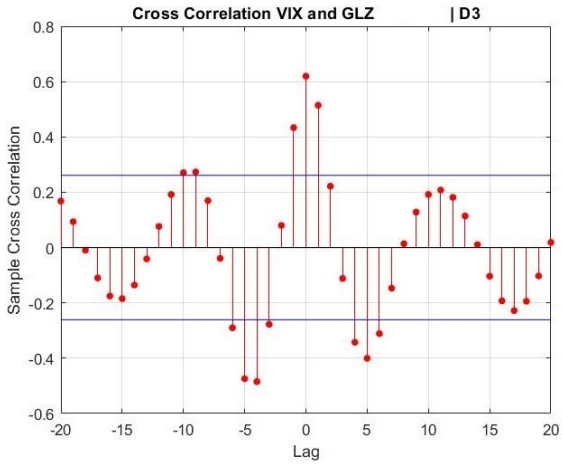
From the figure 3.1.25 we can observe strong positive correlation with mostly leading trends practically in most time horizons as it was with considering the time span of five years. This was caused due to the same reasons: high volatility of any volatility indexes as these kinds of financial instruments are not considered for long-term investment strategies and also might work as indicators of the decline in the main assets.

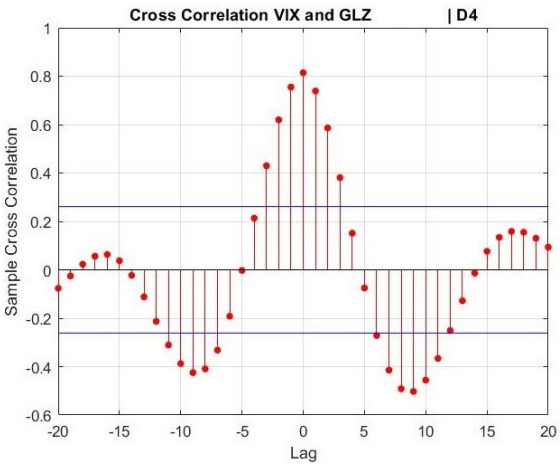
After applying wavelet decomposition and extracting decomposed data, we studied possible correlation between aggregate data and different levels of decomposition using cross-correlation analysis:


Cross Correlation VIX and GLZ
Sample Cross Correlation
Lag


Cross Correlation VIX and GLZ | D1
Sample Cross Correlation
Lag


Cross Correlation VIX and GLZ | D2
Sample Cross Correlation
Lag


Cross Correlation VIX and GLZ | D3
Sample Cross Correlation
Lag


Cross Correlation VIX and GLZ | D4
Sample Cross Correlation
Lag

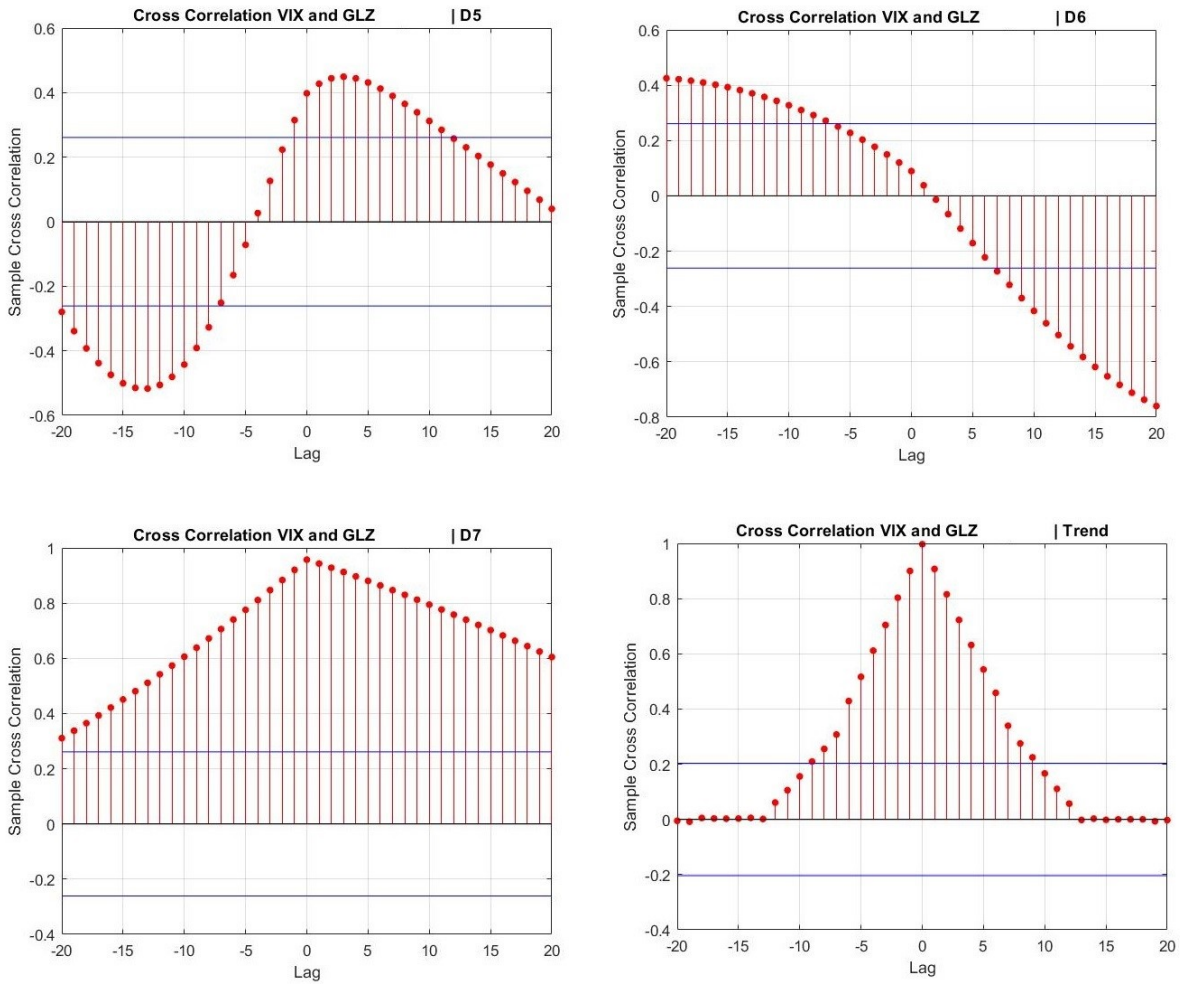


Figure 3.1.26 Cross-correlation analysis of VIX and GVZ monthly observations

Cross correlation between the aggregate data is insignificant and dynamic from D1 to D3, single peaks reach significant level between lags -5 and 5 but it does not influence the main dynamic.

At the higher level of decomposition the dynamic takes more positive tendency exceeding the confidence intervals, but then changes into negative again. Such changing dynamic can be explained by some other factors in the market that affect trading strategies.

Dynamics at the D7 and the main trend are similar as well, VIX and GVZ are positively correlated but to a lesser extent than from weekly movements. The main trends are strongly and positively correlated but for a shorter time period within the time span of ten years compared to the five years' time span.

4. Conclusions

One of the main properties of wavelet decomposition is the ability to decompose the input signal so that the output signal is a function of both time and scale, providing us with simultaneous information from both the time and frequency domains.

The dynamics of the interaction of financial assets can be studied by decomposing the data about these assets into several levels of resolution along the time scale (for example, from analysis of the short-term timescale to the long-term timescale), which can provide useful information for investors with different trading horizons.

Scale decomposition of the original series into different sets of wavelet coefficients associated with different scales allows you to choose the time scale of interest, looking at the contribution of the energy coefficients at a particular scale to the total energy in the series. Moreover, the transformed time series and the highlighting of the main trend makes it possible to predict the further movement of the trend. We can also study market interdependencies from a timeline perspective using wavelet cross-correlation techniques. Analyzing asset interactions at different multiscale resolutions allows agents dealing with different trading horizons to make decisions based on the timeline of interest.

It should be noted that there are several ways to invest in gold: ETF, futures contracts, and gold producer stocks. In the research, we chose futures contracts as the instrument closest to gold in terms of value, since there is an interest rate for fund management included in the ETF, and shares of gold producers cannot accurately reflect the movement of gold prices, since they are influenced by the company's management and reputation.

In the research we focused on studying interdependency between volatility in the stock market, which is expressed by movements of the S&P500 index, and using gold as a defensive asset against volatility.

The main goal was to check the validity of this assumption, to see on what time horizons this strategy might work and for how long investors tend to rebalance their investment portfolios during periods of high volatility. Apart from the main data sets to study, we additionally took into consideration volatility indexes data sets: VIX and GVZ.

We studied different time spans (from two to ten years) with different frequencies of observations (from daily to average monthly movements). Throughout the whole period we detected mostly negative correlation between S&P and gold futures' prices, thus investors tend to buy gold futures when the market trend is in decline and sell when it is on the rise. This assumption is supported by movements in VIX and GVX: VIX tends to rise when S&P is in decline and the opposite; the same is applicable to GVZ.

After studying the shortest time span with two years of observations with the daily frequency, we came to a conclusions we discovered that the correlation between the assets is mostly absent as expected and that gold does not work as a defensive asset against volatility in markets on short time horizons within 64 days, more likely investors tend to consider buying gold in terms of defensive strategy for mid-term time horizons form 64 to 128 days of volatility.

Studying the time span with the weekly frequency of observations, cross-correlation with one period lag between aggregated data and also between each level of wavelet decomposition, led to pronounced correlation between S&P and gold futures in the mid-term time horizon between 8th and 32d periods.

The same analysis with monthly frequency of observations led to the fact that positive correlation can be seen only on a 4 period time horizon. With bigger data sets, the results were broadly similar: investors tend to consider gold futures as a defensive asset for the mid-term investment time horizon from 4 to 8 months. Besides, changes in trading strategies and thus portfolio rebalances were done less than throughout five years.

Correlations between VIX and GVZ are high and positive as both instruments are mostly used for trading strategies and inherently have high levels of volatility, in other words if we compared

any other volatility indexes (e.g., oil prices volatility index and VIX) the result would be broadly similar.

In fact, gold can be used as a defensive asset in periods of high volatility on the market but the strategy is effective for investments only on mid-term time horizons. In calmer times in the market, gold is not an attractive asset for investment because it does not bring passive income - neither interest nor dividends. In this regard, it is better to consider reliable state or commercial bonds for long-term investments.

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