

UNIVERSITÀ POLITECNICA DELLE MARCHE

FACOLTÀ DI ECONOMIA "GIORGIO FUÀ"

Bachelor's Degree Course in Digital Economics and Business

BALANCING GROWTH AND ENVIRONMENTAL DAMAGES: A RAMSEY-BASED APPROACH

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Academic Year 2025/2026

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INTRODUCTION

For centuries, philosophers and economists have studied economic growth in an attempt to understand its characteristics, its causes and, most importantly, why some economies are larger and more prosperous than others. Like many major scientific questions, this remains an active field of study.

At the same time, economic growth has often been accompanied by negative effects and externalities. A particularly significant example is the Industrial Revolution in England, which generated profound economic, political and social transformations. Although it greatly increased production and living standards over time, it also produced serious social costs, environmental degradation and widespread pollution.

Environmental problems had long been observed, but they gained particular academic and political relevance during the 1960s and 1970s. Today, the relationship between economic activity and the environment is a prominent topic across several fields of study.

The objective of this thesis is to provide a simple but meaningful examination of this relationship through the construction and numerical implementation of a mathematical growth model. The model is based on the Ramsey model, a well-established growth framework that has influenced economic theory for decades.

The central question of the thesis is therefore how an exogenous reduction in emissions intensity affects the transition between environmental Ramsey steady states, and how these dynamics differ when the reduction occurs at different speeds.

Chapter 1 provides a brief theoretical introduction to economic growth, presenting some of the most important theories and discussing the relationship between growth and the environment.

Chapter 2 examines the theoretical foundations of the model and presents its mathematical derivation.

Finally, Chapter 3 discusses the numerical implementation of the model in Dynare and analyses the resulting transition paths and economic implications.

CHAPTER 1: DEFINING ECONOMIC GROWTH AND ITS ENVIRONMENTAL ISSUES

1.1 WHAT IS ECONOMIC GROWTH?

Economic growth has been defined in many different ways throughout history, making it difficult to identify a single definition that captures all of its dimensions. Nevertheless, many definitions share certain common elements. In particular, they usually refer to an increase in a country's wealth, productive capacity or supply of goods and services, often accompanied by a broader transformation in the structure of the economy (Kuznets, 1973; Cornwall, 2023). This transformation may be economic, political or social, although in practice it generally involves a combination of all three.

Economic growth is also uneven, both within individual countries and across countries (Cornwall, 2023). Its benefits and costs are not distributed equally among regions, sectors or social groups. In this sense, Kuznets described modern economic growth as a form of controlled revolution, since the structural changes generated by growth may create tensions and conflicts among groups that are affected differently by the transformation of the economy.

The uneven nature of growth can also be understood through the convergence-club hypothesis. According to this hypothesis, economies do not necessarily converge towards the same level of income. Instead, countries with similar structural characteristics, initial conditions and levels of development tend to converge towards one another (Galor, 1996). Empirical support for this hypothesis has been provided through different econometric methods, including the analysis conducted by Bartkowska and Riedl (2012).

Another influential interpretation of economic growth was proposed by Rostow (1959), who divided the development process into a sequence of stages. The first stage is the

traditional society, in which limited technology and low productivity prevent sustained economic growth. A related idea is discussed by Akcigit (2026), who argues that economic growth before the modern era was generally episodic rather than continuous. Periods of progress were frequently interrupted by wars, diseases and the inability of societies to preserve and build upon earlier advances.

The second stage is the preconditions for take-off. During this phase, important structural changes begin to occur, creating the economic, institutional and technological foundations required for sustained growth. According to Rostow, this process first emerged clearly in Britain. Mokyr (1990) provides a complementary interpretation by arguing that societies require both propositional knowledge, which explains why natural and technical processes work, and prescriptive knowledge, which concerns how such knowledge can be applied in practice.

The third stage is the take-off, during which rapid economic growth becomes concentrated in a limited number of leading sectors. Investment rises, production expands and growth becomes increasingly self-sustaining.

The fourth stage is the drive to maturity. At this point, a society has successfully applied modern technology to most of its available resources and productive activities. New leading sectors progressively replace older ones, partly because established technologies and industries face diminishing returns. This process is consistent with the Schumpeterian idea of creative destruction, according to which economic growth is driven by innovations that replace or render obsolete older technologies, firms and production methods (Schumpeter, 1942).

The final stage is the age of high mass consumption. At this stage, society has achieved a sufficiently high level of productivity and income to direct its resources towards different

objectives. Rostow identifies three main possibilities: expanding public welfare and social protection, increasing private consumption, or investing resources in geopolitical and military power.

1.2 GROWTH THEORIES

Economic growth theories are commonly divided into two broad categories: exogenous and endogenous growth theories (Gould and Ruffin, 1993). The distinction concerns whether the fundamental source of long-run growth, particularly technological progress, is determined within the model or treated as an external factor.

1.2.1 The Solow–Swan Model

The Solow–Swan model (Solow, 1956; Swan, 1956) is one of the foundations of modern growth theory. It assumes a neoclassical production function characterised by constant returns to scale and positive but diminishing marginal products of capital and labour.

Expressed in per-worker terms, output is given by $y = f(k)$where y denotes output per worker and k denotes capital per worker. The evolution of capital per worker is described by the central equation:

$$\dot{k} = sf(k) - (n + \delta)k$$

where $sf(k)$ represents actual investment per worker, s is the saving rate, n is the population-growth rate and δ is the depreciation rate. The term $(n + \delta)k$ represents the break-even level of investment required both to replace depreciated capital and to provide capital for new workers.

The economy converges toward a unique steady state where investment exactly equals required investment $sf(k) = (n + \delta)k$. A higher saving rate or a higher level of technology increases steady-state output per worker, whereas faster population growth or a higher depreciation rate reduces it. However, these changes affect the long-run level of

output per worker rather than its permanent growth rate. The model therefore delivers an important implication: capital accumulation alone cannot sustain long-run growth in output per worker. Because capital is subject to diminishing marginal returns, each additional unit of capital produces a smaller increase in output than the previous one. An economy may become richer through capital accumulation, but this process gradually slows as it approaches its steady state.

The Solow model implies that if rates of growth differ among countries, it is only because the countries are at different stages of movement toward the steady state. Rich countries should grow at a slower pace than poor ones; accordingly, over time, the per capita incomes of the rich and poor countries should converge.

Nevertheless, an important limitation of the Solow model is that the ultimate source of sustained per-worker growth, technological progress, is not explained within the model. Instead, it is introduced as an exogenous process that is independent of the decisions made by households and firms (Gould and Ruffin, 1993).

1.2.2 The Romer Model

Endogenous growth theory emerged partly in response to this limitation. Romer (1986) argued that traditional growth theory could not fully reconcile its predictions with several empirical observations, including persistent differences in growth rates across countries and the absence of an automatic tendency towards complete international convergence.

In endogenous growth models, technological progress results from purposeful economic activity rather than arising from an unexplained external process. Firms and individuals intentionally invest in research, education and knowledge creation because they expect to receive an economic return.

Romer (1986) emphasised that knowledge accumulation can generate spillovers. New discoveries do not only benefit their original creators; they may also increase the productivity of other firms and reduce the cost of future innovation. Knowledge therefore creates positive external effects that can offset the diminishing returns associated with physical capital.

Romer (1990) developed this idea further by explaining long-run growth through the deliberate creation of new ideas and new varieties of intermediate products. Ideas are non-rival: the use of an idea by one firm does not prevent another firm from using the same idea. Consequently, knowledge can be used repeatedly without being depleted, allowing its accumulation to avoid the diminishing returns that eventually stop per-worker growth in the basic Solow model.

In a simplified representation, the stock of knowledge evolves according to:

$$\dot{A} = \lambda H_A A$$

where A is the existing stock of ideas, H_A is the amount of human capital employed in research and λ measures the productivity of research activity. The presence of A on the right-hand side means that the existing stock of knowledge makes the creation of additional knowledge easier. Previous discoveries therefore contribute to future innovations, producing an intertemporal knowledge spillover.

Innovators receive temporary or permanent monopoly rights over the intermediate products created through their inventions. These rights allow them to recover the costs of research and create an incentive to innovate. Assuming constant monopoly profits π, a constant interest rate r and an infinitely lived patent, the simplified present value of an innovation can be written as:

$$V^{\text{Romer}} = \frac{\pi}{r}$$

(Akcigit, 2026)[1].

The Romer model therefore makes technological progress endogenous by linking it to intentional research decisions and the expected profits from innovation. Nevertheless, its variety-expansion mechanism does not fully capture another important feature of capitalist development. Innovation is not only cumulative, but also competitive. New technologies do not always coexist with previous technologies; they frequently replace them, causing older products, firms and production processes to disappear.

1.2.3 The Aghion–Howitt Model

Aghion and Howitt (1992) developed a model of endogenous growth based on Schumpeter's concept of creative destruction. Unlike models in which innovation primarily increases the number of available products, the Aghion–Howitt model assumes that innovation improves the quality or productivity of existing technologies and replaces the current technological leader.

Suppose that the current technological frontier has productivity A_t. A successful innovation raises productivity to:

$$A_{t+1} = (1 + \gamma)A_t$$

where $\gamma > 0$ represents the proportional improvement in technology. It follows that:

$$A_{t+1} - A_t = \gamma A_t$$

Each innovation therefore builds upon the existing technological frontier. Earlier discoveries determine the productivity level from which later innovators begin, generating an intertemporal spillover effect.

[1] This formula and the following in this subchapter come from (Akcigit, 2026)

However, technological leadership is only temporary. A successful innovator obtains a monopoly position and earns profits from the use of the new technology. These profits disappear when another firm develops a superior innovation and replaces the incumbent. If τ denotes the arrival rate of future innovations, the simplified value of technological leadership can be expressed as:

$$V^{\text{Aghion--Howitt}} = \frac{\pi}{r + \tau}$$

Unlike in the simplified Romer formulation, the value of an innovation is discounted not only by the interest rate r, but also by the risk τ that the incumbent technology will be displaced. A higher risk of creative destruction reduces the expected duration of monopoly profits and therefore lowers the value of becoming the technological leader.

The model consequently captures two opposing forces. The possibility of earning monopoly profits encourages firms to invest in research, while the arrival of subsequent innovations continuously destroys existing monopoly positions. Economic growth results from this repeated process of technological improvement, market entry and displacement.

1.3 GROWTH AND THE ENVIRONMENT

1.3.1 Environmental issues

Kuznets (1973) acknowledged that economic growth can generate unintended consequences, both positive and negative. As the pace of economic transformation accelerates, the number and complexity of these consequences may also increase. Growth therefore cannot be understood solely as an expansion of output and income; it is a process of structural change that may produce significant social, economic and environmental trade-offs.

Although Kuznets did not focus explicitly on pollution or fossil-fuel use, environmental degradation and climate change have become central concerns in contemporary debates

on economic growth. Climate change is now regarded as one of the most serious threats facing societies and ecosystems (IPCC, 2023). Environmental risks also occupy a prominent position in the World Economic Forum's risk outlook. Extreme weather events are ranked among the most severe global risks over the coming decade, while pollution also appears among the most important long-term threats (World Economic Forum, 2025).

The IPCC (2023) reports that human activity has caused widespread changes in the atmosphere, oceans, cryosphere and biosphere. Climate change is already increasing the frequency or intensity of several extreme events, including heatwaves, heavy precipitation and droughts, producing losses to ecosystems, infrastructure, livelihoods and human health. Without sustained mitigation, these damages are expected to become progressively more severe and may also impede economic growth over the course of the century.

1.3.2 Past and Present developments

One of the most influential early studies of the interaction between economic expansion, resource use and environmental degradation was *The Limits to Growth* (Meadows et al., 1972). Commissioned by the Club of Rome, the study used the World3 computer model to examine twelve possible trajectories for the global economy between 1972 and 2100. The model considered interactions among population, industrial production, food production, technological development, non-renewable resources and pollution.

The most widely discussed trajectory was the business-as-usual scenario. Under this scenario, industrial production and living standards initially continue to rise. However, as non-renewable resources become increasingly scarce, a growing share of industrial capital must be devoted to resource extraction. This leaves fewer resources available for

food production, public services and the replacement of productive capital. The resulting decline in industrial and agricultural capacity eventually contributes to falling living standards and population. Subsequent comparisons have argued that current observed global trends have remained relatively close to the business-as-usual trajectories described by the model (Turner, 2014; Herrington, 2021).

A different interpretation of the relationship between growth and environmental degradation emerged from the Environmental Kuznets Curve hypothesis. Grossman and Krueger (1993) suggested that the relationship between income and environmental quality may change as an economy develops. During the early stages of development, rising production may increase pollution. After a certain income threshold is reached, however, greater public demand for environmental quality, improved technology and stronger institutions may cause pollution to decline. This produces the well-known inverted-U-shaped relationship between income and environmental degradation.

Nevertheless, the Environmental Kuznets Curve is primarily an empirical hypothesis rather than a universal economic law. Evidence of an inverted-U relationship has been found for certain pollutants, but not for all forms of environmental degradation. Moreover, there is nothing inevitable or optimal about the turning point of the curve. As Panayotou (2003) explains, environmental improvements do not arise from higher income alone. They depend on policy responses, technological change, environmental legislation and the development of institutions capable of responding to society's demand for a cleaner environment.

Policy can therefore bring the turning point forward, reduce the height of the curve and limit the environmental damage accumulated during development. This is particularly important because low-income countries may require decades to reach the income level

at which pollution begins to decline. By that stage, the cumulative environmental costs may be considerable. Active environmental policies during the earlier stages of development may consequently be justified on economic as well as ecological grounds (Panayotou, 2003).

Dasgupta (2021) offers a broader criticism of the idea that conventional economic growth is first required to finance environmental protection. This argument may be paradoxical because growth, when based on increasing material and energy use, can itself intensify pressure on the biosphere. Sustainable economic progress should therefore be assessed not solely through GDP, but through changes in inclusive wealth, which includes produced capital, human capital and natural capital. An economy cannot be considered genuinely prosperous if rising output is achieved through the continued depletion of its natural assets.

Recent empirical research suggests that environmental protection and economic performance are not necessarily incompatible. Hysa et al. (2020) find that circular-economy practices associated with environmental sustainability can contribute positively to economic growth. This indicates that improvements in resource efficiency, recycling and sustainable production may support rather than obstruct economic development.

Using data from nine Canadian provinces, Chen et al. (2023) find that renewable electricity generation has a sizeable and positive long-run effect on economic growth. Their results indicate unidirectional causality from renewable energy generation to GDP, with the effect becoming stronger during periods of economic expansion.

Similarly, Acheampong and Opoku (2023) find that environmental degradation generally has a negative effect on economic performance, although the relationship is nonlinear. Emissions may initially accompany and support increases in production, but once certain

thresholds are exceeded, further environmental degradation reduces GDP per capita. These findings suggest that the economic benefits associated with pollution-intensive development may be temporary, while the long-run effects of environmental deterioration can become increasingly harmful.

Alongside this empirical literature, economists have developed theoretical growth models that explicitly incorporate environmental damages. One of the most prominent examples is the Dynamic Integrated Climate–Economy model, commonly known as DICE (Barrage and Nordhaus, 2023).

The DICE model extends the Ramsey growth framework by introducing greenhouse-gas emissions, climate damages and mitigation costs. A social planner maximises discounted, population-weighted utility from consumption:

$$W = \sum_{t=1}^{T_{\max}} U\,[c(t)]L(t)\Pi(t)$$

where $c(t)$ is consumption per person, $L(t)$ is population and $\Pi(t)$ is the discount factor. subject to the net economic output:

$$Q(t) = [1 - \Lambda(t)][1 - \Omega(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}$$

where $\Lambda(t)$ represents the share of output absorbed by emissions-mitigation costs and $\Omega(t)$ represents the share of output lost because of climate damages. The planner therefore faces an intertemporal trade-off. Reducing emissions requires sacrificing part of current output and consumption, but it also limits future climate damages and preserves future productive capacity. The optimal policy balances these present costs against the future economic and environmental benefits of mitigation.

CHAPTER 2: THEORETICAL FRAMEWORK AND MODEL DERIVATION

2.1 ASSUMPTIONS OF THE BENCHMARK ECONOMY

The economy is closed, contains no government sector and produces a single homogeneous good. This good can either be consumed or invested in the accumulation of physical capital. Consequently, aggregate output is allocated according to

$$Y(t) = C(t) + I(t)$$

Physical capital depreciates at the constant rate $\delta > 0$, so its aggregate law of motion is:

$$\dot{K}(t) = Y(t) - C(t) - \delta K(t)$$

The one-sector assumption implies that consumption and investment goods are physically identical: one unit of output that is not consumed becomes one unit of gross investment.

Output is produced using physical capital, labour:

$$Y(t) = F\big(K(t), L(t)\big)$$

Technological progress is excluded in order to keep the model simple. Population growth n is retained initially in the continuous-time Ramsey formulation, since this provides the standard per-worker representation. In the discrete environmental model subsequently implemented in Dynare, population growth is set equal to zero.

The production function is assumed to exhibit constant returns to scale in capital and labour. Capital has a positive but diminishing marginal product:

$$\frac{\partial F}{\partial K} > 0, \qquad \frac{\partial^2 F}{\partial K^2} < 0$$

The standard Inada conditions are also imposed:

$$\lim_{K \to 0} \frac{\partial F}{\partial K} = \infty, \quad \lim_{K \to \infty} \frac{\partial F}{\partial K} = 0$$

and analogously for labour,

$$\lim_{L\to 0}\frac{\partial F}{\partial L}=\infty,\quad \lim_{L\to\infty}\frac{\partial F}{\partial L}=0$$

These assumptions imply that capital is highly productive when it is scarce but becomes progressively less productive as it accumulates.

Let lower-case variables denote quantities per worker:

$$k(t)=\frac{K(t)}{L(t)},\quad c(t)=\frac{C(t)}{L(t)},\quad y(t)=\frac{Y(t)}{L(t)}$$

Constant returns to scale allow output per worker to be written as $y(t)=f\big(k(t)\big)$.

Households are identical and can therefore be represented by a single infinitely lived representative household. Labour is supplied inelastically, while the household chooses its consumption path to maximise lifetime welfare. Utility is increasing and concave in consumption:

$$u'(c)>0,\quad u''(c)<0$$

2.2 THE RAMSEY OPTIMISATION PROBLEM

Before deriving the environmental model, it is useful to briefly introduce the standard Ramsey growth model that provides its theoretical foundation.

The representative household chooses the path of consumption to maximise discounted lifetime utility:

$$U=\int_0^{\infty} u\left[c(t)\right]\cdot e^{nt}\cdot e^{-\rho t}dt$$

where $\rho>0$ is the rate of time preference. The term e^{nt} accounts for the number of households members, while $e^{-\rho t}$ discount future utility.

The optimisation problem is subject to the per-worker resource constraint:

$$\dot{k}=f\big(k(t)\big)-c(t)-(n+\delta)k(t)$$

with $k(0)>0$.

The present-value Hamiltonian is:

$$\mathcal{H}(t) = e^{-(\rho-n)t}u\big(c(t)\big) + \nu(t)\big[f\big(k(t)\big) - c(t) - (n+\delta)k(t)\big]$$

The first-order conditions for a maximum of U are:

$$\frac{\partial \mathcal{H}}{\partial c} = e^{-(\rho-n)t}u'(c) - \nu = 0$$

$$\dot{\nu}(t) = -\frac{\partial \mathcal{H}}{\partial k}$$

Under CRRA utility $u(c) = \frac{c^{1-\theta}}{1-\theta}$ and combining these equations we obtain the Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\big[f'\big(k(t)\big) - \delta - \rho\big]$$

This equation determines the optimal evolution of consumption. Consumption grows when the net return to capital, $f'(k) - \delta$, exceeds the household's rate of time preference. A larger θ reduces the response of consumption growth because households are less willing to substitute consumption across time.

Together with the initial condition $k(0)$ and the transversality condition, the resource constraint and Euler equation determine the optimal paths of $k(t)$ and $c(t)$ towards the steady state.

The first-order conditions must be accompanied by the transversality condition:

$$\lim_{t\to\infty} \nu(t)\, k(t) = 0$$

Using $\nu(t) = e^{-(\rho-n)t}u'\big(c(t)\big)$, this can also be written as:

$$\lim_{t\to\infty} e^{-(\rho-n)t}\, u'\big(c(t)\big)k(t) = 0$$

The condition prevents the household from accumulating capital indefinitely without ever using it to finance consumption. It also rules out infeasible paths based on permanently rolling over debt.

The standard Ramsey model therefore describes optimal consumption and capital accumulation but assigns no welfare cost to pollution. The next section extends the model by adding pollution to the household's objective function and introducing a law of motion for the pollution stock.

2.3 CONTINUOUS-TIME ENVIRONMENTAL RAMSEY MODEL

The environmental extension modifies the Ramsey objective function by introducing disutility from pollution:

$$\max \int_0^{\infty} e^{-(\rho-n)t} \left[u(c) - v(p)\right] dt$$

subject to

$$\dot{k} = f(k) - c - (n+\delta)k$$
$$\dot{p} = \phi f(k) - \eta p$$

Here p denotes the pollution stock, $\phi f(k)$ is emissions created by production, $\phi > 0$ is the emissions intensity of output, and $\eta > 0$ represents the natural absorption or decay rate of pollution.

The present-value Hamiltonian is:

$$\mathcal{H} = e^{-(\rho-n)t}[u(c) - v(p)] + \nu[f(k) - c - (n+\delta)k] + q[\phi f(k) - \eta p]$$

The multiplier ν is the shadow value of capital, and q is the shadow value associated with pollution. Since pollution reduces welfare, q will normally be negative under this sign convention.

The first-order condition with respect to the control variable c is:

$$\frac{\partial \mathcal{H}}{\partial c} = e^{-(\rho-n)t} u'(c) - \nu$$
$$\nu = u'(c) e^{-(\rho-n)t}$$

Derivative of ν with respect to time:

$$\dot{\nu} = u''(c)\dot{c} \cdot e^{-(\rho-n)t} + u'(c)\left[-(\rho-n)e^{-(\rho-n)t}\right]$$
$$\dot{\nu} = e^{-(\rho-n)t}[u''(c)\dot{c} - (\rho-n)u'(c)]$$

dividing by $\nu = u'(c)e^{-(\rho-n)t}$ gives:

$$\frac{\dot{\nu}}{\nu} = \frac{e^{-(\rho-n)t}[u''(c)\dot{c} - (\rho-n)u'(c)]}{u'(c)e^{-(\rho-n)t}}$$
$$\frac{\dot{\nu}}{\nu} = \frac{u''(c)}{u'(c)}\dot{c} - (\rho-n)$$

The capital costate equation is:

$$\dot{\nu} = -\frac{\partial \mathcal{H}}{\partial k}$$
$$\dot{\nu} = -\nu[f'(k) - (n+\delta)] - q\phi f'(k)$$
$$\frac{\dot{\nu}}{\nu} = -[f'(k) - \delta - n] - \frac{q}{\nu}\phi f'(k)$$

since $r = f'^{(k)} - \delta$[2]:

$$\frac{\dot{\nu}}{\nu} = -[r-n] - \frac{q}{\nu}\phi f'(k)$$

Equate the two expressions for $\frac{\dot{\nu}}{\nu}$:

$$\frac{u''(c)}{u'(c)}\dot{c} - (\rho-n) = -(r-n) - \frac{q}{\nu}\phi f'(k)$$
$$\frac{u''(c)}{u'(c)}\dot{c} = (\rho-n) - (r-n) - \frac{q}{\nu}\phi f'(k)$$
$$\frac{u''(c)}{u'(c)}\dot{c} = \rho - r - \frac{q}{\nu}\phi f'(k)$$
$$\frac{u''(c)c}{u'(c)}\frac{\dot{c}}{c} = \rho - r - \frac{q}{\nu}\phi f'(k)$$

Assuming again a CRR utility function, we have $\frac{u''(c)c}{u'(c)} = -\theta$:

[2] In the decentralised Ramsey economy, competitive firms rent capital and labour from households and maximise profits. Perfect competition implies that the net return to capital equals the marginal product of capital net of depreciation $r = f'(k) - \delta$

$$-\theta\frac{\dot{c}}{c} = \rho - r - \frac{q}{v}\phi f'(k)$$
$$\theta\frac{\dot{c}}{c} = r - \rho + \frac{q}{v}\phi f'(k)$$
$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left[r - \rho + \frac{q}{v}\phi f'(k)\right]$$
$$\boxed{\frac{\dot{c}}{c} = \frac{1}{\theta}\left[f'(k) - \delta - \rho + \frac{q}{v}\phi f'(k)\right]}$$

This is the environmental Ramsey Euler equation

The environmental model adds: $\frac{q}{v}\phi f'(k)$.Since pollution is harmful, usually $q < 0$, while normally $v > 0,\ \phi > 0, f'(k) > 0$. Consequently,

$$\frac{q}{v}\phi f'(k) < 0$$

The environmental term therefore reduces the socially desirable return from additional capital accumulation. More capital raises productive capacity, but the associated increase in output also generates additional emissions and future pollution.

The costate equation associated with pollution is:

$$\dot{q} = -\frac{\partial \mathcal{H}}{\partial p}$$
$$\frac{\partial \mathcal{H}}{\partial p} = -e^{-(\rho-n)t}v'(p) - \eta q$$
$$\dot{q} = e^{-(\rho-n)t}v'(p) + \eta q$$

The model is accompanied by two transversality conditions:

$$\lim_{t\to\infty} v(t)\,k(t) = 0$$
$$\lim_{t\to\infty} q(t)\,p(t) = 0$$

The first concerns physical capital, while the second is the corresponding environmental condition.

The continuous-time formulation establishes the economic logic of the model. For the numerical implementation, however, the model is reformulated in discrete time and explicit functional forms are assigned to utility, production and environmental damages.

2.4 DISCRETE-TIME ENVIRONMENTAL MODEL

The discrete-time planner problem is

$$\max_{\{c_t, k_{t+1}, p_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v(p_t)\right]$$

where $0 < \beta < 1$.

The planner chooses the entire sequences of consumption, capital and pollution: $\{c_t, k_{t+1}, p_{t+1}\}_{t=0}^{\infty}$.

the resource constraint is

$$k_{t+1} = f(k_t) + (1-\delta)k_t - c_t$$
$$c_t + k_{t+1} = f(k_t) + (1-\delta)k_t$$

The right-hand side contains all the resources available during period t, $f(k_t) + (1-\delta)k_t$. The term $f(k_t)$ is the output produced during period t. The term $(1-\delta)k_t$ is the part of the current capital stock that survives depreciation. The left-hand side shows how those resources are used $c_t + k_{t+1}$. Some resources are consumed, while the rest become capital for the following period.

Pollution evolves according to

$$p_{t+1} = (1-\eta)p_t + \phi f(k_t)$$

The term $(1-\eta)p_t$ is the amount of the existing pollution stock that remains after natural absorption.

For example, if $\eta = 0.10$, then 10% of pollution is naturally absorbed during one period, and 90% remains

$$(1-\eta)p_t = 0.90p_t$$

The term ϕ is the emissions intensity of output. If $\phi = 0.10$, then each unit of output produces 0.10 units of emission.

At the beginning of period t, the planner inherits k_t and p_t. Capital produces output $f(k_t)$. The planner chooses consumption c_t, and the remaining resources become k_{t+1}. Production also creates emissions, which contribute to the pollution stock p_{t+1}.

2.4.1 Functional forms

CRRA utility function:

$$u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}$$

Its marginal utility is:

$$u'(c_t) = \frac{\partial}{\partial c_t}\left[\frac{c_t^{1-\theta}}{1-\theta}\right]$$
$$u'(c_t) = \frac{1}{1-\theta}(1-\theta)c_t^{-\theta}$$
$$u'(c_t) = c_t^{-\theta}$$

Cobb–Douglas production function:

$$f(k_t) = k_t^{\alpha}$$

The marginal product of capital is:

$$f'(k_t) = \frac{\partial k_t^{\alpha}}{\partial k_t}$$
$$f'(k_t) = \alpha k_t^{\alpha-1}$$

A convenient choice for the Pollution-damage function is the quadratic function:

$$v(p_t) = \frac{\chi}{2}p_t^2$$

where $\chi > 0$ measures how strongly the planner dislikes pollution. The quadratic function means that pollution damages rise more than proportionally as the pollution stock becomes larger.

The marginal damage from pollution is:

$$v'(p_t) = \frac{\partial}{\partial p_t}\left(\frac{\chi}{2}p_t^2\right)$$
$$v'(p_t) = \frac{\chi}{2} \cdot 2p_t$$
$$v'(p_t) = \chi p_t$$

The complete planner problem is now:

$$\max \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\chi}{2}p_t^2\right]$$

subject to:

$$k_{t+1} = k_t^{\alpha} + (1-\delta)k_t - c_t$$
$$p_{t+1} = (1-\eta)p_t + \phi k_t^{\alpha}$$

The initial values k_0 and p_0 are given. Capital and pollution are state variables because their current values are inherited from the past, while consumption is the control variable directly chosen by the planner.

2.4.2 The intertemporal Lagrangian

Let λ_t be the present-value multiplier associated with the capital constraint and let $q_t$be the present-value multiplier associated with the pollution constraint. The intertemporal Lagrangian is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v(p_t)\right] + \sum_{t=0}^{\infty} \lambda_t \left[f(k_t) + (1-\delta)k_t - c_t - k_{t+1}\right]$$

$$+ \sum_{t=0}^{\infty} q_t \left[(1-\eta)p_t + \phi f(k_t) - p_{t+1}\right]$$

The multiplier λ_t is positive. because an additional unit of available resources or capital increases the planner's welfare.

Pollution is harmful, so the multiplier associated with pollution q_t will normally be negative. An additional unit of pollution reduces welfare.

First-order condition with respect to consumption:

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t u'(c_t) - \lambda_t$$
$$\beta^t u'(c_t) - \lambda_t = 0$$
$$\lambda_t = \beta^t u'(c_t)$$

Using CRRA utility, $u'(c_t) = c_t^{-\theta}$:

$$\lambda_t = \beta^t c_t^{-\theta}$$

This means that the shadow value of capital is equal to the discounted marginal utility of consumption. An additional unit of resources is valuable because it could be transformed into an additional unit of consumption.

First-order condition with respect to capital:

Consider k_{t+1}. In period $t+1$, it becomes productive capital and contributes to resources according to

$$\lambda_{t+1}[f(k_{t+1}) + (1-\delta)k_{t+1}]$$

It also generates emissions in period $t+1$ through

$$q_{t+1}\phi f(k_{t+1})$$

Therefore,

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\lambda_t + \lambda_{t+1}[f'(k_{t+1}) + 1 - \delta] + q_{t+1}\phi f'(k_{t+1})$$
$$-\lambda_t + \lambda_{t+1}[1 - \delta + f'(k_{t+1})] + q_{t+1}\phi f'(k_{t+1}) = 0$$
$$\lambda_t = \lambda_{t+1}[1 - \delta + f'(k_{t+1})] + q_{t+1}\phi f'(k_{t+1})$$

The term $\lambda_{t+1}[1 - \delta + f'(k_{t+1})]$ is the ordinary economic value of investing in capital. The environmental contribution is $q_{t+1}\phi f'(k_{t+1})$. Additional capital raises production by $f'(k_{t+1})$ and each additional unit of output creates ϕ units of emissions. Hence the increase in future pollution generated by marginal capital is $\phi f'(k_{t+1})$.

Since pollution is harmful, $q_{t+1} < 0$ we have

$$q_{t+1}\phi f'(k_{t+1}) < 0$$

Environmental damage therefore reduces the marginal social value of accumulating additional capital.

Finally, we calculate the first-order condition with respect to pollution. The variable p_{t+1} directly affects utility in period $t+1$ through

$$-\beta^{t+1}v(p_{t+1})$$

Part of p_{t+1} survives into the next period through

$$q_{t+1}(1-\eta)p_{t+1}$$

Therefore

$$\frac{\partial \mathcal{L}}{\partial p_{t+1}} = -q_t - \beta^{t+1}v'(p_{t+1}) + (1-\eta)q_{t+1}$$
$$-q_t - \beta^{t+1}v'(p_{t+1}) + (1-\eta)q_{t+1} = 0$$
$$q_t = (1-\eta)q_{t+1} - \beta^{t+1}v'(p_{t+1})$$

This is the discrete-time pollution costate equation. The shadow value associated with pollution reflects both the direct environmental damage generated in the next period and the fact that part of the pollution stock survives into later periods.

2.4.3 Environmental Euler equation

For convenience, it is useful to transform the present-value multipliers into current-value multipliers.

The multipliers λ_t and q_t are present-value multipliers because the discount factor β^t appears in the objective function.

Define the current-value shadow value of capital as:

$$\nu_t = \frac{\lambda_t}{\beta^t}$$

We previously found:

$$\lambda_t = \beta^t u'(c_t)$$
$$\nu_t = \frac{\beta^t u'(c_t)}{\beta^t}$$
$$\nu_t = u'(c_t)$$

with CRRA utility:

$$\nu_t = c_t^{-\theta}$$

Similarly, define the current-value shadow value of pollution as:

$$Q_t = \frac{q_t}{\beta^t}$$

Since pollution is harmful, $Q_t < 0$. The present-value capital condition is:

$$\lambda_t = \lambda_{t+1}[1 - \delta + f'(k_{t+1})] + q_{t+1}\phi f'(k_{t+1})$$
$$\frac{\lambda_t}{\beta^t} = \frac{\lambda_{t+1}}{\beta^t}[1 - \delta + f'(k_{t+1})] + \frac{q_{t+1}}{\beta^t}\phi f'(k_{t+1})$$

Notice

$$\lambda_{t+1} = \beta^{t+1}\nu_{t+1}$$
$$\frac{\lambda_{t+1}}{\beta^t} = \beta\nu_{t+1}$$

Similarly,

$$q_{t+1} = \beta^{t+1}Q_{t+1}$$
$$\frac{q_{t+1}}{\beta^t} = \beta Q_{t+1}$$

Therefore

$$\nu_t = \beta\nu_{t+1}[1 - \delta + f'(k_{t+1})] + \beta Q_{t+1}\phi f'(k_{t+1})$$
$$\nu_t = \beta\{\nu_{t+1}[1 - \delta + f'(k_{t+1})] + Q_{t+1}\phi f'(k_{t+1})\}$$

Since $\nu_t = u'(c_t)$, the environmental Euler equation becomes

$$\boxed{u'(c_t) = \beta\{u'(c_{t+1})[1 - \delta + f'(k_{t+1})] + Q_{t+1}\phi f'(k_{t+1})\}}$$

This is the discrete-time environmental Ramsey Euler equation. The ordinary economic return is:

$$1 - \delta + f'(k_{t+1})$$

The environmental adjustment is:

$$Q_{t+1}\phi f'(k_{t+1})$$

Since $Q_{t+1} < 0$, the environmental adjustment reduces the total social return to capital.

The present-value pollution equation is:

$$q_t = (1-\eta)q_{t+1} - \beta^{t+1}v'(p_{t+1})$$
$$\frac{q_t}{\beta^t} = (1-\eta)\frac{q_{t+1}}{\beta^t} - \beta v'(p_{t+1})$$

Using

$$Q_t = \frac{q_t}{\beta^t}, \quad \frac{q_{t+1}}{\beta^t} = \beta Q_{t+1}$$

we obtain:

$$Q_t = \beta(1-\eta)Q_{t+1} - \beta v'(p_{t+1})$$

Now define the positive current-value shadow cost of pollution as $Q_t = -m_t$and $Q_{t+1} = -m_{t+1}$. This gives

$$-m_t = -\beta(1-\eta)m_{t+1} - \beta v'(p_{t+1})$$
$$m_t = \beta[v'(p_{t+1}) + (1-\eta)m_{t+1}]$$

with quadratic damages $v'(p_{t+1}) = \chi p_{t+1}$. So

$$\boxed{m_t = \beta[\chi p_{t+1} + (1-\eta)m_{t+1}]}$$

The current shadow cost of pollution is equal to the discounted sum of the direct damage caused in the next period and the shadow cost of the pollution that remains after natural absorption.

Using $m_t = -Q_t$, the environmental Euler equation can also be written as

$$c_t^{-\theta} = \beta\left[c_{t+1}^{-\theta}\left(1-\delta+f'(k_{t+1})\right) - m_{t+1}\phi f'(k_{t+1})\right]$$

For Cobb-Douglas production, $f'(k_{t+1}) = \alpha k_{t+1}^{\alpha-1}$, so

$$c_t^{-\theta} = \beta\left[c_{t+1}^{-\theta}(1-\delta+\alpha k_{t+1}^{\alpha-1}) - m_{t+1}\phi\alpha k_{t+1}^{\alpha-1}\right]$$

2.4.4 Discrete transversality conditions

The discrete capital transversality condition is:

$$\lim_{T\to\infty} \lambda_T\, k_{T+1} = 0$$

Using $\lambda_T = \beta^T u'(c_T)$, this can also be written as:

$$\lim_{T\to\infty} \beta^T u'(c_T) k_{T+1} = 0$$

For pollution, the corresponding transversality condition is:

$$\lim_{T\to\infty} q_T \, p_{T+1} = 0$$

Using the current-value multiplier $Q_T = \frac{q_T}{\beta^T}$, this becomes:

$$\lim_{T\to\infty} \beta^T \, Q_T p_{T+1} = 0$$

We do not normally type these infinite-horizon limits directly into the Dynare model. In a finite perfect-foresight simulation, the infinite horizon is approximated by choosing a sufficiently long simulation horizon and terminal conditions consistent with convergence towards the steady state.

2.4.5 The steady state

At the steady state, all variables are constant:

$$\begin{gathered} k_{t+1} = k_t = k^* \\ c_{t+1} = c_t = c^* \\ p_{t+1} = p_t = p^* \\ m_{t+1} = m_t = m^* \end{gathered}$$

Steady-state resource constraint

Begin from

$$c_t + k_{t+1} = f(k_t) + (1-\delta)k_t$$

At the steady state,

$$\begin{gathered} c^* + k^* = f(k^*) + (1-\delta)k^* \\ c^* = f(k^*) + (1-\delta)k^* - k^* \\ c^* = f(k^*) - \delta k^* \end{gathered}$$

With Cobb-Douglas production,

$$c^* = (k^*)^\alpha - \delta k^*$$

Steady-state output

$$\begin{gathered} y_t = k_t^\alpha \\ y^* = (k^*)^\alpha \end{gathered}$$

Steady-state investment

Gross investment is defined as

$$i_t = k_{t+1} - (1-\delta)k_t$$
$$i^* = k^* - (1-\delta)k^*$$
$$i^* = k^* - k^* + \delta k^*$$
$$i^* = \delta k^*$$

Since there is no population or technological growth in the model, steady-state investment is exactly the amount required to replace depreciated capital.

Steady-state emissions

$$e_t = \phi y_t$$
$$e^* = \phi y^* = \phi (k^*)^{\alpha}$$

Steady-state pollution

Begin from

$$p_{t+1} = (1-\eta)p_t + \phi f(k_t)$$
$$p^* = (1-\eta)p^* + \phi f(k^*)$$
$$p^* - (1-\eta)p^* = \phi f(k^*)$$
$$\eta p^* = \phi f(k^*)$$
$$p^* = \frac{\phi f(k^*)}{\eta}$$

With Cobb–Douglas production,

$$p^* = \frac{\phi (k^*)^{\alpha}}{\eta}$$

Thus, for a given k^*, steady-state pollution increases with emissions intensity ϕ and decreases with the natural absorption rate η.

Steady-state environmental shadow cost

The pollution shadow-cost equation is

$$m_t = \beta[v'(p_{t+1}) + (1-\eta)m_{t+1}]$$
$$m^* = \beta[v'(p^*) + (1-\eta)m^*]$$
$$m^* = \beta v'(p^*) + \beta(1-\eta)m^*$$
$$m^* - \beta(1-\eta)m^* = \beta v'(p^*)$$
$$m^*[1-\beta(1-\eta)] = \beta v'(p^*)$$
$$m^* = \frac{\beta v'(p^*)}{1-\beta(1-\eta)}$$

With quadratic damages $v'^{(p^*)} = \chi p^*$

$$m^* = \frac{\beta\chi p^*}{1-\beta(1-\eta)}$$

If pollution is highly persistent, then η is small and $1-\eta$ is large. This makes the denominator smaller, increasing the shadow cost m^*. This occurs because pollution that remains in the environment for many periods generates a longer stream of future welfare losses

Steady-state Euler equation

The environmental Euler equation is

$$c_t^{-\theta} = \beta\left[c_{t+1}^{-\theta}(1-\delta+\alpha k_{t+1}^{\alpha-1}) - m_{t+1}\phi\alpha k_{t+1}^{\alpha-1}\right]$$
$$(c^*)^{-\theta} = \beta\left[(c^*)^{-\theta}(1-\delta+\alpha(k^*)^{\alpha-1}) - m^*\phi\alpha(k^*)^{\alpha-1}\right]$$
$$1 = \beta\left[1-\delta+\alpha(k^*)^{\alpha-1} - \frac{m^*}{(c^*)^{-\theta}}\phi\alpha(k^*)^{\alpha-1}\right]$$
$$1 = \beta\left[1-\delta+\alpha(k^*)^{\alpha-1} - m^*(c^*)^{\theta}\phi\alpha(k^*)^{\alpha-1}\right]$$
$$\frac{1}{\beta} = 1-\delta+\alpha(k^*)^{\alpha-1}\left[1-m^*(c^*)^{\theta}\phi\right]$$

This is the environmental steady-state condition for capital. Substituting the pollution equation $m^* = \frac{\beta\chi p^*}{1-\beta(1-\eta)}$ into the shadow-cost equation gives

$$m^* = \frac{\beta\chi\left[\frac{\phi(k^*)^{\alpha}}{\eta}\right]}{1-\beta(1-\eta)}$$

Substituting, these expressions into the steady-state Euler equation yields the steady-state capital

$$0 = \beta \left[1 - \delta + \alpha k^{\alpha-1} - \frac{\beta\chi\left(\frac{\phi k^{\alpha}}{\eta}\right)}{1-\beta(1-\eta)} (k^{\alpha} - \delta k)^{\theta} \phi \alpha k^{\alpha-1} \right] - 1$$

There is generally no simple closed-form formula. We need to calculate it numerically. We define the steady-state function

$$F(k) = \beta \left[1 - \delta + \alpha k^{\alpha-1} - \frac{\beta\chi\left(\frac{\phi k^{\alpha}}{\eta}\right)}{1-\beta(1-\eta)} (k^{\alpha} - \delta k)^{\theta} \phi \alpha k^{\alpha-1} \right] - 1$$

Then we calculate the positive root satisfying $F(k^*) = 0$. This is a numerical calculation with the illustrative parameters

2.4.6 An illustrative calibration

The model is intended as a theoretical and numerical illustration of how introducing pollution into a Ramsey framework affects the steady state and transitional dynamics. It is not calibrated to reproduce a particular economy, pollutant or atmospheric concentration. The environmental parameters are therefore illustrative and are selected to generate economically meaningful and numerically stable dynamics. All quantities should consequently be interpreted in normalised model units, rather than as literal tonnes of emissions, euros of output or atmospheric concentrations.

The economic parameters are

$$\alpha = 0.30$$
$$\theta = 1.50$$
$$\delta = 0.10$$
$$\beta = 0.95$$

The initial environmental parameters are

$$\phi = 0.10$$
$$\eta = 0.10$$
$$\chi = 0.05$$

The parameter $\phi = 0.10$ means that emissions are proportional to output and equal to 10% of output in the model's normalised units.

The parameter $\eta = 0.10$ means that 10% of the existing pollution stock is naturally absorbed during each period.

Finally, $\chi = 0.05$ determines the strength of the welfare loss generated by pollution through the quadratic damage function.

CHAPTER 3: DYNARE CODE IMPLEMENTATION AND RESULTS

3.1 MATHEMATICAL FRAMEWORK OF THE EXPERIMENT

As the IPCC (2023) emphasises, deep, rapid and sustained mitigation, together with accelerated adaptation, can reduce projected losses and damages to humans and ecosystems while also generating important co-benefits, particularly for air quality and health. Motivated by this, the experiment considers an economy that begins in a steady state characterised by relatively high emissions and gradually moves towards a different steady state with lower emissions. The analysis then compares a faster transition with a slower one.

The constant parameter ϕ is replaced by an exogenous sequence $\{\phi_t\}_{t=0}^{\infty}$. Emission therefore becomes

$$e_t = \phi_t f(k_t)$$
$$e_t = \phi_t k_t^{\alpha}$$

Pollution accumulation becomes

$$p_{t+1} = (1-\eta)p_t + \phi_t k_t^{\alpha}$$

The transition is therefore generated by a decline in the amount of emissions produced per unit of output.

Economically, ϕ_t can represent improvements in clean technology, stricter environmental regulation, a shift towards renewable energy, cleaner production processes, or a combination of these factors. The IPCC(2023), for example, associates net-zero energy systems with substantial reductions in fossil-fuel use, greater electrification, cleaner electricity systems, alternative energy carriers, improvements in energy efficiency and greater integration across the energy system.

Let ω_t denote the exogenous change in emissions intensity. Its evolution can be written as:

$$\phi_t = \phi_{t-1} + \omega_t$$

The sequence ω_t is determined outside the optimisation problem and is therefore taken as given by the planner.

During the green transition, $\omega_t < 0$, so emission intensity declines. Once the target has been reached, $w_t = 0$, ϕ_t remains permanently at its new lower level.

Let the initial and final intensities be:

$$\phi_H \text{ and } \phi_L$$

where $\phi_L < \phi_H$. The cumulative shocks must satisfy

$$\sum_{t=1}^{T} \omega_t = \phi_L - \phi_H$$

If the policy target were stated directly as a proportional reduction r_ϕ in emissions intensity, then

$$\phi_L = (1 - r_\phi)\phi_H$$

and therefore

$$\sum_{t=1}^{T} \omega_t = -r_\phi \phi_H$$

To provide an empirical benchmark, the final reduction in model emissions is based on the EU 2040 climate target of a 90% reduction in net greenhouse-gas emissions relative to 1990 levels (European Commission, 2026; UNEP, 2025). Emissions in the model are not measured in gigatonnes of CO_2-equivalent but in abstract model units. The EU target is therefore used only to calibrate the magnitude of the reduction.

For intuition, initial steady-state emissions can be normalised to

$$e_H = 100$$

so that a 90% reduction implies $e_L = 10$. Thus

$$e_L = 0.10 e_H$$

The initial emissions intensity remains $\phi_H = 0.10$. However, the final emissions intensity ϕ_L cannot be obtained simply by reducing ϕ_H by 90%, because capital is allowed to change between the initial and final steady states.

Since $e = \phi k^{\alpha}$, the 90% total-emissions target requires

$$\phi_L (k_L^*)^{\alpha} = 0.10 \phi H (k_H^*)^{\alpha}$$

Because k_L^* itself depends on ϕ_L, the final emission intensity must be found numerically. The procedure is to first calculate the initial steady state of k using $\phi_H = 0.10$, and then search for the value of ϕ_L whose associated final steady state produces total emissions equal to 10% of initial emissions.

Once ϕ_L has been found, the same final intensity is used in both the slow and fast transition scenarios. Only the speed of adjustment differs.

In the slow transition we suppose the reduction is spread evenly over 50 periods. Then

$$\omega_t^S = \frac{\phi_L - \phi_H}{50} = \frac{\phi_L - 0.10}{50} \quad t = 1, \dots, 50$$

Since $\phi_L < \phi_H$, then $\omega_t^S < 0$.The emissions intensity follows

$$\phi_t^S = 0.10 + t\omega_t^S, \quad t = 0, \dots, 50$$
$$\phi_t^S = \phi_L, \quad t \geq 50$$

In the fast transition the same reduction is completed in 30, periods

$$\omega_t^S = \frac{\phi_L - \phi_H}{30} = \frac{\phi_L - 0.10}{30} \quad t = 1, \dots, 30$$

Thus

$$\phi_t^F = 0.10 + t\omega_t^F, \qquad t = 0, \dots, 30$$
$$\phi_t^F = \phi_L, \qquad t \geq 30$$

Therefore, both transitions converge to the same low-emissions steady state and satisfy the same final total-emissions target. The only difference is the speed of adjustment.

A compact general representation is

$$\phi_t(T) = \phi_H - \min\left\{\frac{t}{T}, 1\right\}(\phi_H - \phi_L)$$

where t represents current period and T represents total duration of the slow or fast transition.

The linear path is an assumed transition scenario rather than an implication of the Ramsey model.

3.1.1 The modified analytical system

The planner's problem becomes

$$\max_{\{c_t, k_{t+1}, p_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v(p_t)\right]$$

subject to

$$c_t + k_{t+1} = f(k_t) + (1-\delta)k_t$$
$$p_{t+1} = (1-\eta)p_t + \phi_t f(k_t)$$

together with the exogenous transition process

$$\phi_t = \phi_{t-1} + \omega_t$$

The emissions identity is

$$e_t = \phi_t k_t^{\alpha}$$

Once emissions intensity varies over time, the relevant intensity is the one applying when k_{t+1} is used for production, namely ϕ_{t+1}. The Euler equation therefore becomes

$$c_t^{-\theta} = \beta\left[c_{t+1}^{-\theta}(1 - \delta + \alpha k_{t+1}^{\alpha-1}) - m_{t+1}\phi_{t+1}\alpha k_{t+1}^{\alpha-1}\right]$$

This modification is economically important because the anticipated future path of clean technology or environmental policy now affects current consumption and investment decisions.

As ϕ_{t+1} declines, an additional unit of capital generates fewer emissions. Consequently, the environmental deduction from the return to capital becomes smaller.

The pollution shadow-cost equation does not need a new term:

$$m_t = \beta[\chi p_{t+1} + (1-\eta)m_{t+1}]$$

Although the equation itself is unchanged, the path of m_t changes because the path of pollution now depends on the time-varying emissions intensity, $p_{t+1} = (1-\eta)p_t + \phi_t k_t^{\alpha}$.

A faster decline in ϕ_t reduces emissions more rapidly and, consequently, leads to a faster decline in pollution and its shadow cost.

Because pollution is a stock, it will generally respond more slowly than emissions. Emissions can decline immediately when ϕ_t declines, but pollution remains in the environment through $(1-\eta)p_t$.

3.1.2 Two different steady states

The economy begins at an old high-emissions steady state and converges towards a new low-emissions steady state

Initial high-emissions steady state

Before the transition $\phi_t = \phi_H$. The initial steady state satisfies

$$k_{t+1} = k_t = k_H^*$$
$$c_{t+1} = c_t = c_H^*$$
$$p_{t+1} = p_t = p_H^*$$
$$m_{t+1} = m_t = m_H^*$$

The resource constraint gives

$$c_H^* = (k_H^*)^{\alpha} - \delta k_H^*$$

Initial steady-state output is

$$y_H^* = (k_H^*)^\alpha$$

Initial emissions are

$$e_H^* = \phi_H (k_H^*)^\alpha$$

Initial pollution is

$$\begin{gathered} p_H^* = (1-\eta)p_H^* + \phi_H (k_H^*)^\alpha \\ \eta p_H^* = \phi_H (k_H^*)^\alpha \\ p_H^* = \frac{\phi_H (k_H^*)^\alpha}{\eta} \end{gathered}$$

The initial pollution shadow cost is

$$\begin{gathered} m_H^* = \beta[\chi p_H^* + (1-\eta)m_H^*] \\ m_H^* - \beta(1-\eta)m_H^* = \beta\chi p_H^* \\ m_H^* = \frac{\beta\chi p_H^*}{1-\beta(1-\eta)} \end{gathered}$$

Finally, initial steady-state capital satisfies

$$1 = \beta\left[1 - \delta + \alpha(k_H^*)^{\alpha-1} - m_H^*(c_H^*)^\theta \phi_H \alpha (k_H^*)^{\alpha-1}\right]$$

Final low-emissions steady state

After the transition has been completed, $\phi_t = \phi_L$. The final steady state satisfies

$$\begin{gathered} c_L^* = (k_L^*)^\alpha - \delta k_L^* \\ e_L^* = \phi_L (k_L^*)^\alpha \\ p_L^* = \frac{\phi_L (k_L^*)^\alpha}{\eta} \\ m_L^* = \frac{\beta\chi p_L^*}{1-\beta(1-\eta)} \\ 1 = \beta\Big[1 - \delta + \alpha(k_L^*)^{\alpha-1} - m_L^*(c_L^*)^\theta \phi_L \alpha(k_L^{)\,\alpha-1} \backslash right] \end{gathered}$$

The final steady state must also satisfy the emissions-reduction target

$$\begin{gathered} e_L^* = 0.10\, e_H^* \\ \phi_L (k_L^*)^\alpha = 0.10\, \phi_H (k_H^*)^\alpha \end{gathered}$$

The steady-state calculation therefore has two stages. First, initial steady-state capital k_H^* is calculated using the known initial intensity $\phi_H = 0.10$, which gives initial emissions $e_H^* = \phi_H (k_H^*)^\alpha$. Second, ϕ_L is determined numerically. For each candidate value of ϕ_L, the associated final steady-state capital k_L^* and final emissions $e_L^* = \phi_L (k_H^*)^\alpha$ are calculated.

The value of ϕ_L is chosen so that $e_L^* = 0.10 e_H^*$. The resulting ϕ_L and k_L^* define the terminal low-emissions steady state. This calibration is carried out before solving the fast and slow perfect-foresight transition paths.

The transition can therefore be represented as:

$$(k_H^*, p_H^*, \phi_H) \longrightarrow (k_L^*, p_L^*, \phi_L)$$

The economy initially lies in the old steady state:

$$k_0 = k_H^*, \quad p_0 = p_H^*$$

At the beginning of the green transition, capital and pollution are predetermined state variables. They cannot jump immediately. Consumption and the pollution shadow cost, by contrast, are forward-looking variables and may respond immediately once the future path of ϕ_t becomes known.

3.2 DYNARE CODE

The following section presents the main components of the Dynare implementation.

```
// Number of years over which emissions intensity declines.
//
// FAST TRANSITION:
//     T_TRANS = 30
//
// SLOW TRANSITION:
//     change this value to 50.
//
// One model period = one year.
@#define T_TRANS = 30


// Total perfect-foresight simulation horizon
@#define T_SIM = 100
```

Figure 1. Duration of the experiment

One model period is interpreted as one year. Therefore, `T_TRANS = 30` represents the fast transition, while the slow experiment is obtained by setting `T_TRANS = 50`.

The simulation horizon is 100 years in both cases. This is deliberately longer than either transition because reaching the final value of ϕ_t does not imply that all endogenous variables have already reached their new steady-state values.

```
////////////////////////////////////////////////////////////
// Variables
////////////////////////////////////////////////////////////

// Endogenous variables
var c k p m y i em;


// Exogenous time-varying emissions intensity
//
// phit represents phi_t
varexo phit;


// Capital and pollution are predetermined states
predetermined_variables k p;
```

Figure 2. Exogenous and endogenous variables

As it is already known, the endogenous variables are

$$c_t, \quad k_t, \quad p_t, \quad m_t, \quad y_t, \quad i_t, \quad e_t$$

These variables are determined within the model. By contrast, `varexo phit` declares the emissions-intensity sequence ϕ_t as exogenous. The planner does not choose ϕ_t. Its entire path is imposed from outside the optimisation problem.

The declaration `predetermined_variables k p` tells Dynare that capital and pollution are state variables. Their values at the beginning of a period are inherited from the past.

```
//////////////////////////////////////////////////////////////
// Economic calibration
//////////////////////////////////////////////////////////////

alph = 0.30;
sig  = 1.50;
delt = 0.10;
bet  = 0.95;


//////////////////////////////////////////////////////////////
// Environmental calibration
//////////////////////////////////////////////////////////////

eta = 0.10;
chi = 0.05;
```

```
// Initial high-emissions intensity
phi_h = 0.10;


// Temporary initialization.
//
// This value is overwritten below after phi_l is found
// numerically from the 90% TOTAL-emissions reduction target.
phi_l = phi_h;
```

Figure 3 and 4. Calibration

The calibration uses the parameter values introduced in Chapter 2. The line `phi_l = phi_h` is only a temporary initialisation required before the numerical calibration. It does not imply that the final emissions intensity is assumed to equal the initial intensity. The code subsequently replaces this temporary value with the numerically determined ϕ_L. Once the model has been specified, the code calculates the initial high-emissions steady state.

```
kss_h = green_kss(alph,sig,delt,bet,phi_h,eta,chi);

css_h = kss_h^alph - delt*kss_h;

yss_h = kss_h^alph;

iss_h = delt*kss_h;

emss_h = phi_h*yss_h;

pss_h = emss_h/eta;

mss_h = bet*chi*pss_h/(1-bet*(1-eta));
```

Figure 5. Initial high-emissions steady state equations

The MATLAB function `green_kss` numerically solves the environmental steady-state Euler equation conditional on $\phi = \phi_H$. This produces k_H^* which is then used to calculate the remaining initial steady-state variables.

On the other hand the purpose of the following block is to find the value of the final emissions intensity ϕ_L that makes final steady-state emissions exactly 90% lower than initial emissions.

```
target_emissions_ratio = 0.10;

target_emissions = target_emissions_ratio*emss_h;
```

```
phi_target_gap = @(phi_candidate) phi_candidate*(green_kss(alph,sig,delt,bet,phi_candidate,eta,chi,false))^alph-target_emissions;


// Search interval for phi_l
phi_lower_bound = 1e-8;


phi_upper_bound = phi_h;


// Check that the interval brackets the solution
phi_gap_lower = phi_target_gap(phi_lower_bound);


phi_gap_upper = phi_target_gap(phi_upper_bound);


if phi_gap_lower*phi_gap_upper > 0
    error('The selected interval does not bracket the target value of phi_l.');
end
```

```
//////////////////////////////////////////////////////////
// Calculate final emissions intensity
//////////////////////////////////////////////////////////

phi_l_calibrated = fzero(phi_target_gap,[phi_lower_bound phi_upper_bound]);
```

Figure 5,6, and 7. New low-emissions steady state

The code first calculates 10% of initial emissions, which defines the target. It then considers different candidate values of ϕ_L, calculates the steady-state capital associated with each candidate, and computes the resulting steady-state emissions.

MATLAB then searches for the value of ϕ_L that makes final emissions exactly equal to the target. Once this value is found, it is passed to Dynare and used as the final emissions intensity for the transition.

The next part of the code specifies where the economy begins, where it ultimately converges, and what happens to emissions intensity between the two steady states.

```
//////////////////////////////////////////////////////////
// Initial high-emissions steady state
//////////////////////////////////////////////////////////

initval(all_values_required);

    phit = phi_h;

    k = kss_h;
    p = pss_h;

    c  = css_h;
    m  = mss_h;
    y  = yss_h;
    i  = iss_h;
    em = emss_h;

end;
```

```
//////////////////////////////////////////////////////////
// Final low-emissions steady state
//////////////////////////////////////////////////////////

endval(all_values_required);

    phit = phi_l;

    k  = kss_l;
    p  = pss_l;
    c  = css_l;
    m  = mss_l;
    y  = yss_l;
    i  = iss_l;
    em = emss_l;

end;
```

```
//////////////////////////////////////////////////////////
// Impose linear path for emissions intensity
//////////////////////////////////////////////////////////

shocks;

    var phit;

    periods 1:@{T_TRANS};

    values (phi_path);

end;
```

Figure 8, 9, and 10. Initial and final emission

The `initval block` places the economy at the initial high-emissions steady state, using the initial values ϕ, capital, pollution, consumption, the pollution shadow cost, output, investment and emissions.

The `endval block` specifies the corresponding final low-emissions steady state and therefore provides the long-run destination of the transition.

Finally, the shocks block imposes the gradual path of emissions intensity by assigning the previously constructed sequence `phi_path` to ϕ_t over the transition period.

Finally, the two perfect-foresight commands prepare and solve the complete transition path of the economy.

```
//////////////////////////////////////////////////////////////
// Prepare perfect-foresight problem
//////////////////////////////////////////////////////////////

perfect_foresight_setup(periods=@{T_SIM});


//////////////////////////////////////////////////////////////
// Solve transition
//////////////////////////////////////////////////////////////

perfect_foresight_solver;
```

Figure 11 and 12. Solving the transition path

The first sets up a simulation lasting `T_SIM` periods, providing the horizon over which the economy adjusts from its initial conditions towards the final steady state.

The second solves the nonlinear model over the entire horizon using the equilibrium equations, initial conditions, terminal conditions and imposed path of emissions intensity. Dynare therefore calculates how consumption, capital, pollution, investment, output, emissions and the pollution shadow cost evolve over time when the future path of ϕ_t is known in advance.

3.3 MAIN RESULTS

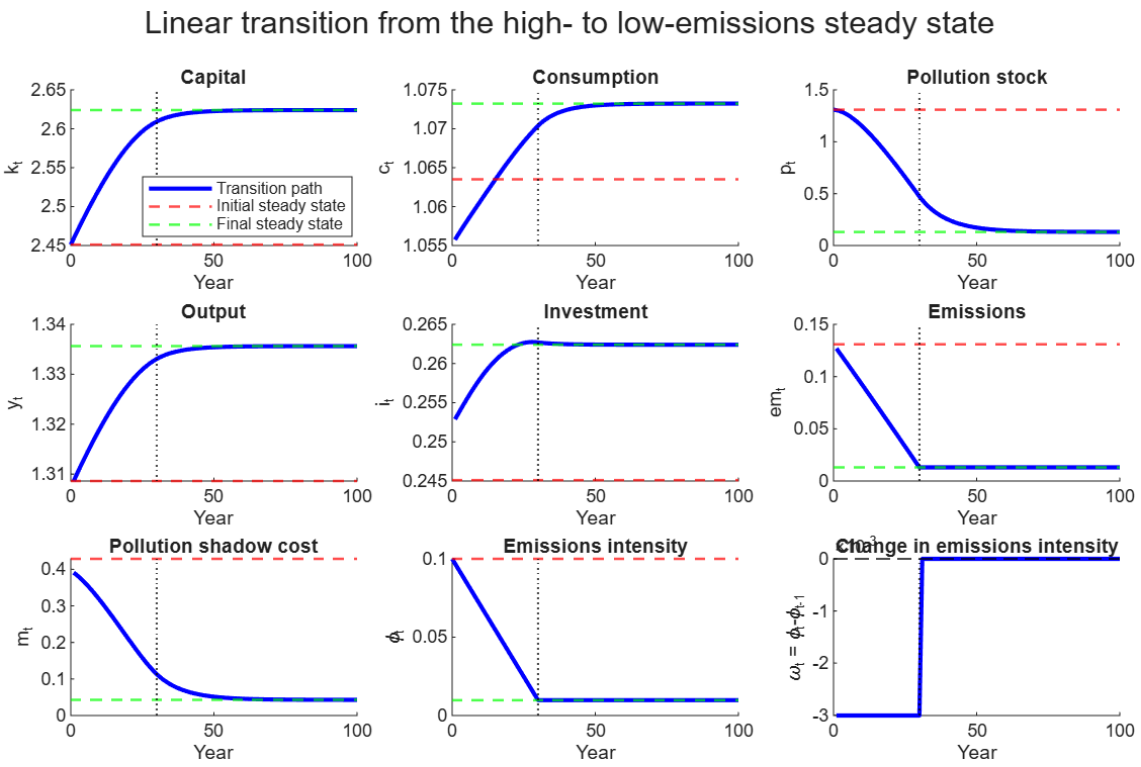


Figure 13. 30-year fast transition to the low-emissions steady state.

Table 1 Environmental steady-state calculations in the fast transition

Variable	High-emissions steady state	Low-emissions steady state
Emissions intensity ϕ	0.100000000	0.009797689
Steady-state capital k*	2.451179348	2.623994116
$F(k^*)$	0.000e+00	0.000e+00

Table 2. Initial and final steady states in the fast transition

Variable	Initial high-emissions steady state	Final low-emissions steady state
Consumption c^*	1.063498971	1.073238867
Output y^*	1.308616906	1.335638278
Investment i^*	0.245117935	0.262399412
Emissions e^*	0.130861691	0.013086169
Pollution p^*	1.308616906	0.130861691
Pollution shadow cost m^*	0.428684849	0.042868485

Table 3. Total-emissions target in the fast transition

Target variable	Value
Final / initial emissions ratio	0.1000
Target emissions reduction	90.0000%
Initial model emissions	0.130861691
Target final model emissions	0.013086169

Table 4. Linear emissions-intensity transition in the fast transition

Transition characteristic	Value
Transition duration	30 years
Total simulation horizon	100 years
Emissions-intensity reduction	90.2023%
Annual change in emissions intensity	-0.003006744
Steady-state emissions reduction	90.0000%
Emissions-target error	0.000e+00

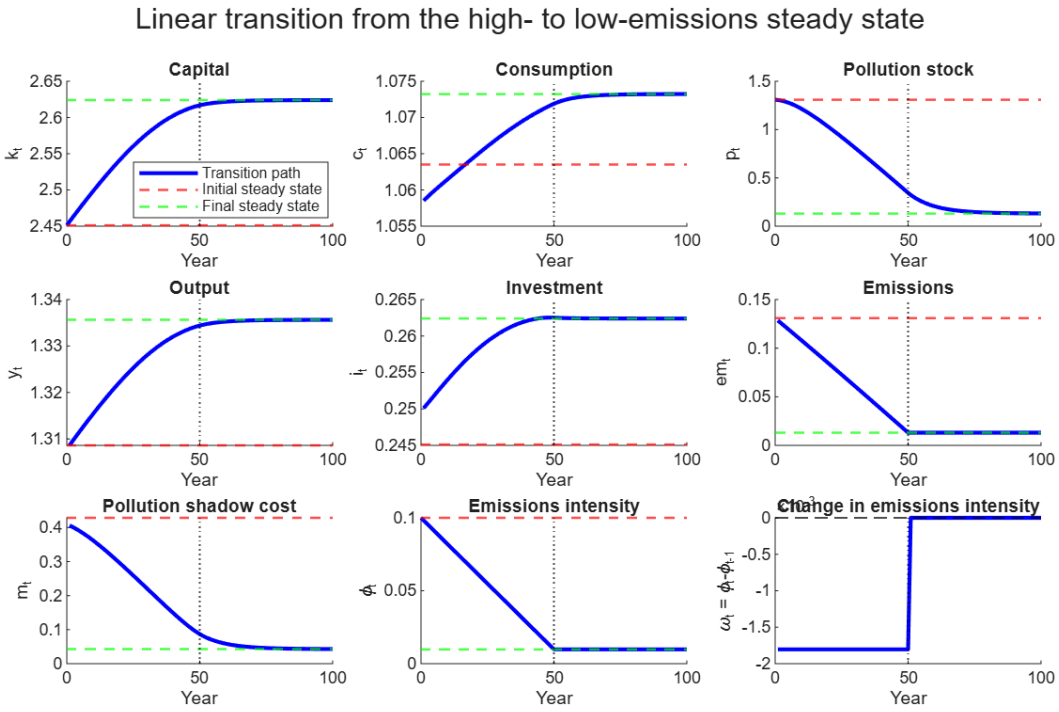


Figure 14. 50-year slow transition to the low-emissions steady state

Table 5 Environmental steady-state calculations in the slow transition

Variable	High-emissions steady state	Low-emissions steady state
Emissions intensity ϕ	0.100000000	0.009797689
Steady-state capital k^*	2.451179348	2.623994116
$F(k^*)$	0.000e+00	0.000e+00

Table 6. Initial and final steady states in the slow transition

Variable	Initial high-emissions steady state	Final low-emissions steady state
Consumption c^*	1.063498971	1.073238867
Output y^*	1.308616906	1.335638278
Investment i^*	0.245117935	0.262399412
Emissions e^*	0.130861691	0.013086169
Pollution p^*	1.308616906	0.130861691
Pollution shadow cost m^*	0.428684849	0.042868485

Table 7. Total-emissions target in the slow transition

Target variable	Value
Final / initial emissions ratio	0.1000
Target emissions reduction	90.0000%
Initial model emissions	0.130861691
Target final model emissions	0.013086169

Table 8. Linear emissions-intensity transition in the slow transition

Transition characteristic	Value
Transition duration	50 years
Total simulation horizon	100 years
Emissions-intensity reduction	90.2023%
Annual change in emissions intensity	-0.001804046
Steady-state emissions reduction	90.0000%
Emissions-target error	0.000e+00

3.3.1 Comparison of the two transition scenarios

As expected, both experiments have exactly the same initial and final steady states. The only difference between them is the speed at which emissions intensity ϕ_t declines. The economy starts from

$$\phi_H = 0.10, \qquad k_H^* = 2.45118,$$

while the numerical calibration gives the final values:

$$\phi_L = 0.009797689, \qquad k_L^* = 2.62399.$$

Therefore, the long-run effects are identical in the fast and slow scenarios.

In the 30-year transition, $\Delta\phi = -0.003006744$ per year, whereas in the 50-year transition, $\Delta\phi = -0.001804046$ per year. In absolute terms, the annual reduction in the fast scenario is therefore approximately $\frac{0.003006744}{0.001804046} \approx 1.667$ times as large. Thus, the 30-year transition requires an annual improvement in emissions intensity that is approximately 66.7% faster than in the 50-year transition.

The central distinction is therefore that transition speed changes the transitional dynamics but not the long-run equilibrium.

3.3.2 Emissions

Initial steady-state emissions are $e_H^* = 0.130861691$. The target is a 90% reduction in total emissions, so final emissions must equal:

$$0.10 e_H^* = 0.013086169$$

The numerical procedure reproduces this value exactly, with the reported steady-state emissions reduction equal to 90% and a numerical target error of zero.

An important result is that emissions intensity itself has to fall by slightly more than 90%. It moves from $\phi_H = 0.10$ to $\phi_L = 0.009797689$, corresponding to a reduction of 90.2023%.

This occurs because output is higher in the final steady state. Output increases from $y_H^* = 1.30862$ *to* $y_L^* = 1.33564$, which is an increase of approximately 2.06%. Since $e = \phi y$, if the economy produces more output, ϕ must fall by slightly more than 90% for total emissions to fall by exactly 90%.

3.3.3 Pollution and the pollution shadow cost

Steady-state pollution falls from 1.308616906 to 0.130861691. This is exactly a 90% reduction. The result follows directly from the steady-state pollution equation

$$p^* = \frac{e^*}{\eta}$$

Since η remains unchanged, a 90% reduction in steady-state emissions mechanically produces a 90% reduction in steady-state pollution.

The same proportional change occurs in the pollution shadow cost because

$$m^* = \frac{\beta \chi p^*}{1 - \beta(1 - \eta)}$$

All the parameters multiplying p^* remain constant. Therefore,

$$p^* \downarrow 90\% \quad \Longrightarrow \quad m^* \downarrow 90\%$$

Pollution is also one of the variables for which the distinction between the fast and slow transitions is particularly important.

In the fast scenario, emissions intensity reaches its final level in year 30. However, pollution does not immediately reach its new steady state in year 30. Instead, the pollution stock continues declining for many years afterwards.

This follows directly from

$$p_{t+1} = (1 - \eta)p_t + e_t$$

Pollution is a stock, so part of the pollution accumulated in previous periods remains in the environment.

In the 50-year scenario, pollution therefore remains high for longer because relatively high emissions are maintained for a longer period. This illustrates an important implication of the model: delaying emissions reductions has persistent environmental consequences because pollution accumulates as a stock.

3.3.4 Consumption, investment, capital and output

The new steady-state consumption level is higher than the initial one. Consumption rises from $c_H^* = 1.06350$ to $c_L^* = 1.07324$, so in the long run the low-emissions economy has approximately 0.92% higher consumption.

However, at the beginning of the transition, consumption falls below its old steady-state level before subsequently recovering and rising above it. There is therefore a short-run consumption sacrifice despite higher long-run consumption. At the same time, investment immediately rises above its old steady-state level. The mechanism can be summarised as

$$\phi_t \downarrow \implies \text{environmental penalty on capital} \downarrow \implies \text{desired future } k \uparrow.$$

The reduction in emissions intensity lowers the environmental cost generated by an additional unit of capital, $m_{t+1}\phi_{t+1}\alpha k_{t+1}^{\alpha-1}$. As ϕ_{t+1} falls and expected pollution also falls, the environmental deduction from the return to capital becomes smaller. Accumulating capital therefore becomes more socially attractive.

To begin accumulating this additional capital, the economy temporarily gives up some consumption. From the resource constraint, $c_t + k_{t+1} = k^{\alpha}t + (1-\delta)k_t$, holding available resources fixed, a larger k_{t+1} requires a smaller c_t.

Investment therefore correctly increases. Since $i_t = k_{t+1} - (1-\delta)k_t$ capital rises whenever gross investment exceeds depreciation, $i_t > \delta k_t$. Investment rises during the early transition, reaches approximately its highest level around the period in which the emissions-intensity target is completed, and then gradually converges towards its final

replacement-investment level. The small decline in investment towards the end of the transition is also economically sensible. Once capital is close to its new steady-state level, the economy no longer requires unusually high net investment.

Because $y_t = k_t^{\alpha}$ and capital increases, output must also increase. The proportional increase in output is smaller than the increase in capital because $0 < \alpha < 1$, so production exhibits diminishing returns to capital.

The adjustment therefore does not require a decline or destruction of the capital stock. Instead, it represents an intertemporal reallocation from current consumption towards investment, allowing the economy to build a larger capital stock and eventually sustain higher output and consumption.

These results are qualitatively consistent with the empirical findings of Hysa et al. (2020), Chen et al. (2023), and Acheampong and Opoku (2023) discussed in Chapter 1, which suggest that environmental improvements and economic performance need not necessarily be in conflict. This comparison should be interpreted cautiously, however, since those studies do not constitute a direct empirical validation of the model

3.3.5 Differences between the fast and slow macroeconomic adjustment

The same general pattern appears in capital, investment and output under both transition speeds, but the adjustment occurs more quickly in the fast scenario.

In the 30-year case, capital approaches $k_L^* \approx 2.624$ relatively quickly. Output and investment consequently approach their new steady-state levels earlier. In the 50-year transition, the same variables follow more gradual paths and converge later.

The short-run consumption sacrifice also appears stronger under the fast transition. Because emissions intensity falls more rapidly, the environmental penalty on capital

declines sooner. The planner therefore has a stronger incentive to accumulate capital earlier.

This produces a larger short-run reallocation of resources towards investment and away from consumption. The fast transition consequently generates a sharper short-run adjustment, whereas the slow transition spreads the adjustment over a longer period.

3.3.6 Important limitation of the experiment

The current formulation assumes that the decline in ϕ_t has no direct resource cost. Cleaner technology or lower emissions intensity is introduced exogenously, without explicitly requiring expenditure on renewable energy, abatement, replacement of carbon-intensive capital, or cleaner production technologies.

The resource constraint remains $c_t + k_{t+1} = k_t^{\alpha} + (1-\delta)k_t$. There is therefore no additional term representing the direct cost of achieving the reduction in emissions intensity. Although the costs of technologies such as wind and solar have fallen considerably over time (IPCC, 2023), this does not imply that decarbonisation is literally costless in terms of capital. The present model simply abstracts from those direct costs.

Consequently, in the model as currently specified, a faster reduction in emissions intensity is largely environmentally beneficial:

$$\phi_t \downarrow \implies e_t \downarrow \implies p_t \downarrow \implies m_t \downarrow.$$

3.3.7 A simplest abatement extension

Let a_t denote the proportion of potential emissions that is abated. Emission would become

$$e_t = (1-a_t)\phi k_t^{\alpha}$$

Pollution accumulation would become

$$p_{t+1} = (1-\eta)p_t + (1-a\ \ )\phi k_t^{\alpha}$$

Suppose abatement uses a fraction $\Psi(a_t)$ of output. The resource constraint would then become

$$c_t + k_{t+1} = [1 - \Psi(a_t)]k_t^{\alpha} + (1 - \delta)k_t$$

This extension would make emissions reduction explicitly costly in terms of current resources and would introduce an additional choice concerning the optimal amount of abatement.

CONCLUSION

The goal of this thesis was to construct an extension of the intertemporal Ramsey model by incorporating an environmental dimension, while at the same time keeping the framework sufficiently simple to clearly identify how pollution affects the behaviour of the main economic variables.

The analysis initially developed a continuous-time environmental Ramsey model in order to establish the economic intuition of the framework. The model was subsequently reformulated in discrete time and explicit functional forms were introduced, allowing it to be implemented numerically in MATLAB and Dynare.

Since much of the current discussion surrounding climate policy concerns the reduction of emissions towards specific long-run targets, the numerical experiment considered a transition from an initial high-emissions steady state to a new low-emissions steady state. The European Union's emissions-reduction target was used as an empirical benchmark for determining the magnitude of this transition. Two different transition speeds were then considered in order to examine how the timing of the reduction in emissions intensity affects the adjustment of the economy.

The results show that the transition does not require a reduction in the capital stock. On the contrary, as emissions intensity falls, an additional unit of capital generates fewer

emissions and therefore a lower environmental cost. Capital accumulation consequently becomes more attractive to the social planner. The economy responds by increasing investment and gradually accumulating a larger capital stock. This adjustment requires a temporary reduction in consumption, so the main short-run sacrifice in the model takes the form of lower consumption rather than lower capital. In the long run, however, the economy reaches a steady state characterised by lower emissions and pollution together with higher capital, output and consumption.

This result also highlights the main limitation of the experiment. The improvement in emissions intensity is imposed exogenously and does not require the economy to devote resources directly to achieving it. In other words, the model abstracts from the costs associated with replacing carbon-intensive technologies, investing in renewable energy or undertaking abatement. As the traditional economic principle suggests, there is no free lunch: a more complete representation of the green transition would need to account explicitly for these costs. One possible extension would therefore be to introduce an abatement decision and an associated resource cost. Further theoretical work could also investigate how the environmental component affects concepts such as the Golden Rule and the dynamics around the steady state, including the shape of the stable arm.

REFERENCES

Acheampong, A.O. and Opoku, E.E.O. (2023) 'Environmental degradation and economic growth: Investigating linkages and potential pathways', *Energy Economics* 123, 106734. doi: 10.1016/j.eneco.2023.106734.

Aghion, P. and Howitt, P. (1992) 'A model of growth through creative destruction', *Econometrica* 60(2), pp. 323–351. doi: 10.2307/2951599.

Akcigit, U. (2026) 'Creative destruction in economic growth', *Scandinavian Journal of Economics* 128(3), pp. 509–541. doi: 10.1111/sjoe.70037.

Barrage, L. and Nordhaus, W.D. (2023) ‘Policies, projections, and the social cost of carbon: Results from the DICE-2023 model’, *NBER Working Paper* No. 31112. Cambridge, MA: National Bureau of Economic Research.

Bartkowska, M. and Riedl, A. (2012) ‘Regional convergence clubs in Europe: Identification and conditioning factors’, *Economic Modelling* 29(1), pp. 22–31. doi: 10.1016/j.econmod.2011.01.013.

Chen, Y., Mamon, R., Spagnolo, F. and Spagnolo, N. (2023) ‘Sustainable developments, renewable energy, and economic growth in Canada’, *Sustainable Development* 31(4), pp. 2950–2966. doi: 10.1002/sd.2561.

Cornwall, J. (2023) ‘Economic growth’, *Britannica Money* 17 May. Available at: https://www.britannica.com/money/economic-growth

Dasgupta, P. (2021) *The Economics of Biodiversity: The Dasgupta Review*. London: HM Treasury.

European Commission (2026) ‘2040 climate target’. *Climate Action*. Available at: https://climate.ec.europa.eu/areas-action/climate-strategies-targets/2040-climate-target_en (Accessed: 21 August 2026).

Galor, O. (1996) ‘Convergence? Inferences from theoretical models’, *Economic Journal* 106(437), pp. 1056–1069.

Gould, D.M. and Ruffin, R.J. (1993) ‘What determines economic growth?’, *Economic Review, Federal Reserve Bank of Dallas* Second Quarter, pp. 25–40.

Grossman, G.M. and Krueger, A.B. (1993) ‘Environmental impacts of a North American Free Trade Agreement’, in Garber, P.M. (ed.) *The Mexico–U.S. Free Trade Agreement*. Cambridge, MA: MIT Press, pp. 13–56.

Herrington, G. (2021) ‘Update to limits to growth: Comparing the World3 model with empirical data’, *Journal of Industrial Ecology* 25(3), pp. 614–626. doi: 10.1111/jiec.13084.

Hysa, E., Kruja, A., Rehman, N.U. and Laurenti, R. (2020) ‘Circular economy innovation and environmental sustainability impact on economic growth: An integrated model for sustainable development’, *Sustainability* 12(12), 4831. doi: 10.3390/su12124831.

IPCC (2023) ‘Summary for policymakers’, in Lee, H. and Romero, J. (eds.) *Climate Change 2023: Synthesis Report*. Geneva: Intergovernmental Panel on Climate Change, pp. 1–34. doi: 10.59327/IPCC/AR6-9789291691647.001.

Kuznets, S. (1973) ‘Modern economic growth: Findings and reflections’, *American Economic Review* 63(3).

Meadows, D.H., Meadows, D.L., Randers, J. and Behrens III, W.W. (1972) *The Limits to Growth*. New York: Universe Books.

Mokyr, J. (1990) *The Lever of Riches: Technological Creativity and Economic Progress*. New York: Oxford University Press.

Panayotou, T. (2003) 'Economic growth and the environment', paper presented at the Spring Seminar of the United Nations Economic Commission for Europe, Geneva, 3 March.

Romer, P.M. (1986) 'Increasing returns and long-run growth', *Journal of Political Economy* 94(5), pp. 1002–1037. doi: 10.1086/261420.

Romer, P.M. (1990) 'Endogenous technological change', *Journal of Political Economy* 98(5, Part 2), pp. S71–S102. doi: 10.1086/261725.

Rostow, W.W. (1959) 'The stages of economic growth', *Economic History Review* 12(1), pp. 1–16.

Schumpeter, J.A. (1942) *Capitalism, Socialism and Democracy*. New York: Harper & Brothers.

Solow, R.M. (1956) 'A contribution to the theory of economic growth', *Quarterly Journal of Economics* 70(1), pp. 65–94. doi: 10.2307/1884513.

Swan, T.W. (1956) 'Economic growth and capital accumulation', *Economic Record* 32(2), pp. 334–361.

Turner, G.M. (2014) *Is Global Collapse Imminent? An Updated Comparison of The Limits to Growth with Historical Data*. MSSI Research Paper No. 4. Melbourne: Melbourne Sustainable Society Institute, The University of Melbourne.

United Nations Environment Programme (UNEP) (2025) *Emissions Gap Report 2025: Off Target – Continued Collective Inaction Puts Global Temperature Goal at Risk*. Nairobi: UNEP. doi: 10.59117/20.500.11822/48854.

World Economic Forum (2025) *The Global Risks Report 2025*. 20th edn. Geneva: World Economic Forum.

BIBLIOGRAPHY

Barro, R.J. and Sala-i-Martin, X. (2004) *Economic Growth*. 2nd edn. Cambridge, MA: MIT Press.

Wälde, K. (2012) *Applied Intertemporal Optimization: Textbook and Solutions Manual*. Mainz: Know Thyself Academic Publishers.