UNIVERSITA’ POLITECNICA DELLE MARCHE

FACULTY OF ENGINEERING

Department of Information Engineering

Master's Degree Course in Biomedical Engineering

Experimental Assessment of the Vibrational Behavior of a Tennis Racket and the Effect of Damping Systems

Relatore:

Prof. Lorenzo Scalise

Laureando:

Lorenzo Ricciardi

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Abstract

Every year thousands of tennis players are subjected to injuries associated with the inflammation of the tendons, due to the great amount of solicitations transmitted from the racket to the arm. The elbow tendinopathy (or epicondylitis), commonly known as “tennis elbow”, is the most common injury in the game of tennis, affecting almost 40% of players.

In this work, we performed a general characterization of the vibrational behavior of a tennis racket and an assessment of the energy transmitted from the racket to the arm of the subject in presence of damping systems, considering exclusively the 0-200Hz range. This low-frequency interval was identified, by previous studies, as the most important for what concern playing related injuries, and in particular, for the tennis elbow.

To fulfill these tasks three experimental tests were designed and performed with the racket “alone” or with one of the latest damping systems applied to it. These tests were designed with different levels of approximation, starting from almost ideal conditions to almost real playing conditions, in order to obtain more comprehensive results.

During the tests, the racket was solicited by an external impulsive force with the vibrations sensed by three accelerometers. The recorded signals were studied in the frequency domain by calculating the spectral energy distribution and the vibrational energy contained in the harmful frequency range.

The three tests produced the same vibrational energy distribution in every condition studied, highlighting the presence of a first vibrational mode inside the harmful range.

Analyzing the vibrational energy content, the first test, which was performed in almost ideal conditions, demonstrated that the damping systems have a statistically significant impact on the transmitted energy. However, the subsequent tests, which were performed in conditions much closer to reality, did not report a significant damping effect in any of the studied cases, showing how the real efficacy of the damping system is still debatable.

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1. INTRODUCTION and AIM

The attempt of improvement of the tennis racket performances has been investigated by multiple fields, with many innovations, especially at highly competitive levels.

Everyday players seek the optimal trade-off between performances and comfort, professional players are mainly, if not exclusively, interested in obtaining maximum efficiency from the tool. This choice is obviously dictated by the fact that they are competing at high levels which forces the players to endure great efforts and a great amount of solicitations for long periods of time. To give a general idea, professional athletes can easily shoot more than one thousand balls during one single game, subjecting their arm to enormous fatigue. This must be considered in the optic of everyday training where the number of shots can be even greater but also repeated.
For this reason, it became evident the necessity to develop an efficient damping system to add or to embed in the racket to reduce as much as possible the vibrations transmitted to the arm of the player.

The first goal was to increase the “physical performance” of the tool, specifically maximizing the transmitted energy and the control of the ball. Only as a second objective, there was the attempt to improve the comfort of the racket, providing a lower impact on the player’s health. This aspect is of primary importance considering that the racket is the interface between the player’s arm and the ball. The arm of the player is subjected to a great number of impacts during a game, these impacts transfer most of their energy to the arm of the subject and so they constitute the first risk for the subject’s health. In fact, energy absorption by the arm is the main source of injuries in tennis, causing lateral epicondylitis, commonly known as “tennis elbow”.

The aim of this work is to present a simple and reproducible experimental method for the measurement of the vibration transmissibility from the string plate to the player’s arm, with and without damping systems.

For the design of a set up able to reproduce, at least ideally, the real impact's conditions and to provide the possibility to measure the key parameters related to the dynamics of the racket, we started from the methods and parameters present in literature.

The set up was designed to allow the reproduction, at least ideally, of the real impact's conditions and to provide the possibility to measure the key parameters related to the dynamics of the racket, starting from the study of the results and parameters present in literature.

Three experiments were designed to propose different levels of ideality, from ideal conditions to real playing conditions.

Despite the limits associated with the high level of assumptions made in the first experiment, and in particular, the difficulty to produce an excitation of the frequencies with flat band until 500Hz, the obtained results are generally in agreement with the literature.

1.1 General Characteristics and Fundamental Parameters of a tennis Racket

The evolution in the design of the tennis racket is continuous and it involves many fields from the materials to the study of the frame's shape. Also, it includes some essential elements strictly related to the transmission of the impact energy and vibration to the hand-arm system.

The structure of a racket comprises a rigid element, the frame, composed by head, throat, handle and by the string plate. The string plate is the part of the racket structure mainly associated with the dynamics of the ball and so to the quality of the shot.

The frame is constituted by a hollow structure, and it guarantees good rigidity to flexion while maintaining a relatively low mass, which is extremely important for the performance over the long period. A racket with greater rigidity provides more power because a lower amount of energy is associated with the deformation of the structure and also the oscillation period is reduced.

The string plate has the primary aim to absorb the kinetic energy, related to the ball's impact, and to convert it into potential energy, associated to the elastic deformation, to successively transfer it back to the ball. For this reason, the design, the materials and the tension of the strings are of primary importance for the final behavior of the racket and consequently of its performances.

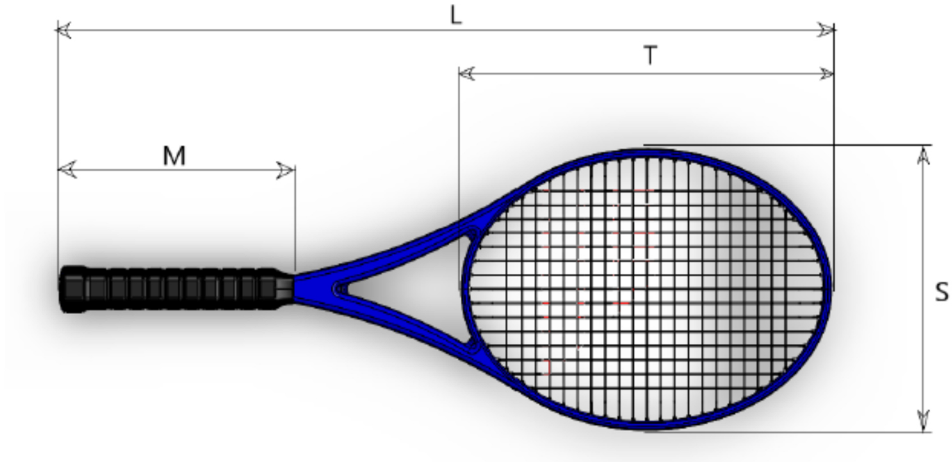


Fig. 1.1: Dimensions and structure of a racket. L (m) 0.652-0.811-M (m) 0.148-0.355-T (m) 0.240-0.493 -S (m) 0.185-0.308 -Mass kg) 0.220-0.427.

The parameters of the racket, that have an influence on the game dynamic and that allow to characterize the transfer of impacts and vibrations to the player, are summarized below:

Center of Mass (COM):

As for any other body, also for the tennis racket it is possible to localize a point in which to imagine concentrating the entire mass of the object. The COM is related to the point of balance, and this is the starting element to design an experimental set up with the racket free from constraints. The position of the COM determines three typologies of rackets:

Head heavy: this typology of rackets is characterized by the large mass at the level of the string plate and it allows players to hit the ball with great power. These rackets are very rigid, and this causes a greater transmission of vibration to the arm. Furthermore, it must be considered that this configuration produces a higher moment of the weight strength with respect to the hand because the arm of the rotation system coincides with the entire length of the racket, causing hand fatigue.

Head light: greater handling is achieved with these rackets that, on the other hand, have a lower shooting power since the mass is mainly localized at the head level. This configuration is the most utilized.

Even balanced: this typology of balance represents a compromise between the two previous types, combining their characteristics and so constituting a tool with good handling and shooting power.

Sweet spot: optimal impact area. It's a region of the string bed that guarantees best performances and minimum discomfort for the player. The extension and location of this area is determined by the string plate dimension, the strings space and by the position of eventual dampers. Hitting the ball at the level of the sweet spot is the most efficient way to impart maximum velocity to the ball and to dampen to the maximum the oscillations that propagate to the hand-arm system. The sweet spot is dependent on a set of areas with different features and so the choice of the impact point, from the player, is related to the type of shoot to perform. The parameters to combine to localize the sweet spot are (fig 1.3):

Center of percussion (COP): it's the surface of the string bed associated with the minimum transmission of energy to the hand of the player. An impact in the COP produces rotational and translational forces that cancel each other out at the level of the hand, causing no effect on this last. Hitting the ball far from this point causes a rotation of the racket with respect to an axis located on the head of the racket. This causes a movement of the racket away from the hand, that is compensated by the player with the development of a great impulsive force at the level of the wrist, typically associated with articular injuries of the ulnar and radial collateral ligaments.

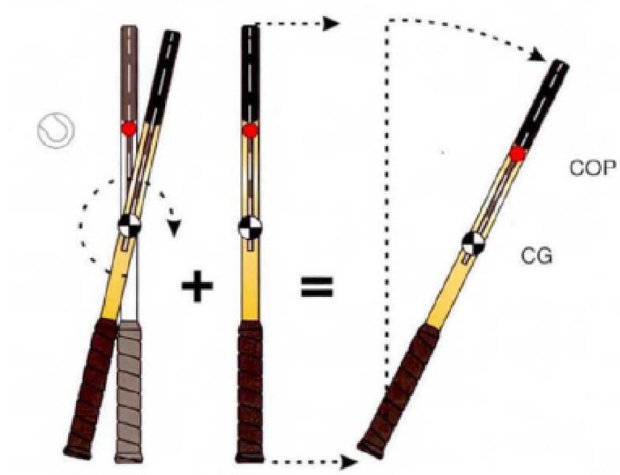


Fig. 1.2: impact on the COP; A-rotation of the system, B-translation of the system, C-zero rotation in correspondence of the hand.

Node: is the point of the string bed that is mostly related to the vibration concept. Every object when excited, by an external input, will vibrate at its natural frequencies and at the corresponding modal forms, which depend on the constraint conditions. The nodes are the points of minimum oscillation, if the ball hits one of these points it will not excite the fundamental mode associated with the maximum contribution of energy.

Coefficient of restitution (COR): this coefficient is a dimensionless indicator of the energy that is naturally restituted to the ball after it has impacted the string plate. It quantitatively represents the ratio between striking and leaving velocities. The loss of energy is related to the elastic deformation of the string bed, to the oscillation of the ball itself after the impact and to the internal vibrations of the frame.

The region of maximum COR is in proximity to the throat of the racket, which is also where the center of mass typically is. If the ball hits in this area the lowest amount of energy will be dissipated in rotational energy while the majority will be retransferred to the ball.

The typical COR values vary from between 0.6 and 0.2, closer to the top of the racket, while the typical values in the central area of the string bed are around 0.4-0.5.

Dead spot: typically located closer to the top of the racket, this point is characterized by the minimum COR value, and it’s exploited by professional players to serve with maximum shooting speed. This point guarantees a higher transmission of energy and in general it’s preferred to the COP for the service. However, hitting this point, also maximizes the transmission of vibrations and so causes fatigue.

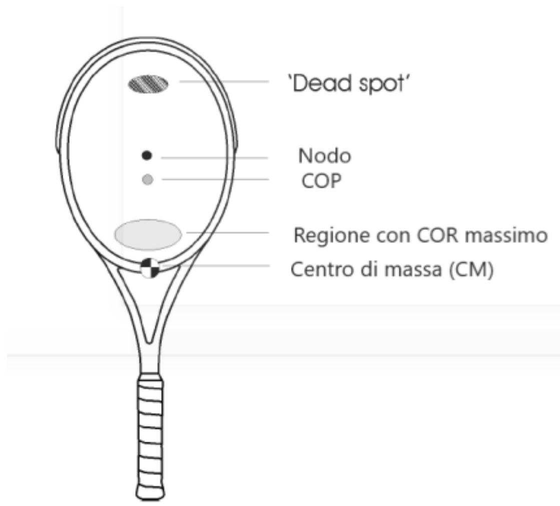


Fig. 1.3: Common positions of the elements that contribute to the definition of the sweet spot.

String tension

The intensity of the vibrations depends on multiple factors, but the string tension is probably the most significant. This parameter measures the tensile force applied to the strings when these are mounted on the frame. The tension of the individual strings has a significant effect on the racket’s frame, contributing to increase the general rigidity of the structure. Although the international unit of measure for the strength is the newton (N), at commercial level the stringing voltage is quantified in pounds (lb), ranging between 40lb (~180N) and 70lb (~312N).

Low values of string tension allow wider strings’ deformation at the instant of impact with the ball, causing an increase of shooting power but a reduction of the ball’s control. Instead, high strings tension’s values improve the string plate stability and so the control on the ball during the strike but reduces the elastic deformation and so the shooting speed.

The deformation's entity of the string bed is not correlated to the kind of ball used to simulate the impact (this assumption will be particularly useful in the first experiment).

The string tension is directly related to the COR, in fact, reducing the string tension the COR will generally increase (fig. 1.4).

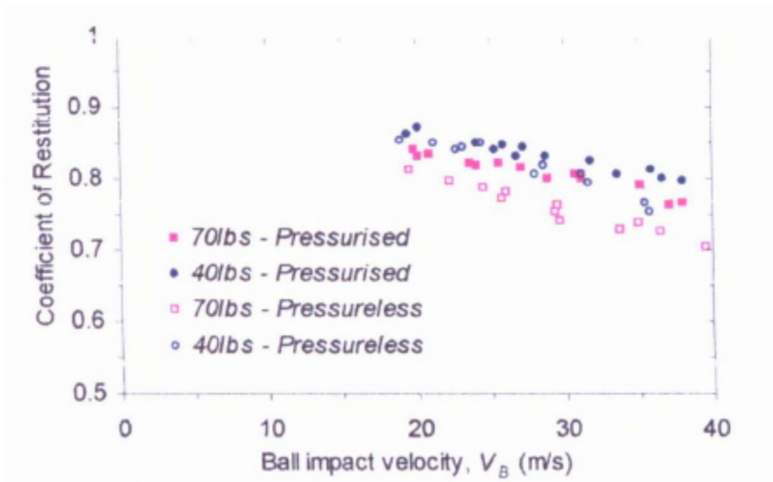


Fig. 1.4: COR for different stringing tensions. Regardless of the type of ball used (whether internally pressurized or not) the COR increases considerably as the pulling force of the strings decreases for each impact speed.

A racket with elevated string tension has higher rigidity that causes it to increase the vibrations, especially at frequency, and therefore to increase the solicitations transmitted to the arm of the subject. Lower string tension rackets have a higher time of contact with the ball, maximizing the damping effect for the vibrations of the frame and of the strings. This is related to the increased deformation of the ball and to the increased time of contact. This last parameter is the starting point to quantify the dynamic response of the "racket system" and for the interpretation of the most involved frequencies in the vibrational phenomenon related to an impact. In fact, increasing the time of contact of the ball with the string bed, allows the racket to transfer more kinetic energy to the ball.

1.1.1 Damping Systems

This paragraph will present the different typologies of vibrational damping systems that can be applied to the tennis racket. Some of these were profoundly studied in literature, in terms of frequency response of the vibrating system after the embedment of such devices.

In every context the damping system's aim is to reduce the decay time of vibrations. It was demonstrated that the most efficient damping system is the hand, however, the influence of the handle is very difficult to quantify because it depends on multiple factors that vary continuously during the game. For this reason, our focus was dedicated to the evaluation of external devices on the oscillation's reduction.

The more rigid tennis rackets vibrate at higher frequencies, so the natural decay of the oscillations is shorter. However, high rigidity frames dissipate a lower amount of energy during the phase of impact, resulting in an increase of vibrations transmitted and so of health risk for the athletes. The concept, at the base of the damping systems, is to introduce a material able to dissipate quickly and efficiently the kinetic energy of vibrations.

The dampers are designed and sized to determine a reduction of the articular injuries related to the transmission of kinetic energy from the impact to the hand-arm system, but they also cause a change in the sound and in the physical perception of the tool by the player. This is a point of disadvantage because athletes are typically unhappy about the change of sensations associated with the use of different devices.

Still today the real efficacy of the damping system is not clearly established even if nowadays these devices are largely diffused. Some previous studies suggest that this could be related to the damping of non-damaging frequencies and so to a placebo effect induced by the production of lower tones.

The main reason for a limited effectiveness of these devices must be related to their lightness. These devices hardly exceed a mass of 10gr, compared to racket masses that can reach even 350gr. So, these systems typically constitute less than 5% of the entire mass system, with many cases where they have even lower impact.

In the second part of this work, the experimental phase, together with the study of the vibrational content of the tennis racket the focus was the evaluation of some of the most used dampers on the reduction of vibrations.

At present, on the market, there are 4 main typologies of tennis racket dampers: string anti-vibration (rubber pad), granular, Fluendo and hybrid. The first three of these typologies are described below while the hybrid one is simply the combination of the string anti-vibration and the granular.

Strings anti-vibration

The traditional and most diffused anti-vibration damper is a small circular device in silicon rubber (Fig. 1.5.A) that is inserted in the string bed close to the throat region. Other types are represented by elastic strings that can be intertwined or fastened into the string plate (Fig. 1.5.B-C).

These devices are studied to reduce as much as possible the vibration at the level of the string plate and to have the minimum effect on the intrinsic characteristics of the rackets (mass, COM, balancing), with a very limited mass of around 2-5gr and a symmetrical placement close to the throat region to reduce as much as possible changes in the center of mass position.

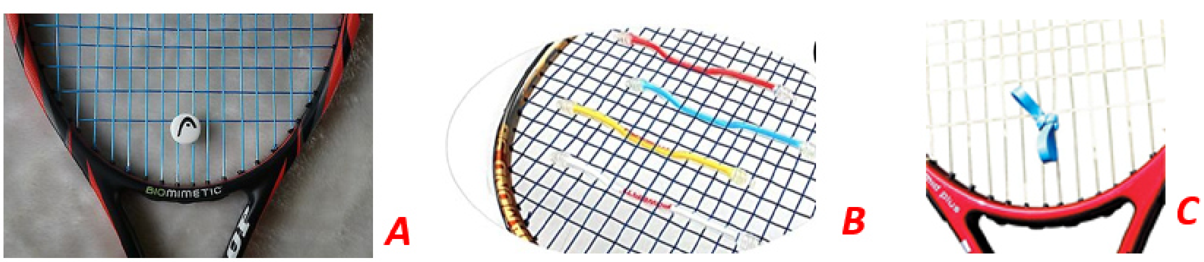


Figure 1.5: Most diffused design of anti-vibrating dampers. A Button. B Worm. C Elastic band.

Granular damping systems

This typology has a very peculiar configuration. It's composed of a "chamber" attached to the racket frame or to the string bed that is partially filled with grains or spheres free to move. The movement of the microspheres is able to dissipate part of the kinetic energy through nonconservative collisions. This device behaves as a system of the second order (inertial system mass-spring-damper) that responds to the force generated from the impact with three distinct contributions (Fig. 1.5):

1- **Inertial force** of the spheres, associated with the second derivative of the sphere's displacements
2- **Viscous damping** of the device, due to the internal irreversibility of the device. This contribution is expressed by the first derivative of the displacement of the spheres.
3- **Elastic reaction** of the constituting material of the damper envelope, directly proportional to the displacement of the excited strings.

The global mass of the device is the sum of the mass of the spheres and of the envelope attached to the racket.

In relation to the considered design variables for the device production, the dissipation of energy can happen in different ways:

- **Inertial dissipation**

  This factor is related to the non-complete fill of the case, which allows a partial movement of the spheres. The impacts of the spheres, between themselves and with the walls of the case, convert the kinetic energy into thermal energy due to the friction. A tradeoff between the number of grains and free space inside the case is necessary to have the greater number of spheres that can properly roto-translate resulting in the maximum energy dissipation.
- **Dissipation for grains rupture**

  A particular intense force can determine an increase of the pressure on some grains causing the rupture of these lasts and resulting in an energy dissipation.

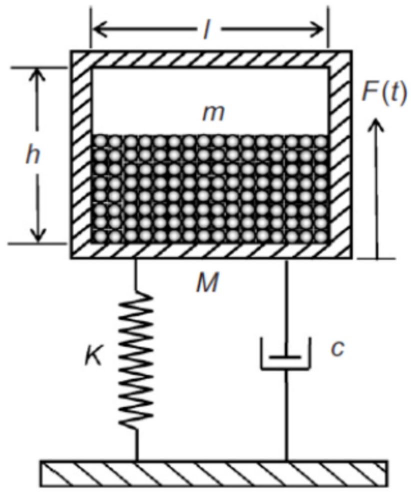


Fig. 1.5: Schematic model of an inertial system associated with a granular damper. K = stiffness of the elastic contribution. C = viscous damping coefficient of the system. m + M = mass of the entire dampener.

Granular damping systems

The Prokennex Kinetic System presented in Fig. 1.6.

This system introduces an evolution of the typical damper concept. This typology of rackets presents embedded in their frames a system of granular dampers. The presence of these secondary moving masses consents to absorb energy and increase exit velocity of the ball. The first effect of this embedded damping system, according to the manufacturer, is the reduction of the transmission of impacts and vibrations to the arm of the player, without impacting on the balance of the racket itself. During the impact the grains move in the opposite direction with respect to the ball, in this way at the moment of impact, they can release their kinetic energy. This phenomenon helps to balance the impulsive force transmitted to the racket frame and so to the hand-arm system. The kinetic system is able also to enlarge the sweet spot by stabilizing the string bed thanks to the reduction of the energy transferred to the frame.

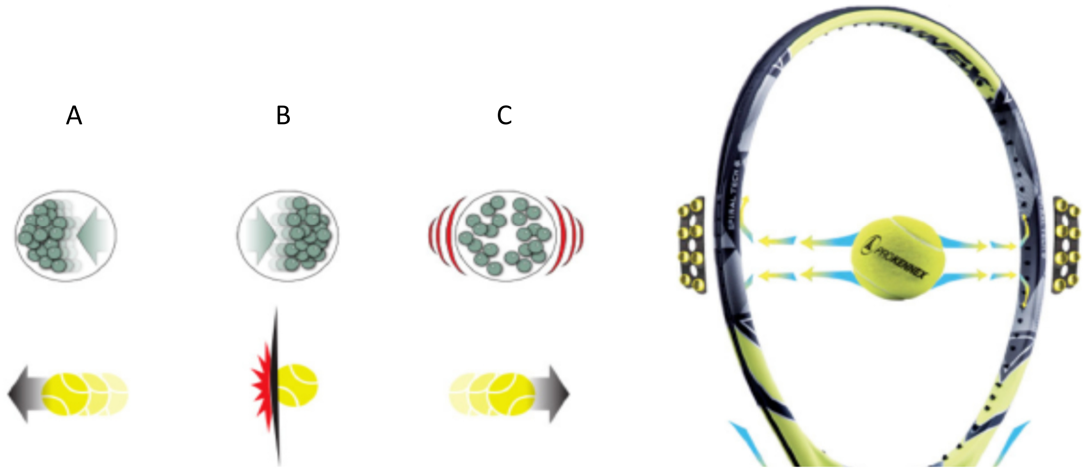


Fig. 1.6: working mechanism of the Kintic dampic system. The figure shows the sphere's displacement before (A), during (B) and immediately after (C) the impact.

Babolat Vibrakill System (Fig. 1.7)

This is, up to now, the unique hybrid damping system. This device has a large silicone body, that is attached to six consecutive vertical strings, and so, involving a very large part of the string bed. The silicone body presents at its center a container with at its inside metallic microspheres, that dissipate energy for friction. The presence of these two main components consents to the damper to dissipate energy in two different modalities, theoretically increasing its efficiency.

Fig. 1.7: Vibrakill system of the Babolat

Fluendo

This device was developed with the aim to create an elastic barrier able to absorb the vibrations coming from the string plate and so to reduce the transmission of solicitations to the olecranon joint and to the wrist joint, reducing articular injuries. Patented in 2018, the Fluendo system is a device made of a silicon polymer of the weight of 15gr, that must be placed at the base of the racket's throat.

Such a device, exploiting the energy generated by the deformation of materials and their shape memory, is able to absorb the vibrational peaks caused by the impact of the ball with the string bed. The decay time, of the fundamental modes, is decreased from 1000 to 820 milliseconds, reducing the intensity of maximum vibrational peak, limiting energy transmission to the arm and the pain perception.

Its higher weight and constitution seem to position the Fluendo as the most efficient external damping system. However, due to these characteristics the Fluendo is also the damper that mostly impacts the intrinsic features of the rackets. In fact, its weight and its particular point of application cause an important displacement of the COM of all the rackets that are not "head light" (fig. 1.8).

Figure 1.8: Damper Fluendo and its mounting on the racket frame.

1.2 Dynamic behavior of a racket

To understand the dynamic response of the racket system in the transmission of vibrations, it is necessary to identify the most representative analytical model. The differential equation associated with the response of the racket after the impact with the ball presents a term related to the second derivative of the displacement of the string bed.

$$f = m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx \qquad \text{[a]}$$

In equation [a], f is the impulsive force associated with the impact (input quantity), m the mass of the vibrating system, c and k respectively the damping coefficient and the stiffness associated with geometric and constructive parameters of the racket, while with x we indicate the displacement of the ropes following the impact (output quantity).

The frame's stiffness (k) is the parameter that most of all influences the dynamic response of the system since the mass changes little between the various existing racquet models.

The structural analysis of the racket shows different modes of vibration, both flexural and torsional, but in both cases the resonance frequency ω_n can be calculated as follows:

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \text{[b]}$$

From the following formula it is possible to deduce that very rigid (high k) or particularly light (low m) racquets determine the increase of the ω_n and with it of the passband of the system. So, the choice of materials and design directly influence the constants of the dynamic of the racket.

The frequency response function of the system can be studied in terms of module and phase and presents the following trend (fig.1.9):

$$\frac{x}{f}(\omega) = \left|\frac{x}{f}(\omega)\right| \left\langle \frac{x}{f}(\omega) \right. \qquad \text{[c]}$$

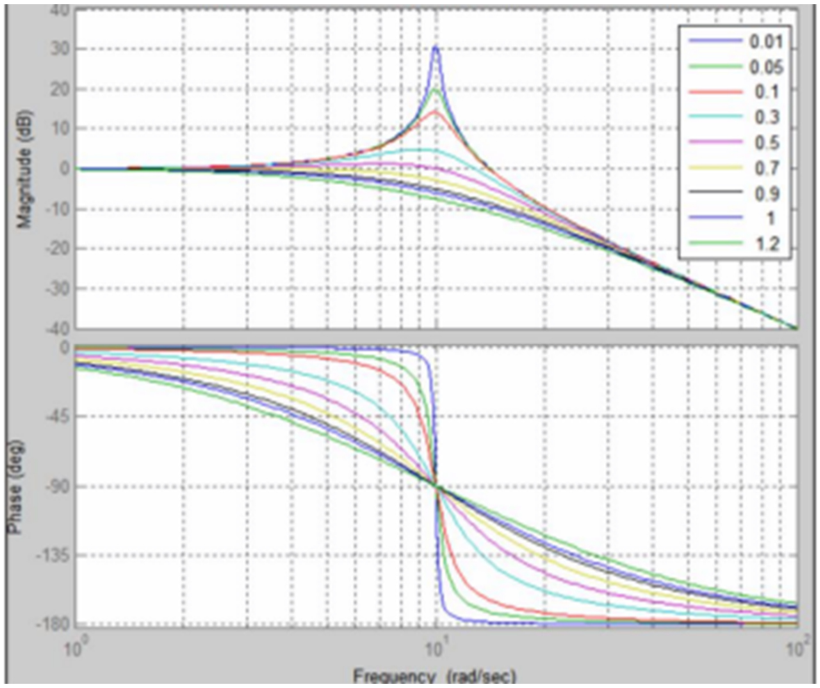


Fig. 1.9: Frequency response of system of the second order. Both the module and the phase trend are represented for different damping values. The racket is considered as an under damped system with 0<ζ<1. The abscissa axis is common to both graphs and represents the axis of normalized frequencies for the resonant frequency of the ω_n system.

From the graphs shown in Figure 1.9, it is evident how the passing band of the system is linked to low frequencies where there is the amplification of the displacement module and the almost absence of phase shift. In particular, the phase shows how for very low frequency values the time delay is negligible, while for pulsations close to ω_n the phase is linear with respect to the frequency. At the ω_n the system greatly amplifies the movement of the string bed (the fundamental mode of vibration of the racket is obtained) while for very high frequency values the racket is not able to transfer the oscillations (the modulus of the transfer function is canceled). From the following evaluation, it is clear that the different damping systems described above must have an appreciable effect at low frequencies (generally up to 200Hz) in order to reduce the transmission of energy to the athlete's arm. Figure 1.10 shows the shape of the oscillations and the position of the nodes associated with the first vibrational mode, the main cause of joint injuries. The knots are located in the center of the racquet head and at the top of the handle.

Furthermore, it is of primary importance to distinguish between the two possible border conditions: handle rigidly held still (stuck) or “free free” condition.

For rackets free to oscillate there are two nodes: one close to the center of the string bed and another in the low part of the handle. The anti-nodes, instead, are located at the top extremities of the head, at the lowest extremity of the handle and in proximity of the throat of the racket.

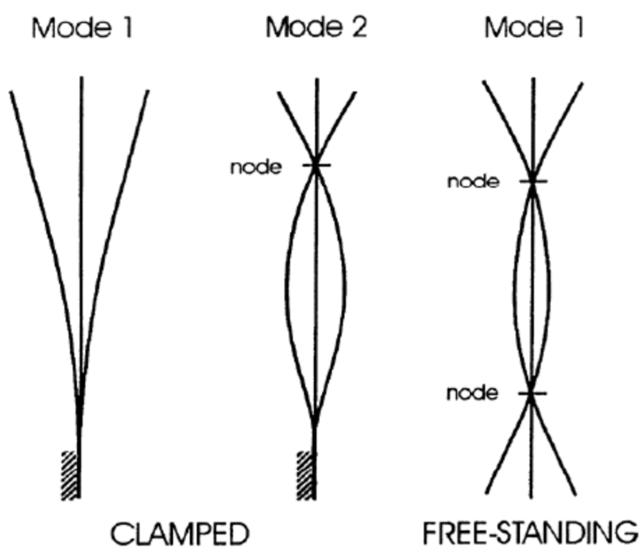


Fig. 1.10: Oscillations of a string plate for two boundary conditions. The first two images are associated with rackets constrained at the level of the neck, while the third is obtained by exciting the system which is left free to vibrate.

The racket presents its first oscillation node inside the range 25-40Hz. In this range, the vibrations occur only for racquets tied to the handle, while it is completely absent in hand-held racquets. This indicates that under playing conditions the grip force is not able to reproduce the condition of a constrained handle.

The first vibrational modes, for suspended tennis rackets, occur for frequencies between 100Hz and 200Hz: this is the superior frequency limit associated with undesired effects for the athlete. It is demonstrated that the energy transferred from the racket to the hand-arm system with such frequency constitutes the most evident contribution related to the appearance of lateral epicondylitis, and for this reason, this range is the focus of this study.

The second mode occurs at frequencies about three times those associated with the fundamental mode; therefore, the corresponding range goes from 300Hz up to a maximum of 550 / 600Hz. (Typically we consider the frequency of 373Hz as a reference).

Figure 1.11 shows the curves associated with the distribution of the knots on the string bed for the first (129 Hz) and the second (373Hz) fundamental mode. Both modes have nodes positioned at the handle of the racket so the vibrations associated with them will be perceived by the player if the corresponding frequencies are excited.

This observation justifies, in the experimental set ups presented in this paper, the choice of positioning accelerometers right at the handle level, in order to verify the effect of the dampers on the first two fundamental modes of oscillation.

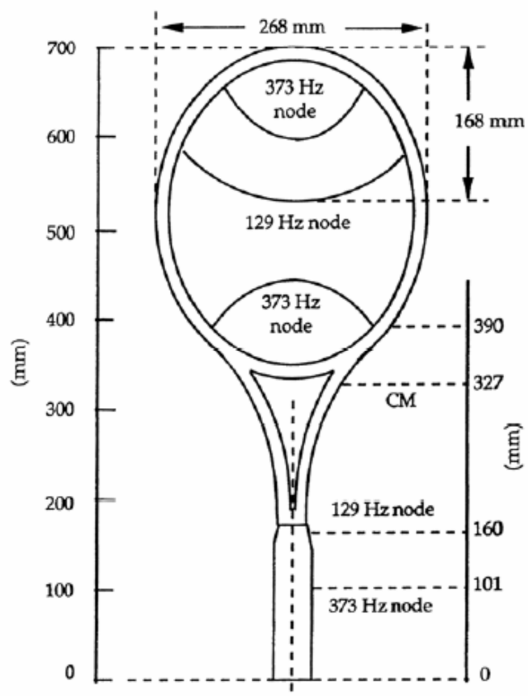


Fig. 1.11: Mean disposition of the nodal lines in a "head light" tennis racket. With 129 Hz is indicated the first vibrational mode, while with 373 Hz the second.

In the first experiment the excitation points on the string bed were selected to involve the areas in the passage of nodal lines, with the aim to simulate different impact conditions. In the other tests the control on the impact point was limited with respect to the first one, due to the more complex set up, for this reason it was decided to keep all the shots around the sweet spot.

The only (theoretical) condition for which the athlete can avoid the transmission of energy to the hand is to hit the ball in the exact point corresponding to the COP of the racket (Fig. 1.3).

However, this is an extremely localized area, so it is always necessary to consider the passage of a, albeit minimal, amount of vibrations to the arm of the player.

Following, there is a brief description of the handling effect on the dynamic behavior of the racket since the handle represents the region of direct shock transmission from the frame to the hand-arm system.

The grip force, also conditioned by the binding reaction of the handle that rotates with respect to the athlete's hand, is symmetrical and increases or decreases according to the different phases of the game.

The trend of the grip force over time (Fig. 1.12) is characterized by the presence of two local maximums. The first is given by the sum of two contributions: the player increases the grip to make the hand-racket system more rigid in preparation for the impact with the ball coming at high speed, plus the acceleration of the racket during the shot generates a binding reaction on the tennis player's hand. The second peak is the consequence of the athlete's attempt to regain control of the racket after the impact with the ball, which has caused a partial and temporary loss of control by the player. The instant associated with the zero of the abscissa axis coincides with the impact between the string bed and the ball, so the intensity of the grip force is represented in the very short time intervals preceding and following the impact.

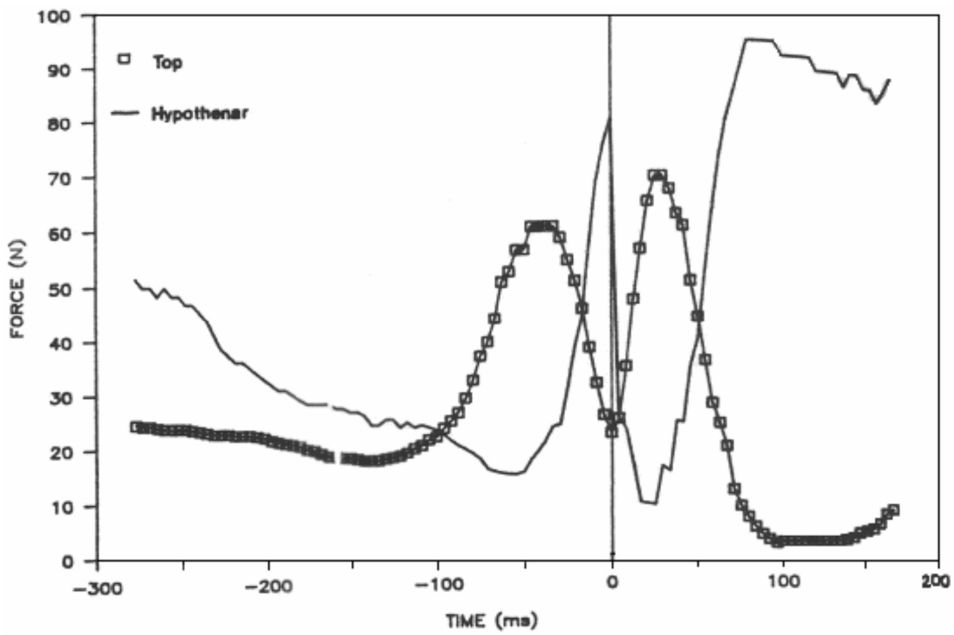


Fig. 1.12: Time course of the gripping force measured by two piezoelectric load cells. The first positioned at the level of the upper portion of the thumb, the second at the hypothenar eminence. In both cases the presence of two local maxima is evident.

The grip force (which generally does not exceed 100 N) must be quantified and analyzed in order to be related to the overall structural damping of the racket.

Once transmitted to the arm, the vibrating energy is absorbed by the soft tissues present in the forearm (tendons, muscles and ligaments) causing the main inflammations.

These are the main reasons that have determined over time the need to design devices (active or passive) that dampen the oscillations of the string bed directly in the vicinity of the impact point. Their reduced mass (5-10g) is not able, however, to absorb the energy associated with the first vibratory modes of the racket since from [c] it is clear that the addition of very small masses to the system does not lead to a reduction appreciable bandwidth (ω_n remains quite large since the mass is the denominator). So, it is intuitive to think that the dampeners only influence frequencies greater than 180 Hz, and therefore not directly related to joint injuries.

1.2.1 Time of contact between the ball and the string plate

The measurement of the contact time of the ball on the string bed following an impact is the main information to obtain the dynamics of the decay over time of the oscillations of the vibrating system.

First, it is necessary to define the concept of damping ratio ʐ which quantitatively represents the damping percentage of the system (c) with respect to the critical damping of the structure (2√ (k · m)):

$$ʐ = \frac{c}{2\sqrt{k \cdot m}} \qquad [d]$$

For tennis rackets ʐ assumes maximum values of 5% for the string bed and 50% for the frame; therefore, the combination of the two damping ratios has always resulted in an overall under-damped system (0 <ʐ <1). By solving the differential equation [a] for under-damped systems and considering an impulsive force as input, we obtain (fig.1.13):

$$x(t) = Ae^{-\omega_n ʐ t} \sin\left(\omega_n \sqrt{1 - ʐ^2}\, t\right) \qquad [e]$$

Where A is the constant of integration associated to the initial conditions of the string bed (at the time t=0 the deformation is also considered equal to zero).

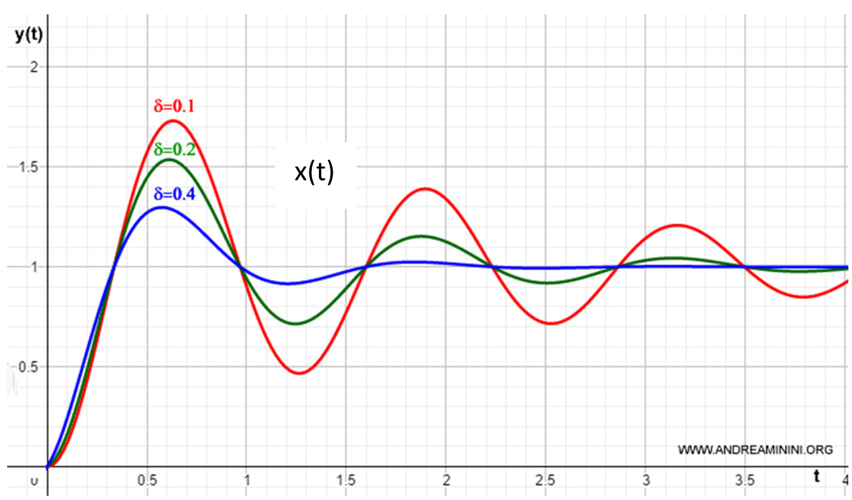


Fig. 1.13: Time domain response of a generic under-damped second order system to an impulsive input. Different curves are shown for different damping ratio values.

As the residence time of the ball increases, the frequency of the oscillations of the string bed decreases and consequently there is an increase in the time required to dampen the vibrational energy.

Generally, it is estimated that the dwell time of the ball on the strings is 5ms, but this can be increased or decreased according to the stringing tension, which alters the value of ʐ and therefore the damping that the system manifests against the vibrations generated, during the impact.

Considering that the frequency is defined as the reciprocal of the period $f = \frac{1}{T}$, a maximum frequency content of about 200Hz corresponds to a time of 5ms, so that vibrations with f> 200Hz are dampened before the ball leaves the string bed.

The waves travel from the point where the impact occurred towards the racket frame, and from there they are reflected back. If the frequency of the vibrations were lower than 200Hz, the reflected wave will reach the point of contact after the ball has left the impact surface and could not be dampened by its elastic retraction. If, on the other hand, the frequency was higher than 200Hz, the reflected wave would reach the point of impact before the ball moves away from the string bed causing a significant reduction in the energy content associated with these oscillations.

The different experimental approaches will now be described to trace the contact time of the ball on the string plate, all strongly correlated to the amplitude of the deformation of the same and to the frequency of the fundamental mode of vibration.

The first experimental set up involves the use of a high-resolution camera (3500 frames per second) which allows to measure the displacement of the string bed, following the impact with the ball, by evaluating the instantaneous position of a marker placed at the COP of the racket and integral with the oscillations of the central strings. The ball must travel at speeds between 15 m / s to 45 m / s in the various shots. The racket is constrained at the level of the head and this boundary condition has consequences on the location of the nodes and on the amplitude of the oscillations. The temporal trend of the string bed deflection is shown in fig. 1.14:

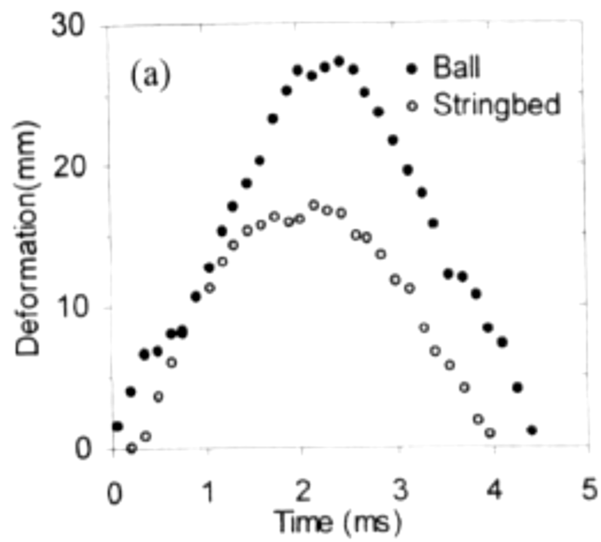


Fig.1.14: Temporal trend of the deformation of a string bed tensioned to 70 lb following the impact with a pressurized ball fired at 25 m/s.

The maximum deformation is about 18mm (which coincides with the initial amplitude of the oscillations of the string bed) and the contact time of about 4ms.

The second method aims to study the overall dynamic behavior of the string bed through the use of an LVDT differential transformer connected to one of the two central strings of the string bed. The ball, fired at about 15 m / s, generates the oscillation of the system (fig.1.15):

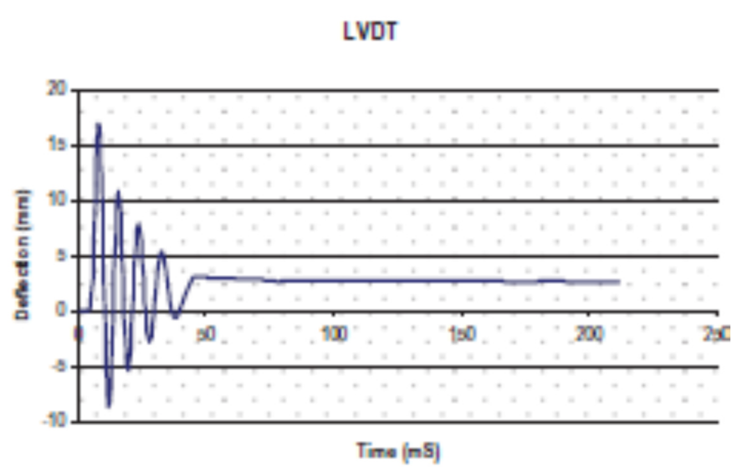


Fig.1.15: Decay over time of the oscillations of the string bed measured through an LVDT deflection sensor. The reported signal has already been demodulated and filtered.

By observing the first oscillation in more detail (fig.1.15) it is possible to estimate the contact time of this case (about 7/8ms). At the maximum displacement (about 12mm) the speed of the ball is zero and all the kinetic energy prior to the impact is transformed into elastic potential energy of the string bed and the ball. In the descending phase of the curve, the energy stored by the string bed is transformed into acceleration of the ball, determining its exit speed.

The third method of evaluation of contact time leveraged both a numerical and experimental approach. In the first case, modelling the string plate as a simple harmonic oscillator, the contact time of the ball exactly coincides with half of oscillation period of the system ($T = 2\pi\sqrt{\frac{m}{k}}$). The spring constant k is function of the tension of the strings, of their size, of the material and of the geometric characteristics of the racket. Considering that the ball has a mass of about 60gr and k≅ 3.5 x 104 N / m the contact time is about 4.5ms.

The experimental approach involves a set up with a laser, a photodetector and an oscilloscope arranged as in figure 1.16. The ball can be thrown perpendicular to the string bed or dropped freely.

When the ball passes along the laser beam, the detector does not record any signals. From the position of the laser with respect to the surface of the string bed, the light signal will pass only when the ball is in contact with the string bed. When the ball detaches from the impact surface, it intercepts the laser signal and blocks its propagation.

Although the measured time depends on many factors, such as the speed of impact or the tension of the strings, the average value, obtained as a result of repeated measurements in different experimental conditions, is around 5ms and this seems to confirm the values measured with the previous techniques.

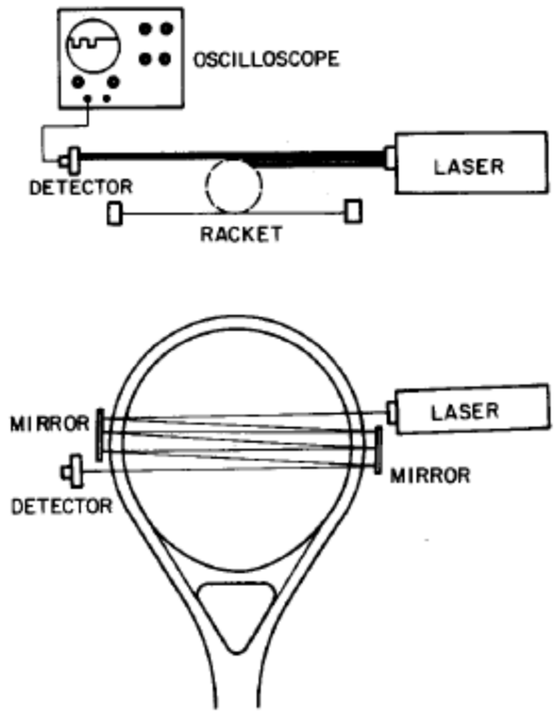


Fig. 1.16: Measuring bench used to quantify the residence time of the ball on the string bed. The arrangement of the laser-detector pair is such as to guarantee the passage of the light beam only when the ball is in contact with the impact surface

With the different measurement methods, it has been verified that the contact time assumes values between 4ms and 8ms and this interval corresponds in frequency to the range 125-250Hz. Therefore, the maximum vibrational energy is associated with harmonics with frequencies between 125Hz and 250Hz, which are also mainly responsible for joint injuries.

The purpose of this thesis is to measure the possible effect of the main damping and grip systems in this frequency range.

1.3 Consequences of vibrations transmission to the hand-arm system

Lateral epicondylitis, commonly known as "tennis elbow", is an injury caused by large and repeated impact forces transmitted to the arm through the player's grip of the racket. The transfer of shocks and vibrations occurs when the forearm muscles are tightly contracted, increasing the overall stiffness of the racket-arm system (the coefficient k of equation [a]). In the relation used to calculate the damping coefficient in a second order system (equation [d]), an increase of stiffness (hence an increase in k) at denominator, leads to a reduction in the damping (ʒ) of the system with the consequent increase in the amplitude of the resonance peak of the system (fig.1.17):

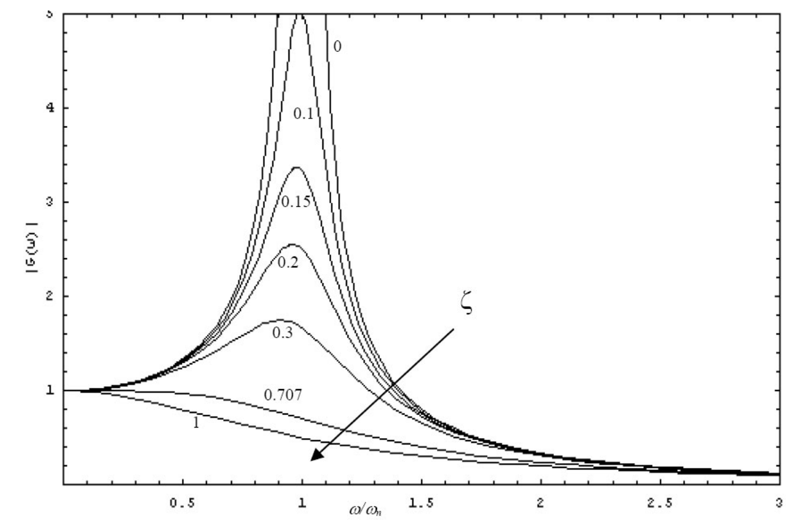


Fig. 1.17: Module of the FRF of a second order vibrating system as function of different values of ʒ (with 0 <ʒ <1). As the damping ratio decreases, the resonance peak amplitude increases

Therefore, the wrist extensor muscles in the forearm transfer a significant amount of energy to the origin of the tendon in the lateral epicondyle of the elbow in this situation (Fig. 1.18).

The absorption of shocks and vibrations by the tendons generates inflammation that presents the typical symptoms of epicondylitis, which compromise the athlete's performance and require treatment and rehabilitation.

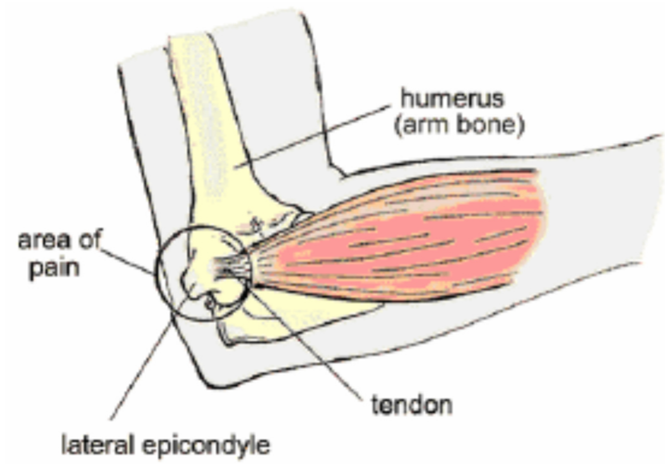


Fig. 1.18: Anatomical diagram of lateral epicondylitis. The traction of the muscle fibers of the wrist extensors (fleshy mass in red in the figure) represents a good mean for the transmission of vibrations (high stiffness k) towards the tendon that is inserted on the epicondyle of the humerus.

Some specific shots, such as the backhand, determine a greater degree of energy transfer to the player as the muscle contraction is maximum (to ensure greater impact power and therefore a large amount of kinetic energy transferred to the ball), and the system is at the minimum damping.

To limit the risk of injury to the upper limbs, it is necessary to design damping systems to be placed on the racket which, with their mass and compliance, absorb part of the impact energy.

However, past studies have experimentally assessed that the hand has the main effect on the dynamic response of the racket in terms of vibration damping: the more vigorous the grip, the greater the energy transferred to the player's hand and forearm due to the maximum muscle contraction and with it also the rigidity of the racket-arm system.

Three simple and reproducible experimental approaches will be introduced to estimate the effect of the dampers, to the “isolated” racket and in addition to the hand effect, on the transmissibility of vibrations to the athlete's wrist and elbow.

2. ANALYSIS OF VIBRATION SIGNALS

This chapter provides a brief description of the main mechanism for the analysis and interpretation of data associated with the generation, transmission and dampening of the vibrations through a structure, in this specific case the tennis racket.

The acquisition of data through accelerometers, which the sensors utilized in this work, is in the time domain, therefore the first data evaluation must be executed with the time variable. The parameters of interest are (fig. 2.1):

- Amplitude
  It's maximum measured value; it doesn't consider the temporal duration of the peak, for this reason it's not correlated to the event's energy.
- Root Mean Square (RMS)
  The root of the quadratic mean of the signal is directly proportional to the vibrational energy.
- Peak to peak value
  The maximum excursion of the measured acceleration; it's correlated to the maximum deformation at which the vibrating structure is subjected to.

The vibration can be interpreted as the oscillation of the structure of interest around a point of equilibrium, in the case of the tennis racket the string bed in the point of impact and the handle for the transmission to the arm in the following phase. The number of cycles that are completed in the time unit is the vibration frequency and it is measured in Hz.

The signals obtained by attaching the accelerometers to the racket frame are the result of the combination of more harmonics (or sinusoidal functions) at different frequencies. Some of these harmonics carry significant information about the vibrational energy transfer, while the other harmonics are mainly related to noise, which can be considerably reduced by the applications of filters at precise cut-off frequencies. In every case, the effect on the arm of the player of the vibration's transmission is related to the propagation frequencies.

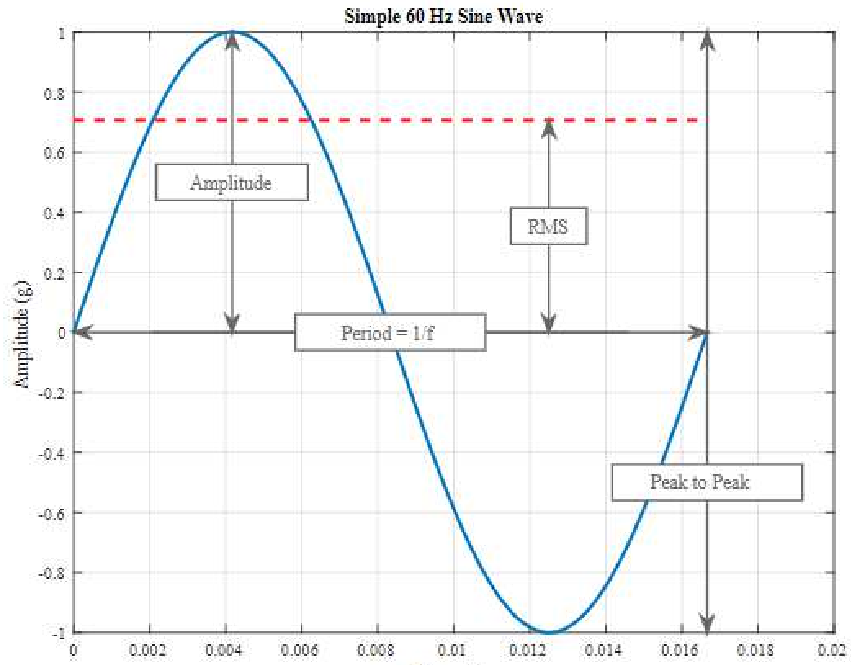


Fig. 2.1: Principal features of a sinusoidal signal, which is the typology of signal recorded by an accelerometer.

Now, a description of the principal spectral analysis techniques will follow.

2.1 Fast Fourier Transform algorithm (FFT algorithm)

Fourier analysis allows to trace the individual elementary components that constitute a signal, starting from the original waveform, which is considered as the sum of simple sinusoids with different periods. The result of this analysis is the amplitude of the acceleration (therefore of the vibration) as a function of the frequency, useful for better interpreting the measured acceleration profile. Fourier analysis "tests" the presence in the acquired signal of each frequency component. The “Fast Fourier Transform algorithm” allows to perform a fast, but at the same time efficient frequency composition analysis.

The number of discrete frequencies that are evaluated as contributions of a Fourier transform is directly proportional to the number of samples of the starting sample. Indicating N as original length of the signal, the number of bin (lines) in frequency is N/2 and their distance is exactly Δf, where Δf indicates the ratio between sampling frequencies (F_s) and the number of samples N ($\Delta f = \frac{F_S}{N}$). In this way, the higher evaluated frequency coincides with the Nyquist frequency ($F_s/2$).

$$F_s \geq 2F_{max}$$

Where F_{max} expresses the highest frequency of the signal, avoiding possible aliasing error (under sampling of the signal).

In the case of the tennis racket, furthermore, it's preferrable to normalize the acceleration at its maximum value to guarantee comparability of the recorded results.

2.2 Power Spectral Density (PSD)

The PSD or power spectrum is a representation of the signal's frequency components distribution, which presents lower complexity and so it is easier to interpret with respect to the Discrete Fourier Transform (DFT). It reports the contribution of each frequency component of a voltage signal ($P = V^2$IR) in proportion of the total signal power. From the DFT, it is obtained by averaging over the *n* samples the mean squared amplitude of each frequency component, in order of digitalization.

Assuming the PSD to be composed by k elements, each of these elements (PSD(k)) represent a measure of the signal power depending on the frequencies in a band *df*, centered at *k df.* An advantage of the PSD is related to the results of the analysis which are real numbers and are expressed as squared signal units per frequency units (e.g. V^2 Hz^{-1}, $mmHg^2$ Hz^{-1}), and so, they can be graphically represented in a single plot. A consequence of this analysis is the loss of information from the DFT, in particular the phase information.

A different perspective of the PSD is related to the signal variance frequency distribution. In fact, computing by the integral of the PSD it is possible to obtain the variance of the original signal.

In general, the FFT is the reference algorithm for the analysis of vibrations when there is a finite number of dominant harmonics. If there are many frequencies excited at the same time, the power spectral density of the signal (PSD) is used. The key aspect of a PSD that makes it more useful than an FFT for random vibration analysis is that the amplitude value associated with each frequency is normalized to the width of the bin, so that the vibration levels of signals of different length and with different sampling frequencies can be compared.

The majority of the signal's energy is associated with the resonant frequencies of the structure, always considering the racket as a system of the second order, whereby, from PSD analysis it is possible to evaluate the spectral components mainly responsible for the transmission of vibrations to the hand-arm system.

The name of this algorithm is self-explanatory about the information that can be obtained from its application:

- Power: the amplitude of this measurement is the root mean square value of the analyzed signal (RMS):

$$\mathrm{PSD} = \sqrt{\frac{1}{T}\int_0^T a(t)^2 dt}$$

  Where with T we indicate the sampling period, while with a (t) the measured acceleration.
- Spectral: PSD is a function of frequency (measured in Hz).
- Density: the width of the PSD is normalized over a single bandwidth.

The following figure (Fig. 2.2) shows an example of PSD obtained by measuring the vibrations generated as a result of the impact of the ball on the string bed.

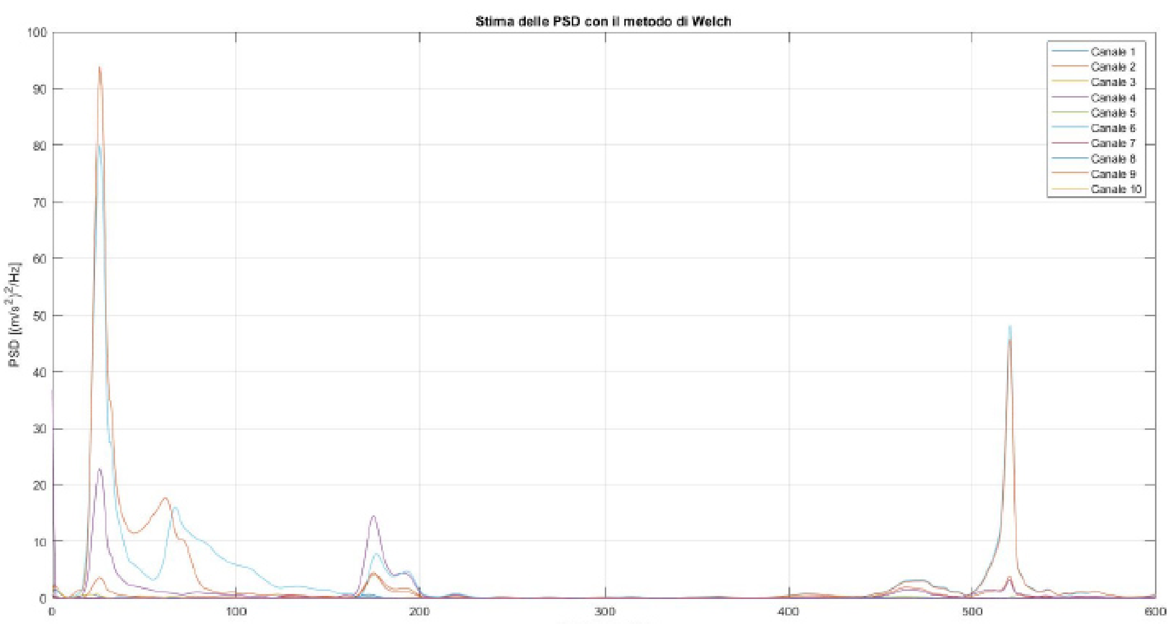


Fig. 2.2: PSD without damping devices, with ball hitting the central region of the racket.

2.3 Computation of reverberation time (t_{60})

The official definition (UNI EN ISO 354:2003) is related to the acoustic phenomenon and is the following: "Time needed to reduce the density of acoustic energy of 60dB from the instant in which the stationary source is switched off".

Such a method can be applied also in dynamics, in fact, in a system of the second order (as we consider the racket) the vibrations show a comparable decay over the time after the initial excitation. In the case of the $T_{60,}$ it must be interpreted as an indicator of the speed of extinction of the vibrations over the time, after removing the source of the impulse. It is a variable strongly related to the damping of the structure, and so, the presence of different types of dampeners and the effect of the handling plays a major role on the value of this variable. A low t_{60} indicates an elevated dampening associated with a fast and efficient oscillation decay.

The reverberation time is calculated through the application of a broad band excitation signal (impulse), to be able to excite most of the resonance frequencies. Starting from a constant equivalent level and after removing the source, the curve of the decay can be approximated to a straight line through the linear interpolation of the obtained points. The T_{60} can be obtained by measuring the time necessary to the signal level to decay of 60dB, taking into account an initial and final discard of 5dB (Fig. 2.3).

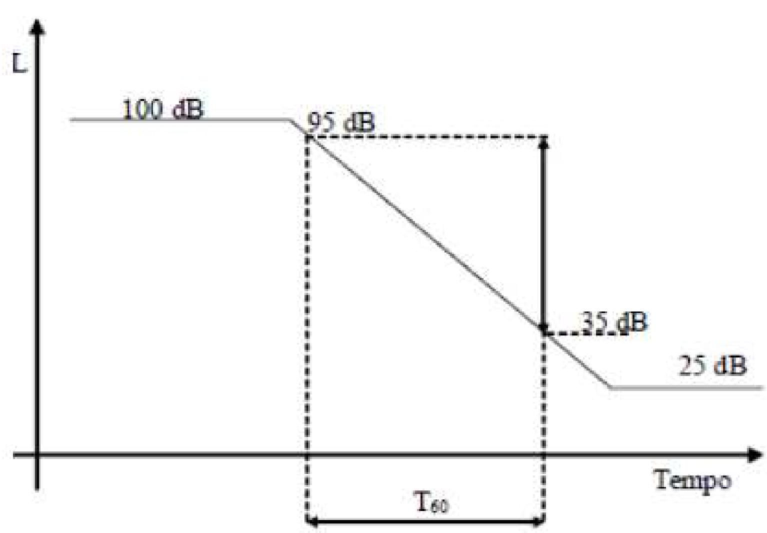


Fig. 2.3: Definition and graphic interpretation of the t60 with linear decay curve.

In the case in which the total decay is shorter than the 60dB, it is possible to consider variables such as t_5, t_{10}, t_{20}. These variables are computed with the same principle of the t_{60} since the trend is linear and coincident, so, applying the opportune proportions it is possible to quantify the reverberation as defined by the normative.

Despite its value this variable was not computed because this work was focused mainly on the transmission of the energy. Future analysis could include the t_{60} computation to complete a more comprehensive evaluation.

3. METHODS

Three different experimental tests were conducted to fully evaluate the behaviour of the tennis racket and the efficiency of the damping systems.

The first experiment, the laboratory test was designed in almost ideal conditions (impulsive input forces, racket free to vibrate) to allow a wide and comprehensive analysis of the tool and of the damping systems. However, these conditions were very far from reality and the result from this trial must be weighted.

The second experiment, racket handling test was designed closer to real conditions by including the handling of the tennis racket by a subject and an excitation with the tennis ball launched through a shooting system.

The third trial, operative condition test was proposed to simulate, as much as possible, real playing conditions, providing almost free movement to a subject, hitting balls launched at a limited speed.

The experimental methods that will be described below are presented as simple and repeatable approaches to measure the vibrational energy in different conditions and points of the racket following the solicitation of the string bed.

The results of these tests were used to describe and evaluate the effect of the different damping systems on the propagation of the oscillations up to the handle of the racket and then to the athlete's hand. The results obtained were statistically analysed and compared with similar data and information present in literature, in order to carry out an exhaustive evaluation of the obtained parameters while, at the same time, confirming the validity of the illustrated methods.

A single tennis racket was used in this work, an “head light” racket, the most widespread and studied type. This is a limitation to our study that, however, leaves space to further analysis.

For this study, 4 different dampers were selected, one for each typology and a fifth system that included the contemporary application of two rubber pads, to see if increasing the mass also increases the damping effect (Table 1).

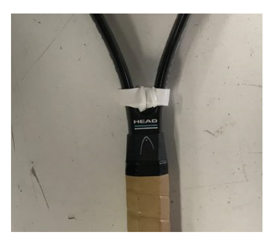
Damper	Typology	Material	Weight (g)	Dimensions (cm)	Application point	Image
Damper 1	String anti-vibratio (rubber pad)	Sintetic rubber	1-2	2 x 2 x 0.5	string bed	
Damper 2	String anti-vibratio (double rubber pads)	Sintetic rubber	3-4	2 x 2 x 0.5	string bed	
Damper 3	Granular-Hybrid	Sintetic rubber	5	6 x 1 x 0.5	string bed	
Damper 4	Elastic band	Sintetic rubber	3	10 x 0.5 x 0.2	Frame or string bed	
Damper 5	Fluendo	Siliconic rubber	15	7 x 4 x 1.5	frame	

Table 1: list of all the dampers used, their characteristics and an image.

3.1 Equipment and Programs used

3.1.1 Equipment

Instrumented Hammer

To evaluate the dynamic of the racket after an impact, it was necessary to apply a load able to simulate the effect of the ball. The impact of the ball can be described as a force that acts for a very short time, of the order of milliseconds, and on an extended area. To simulate such phenomena, in ideal "free free" racket's condition (first experiment), the instrumented hammer was selected as a stimulation tool.

This instrument has integrated load cells on its tip, which allow it to measure the intensity and dynamic characteristics of the force supplied to the structure. The canonical input that can be generated can be mathematically represented by a Dirac delta δ (t) (Fig.3.5B). This has a linear spectral distribution energy of up to about 200Hz, which allowed to excite frequencies up to about 500Hz albeit with an evident reduction in energy.

The amplitude of the excited band is function of the duration of the impulse: shorter duration corresponds to higher excitation frequencies. The control on this factor is mainly related to the typology of the tip that is used. In particular, increasing the tip rigidity, the deformation derived from the impact and the contact time are of lower entity. Since in our work a support made of hard plastic was used to obtain the max relative rigidity between the impacting surfaces, a metallic head was chosen.

Despite, the amplitude values of the applied forces are not comparable with the normal game conditions, a general evaluation of the tennis racket behavior and of the effect damping systems was possible, by making opportune considerations.

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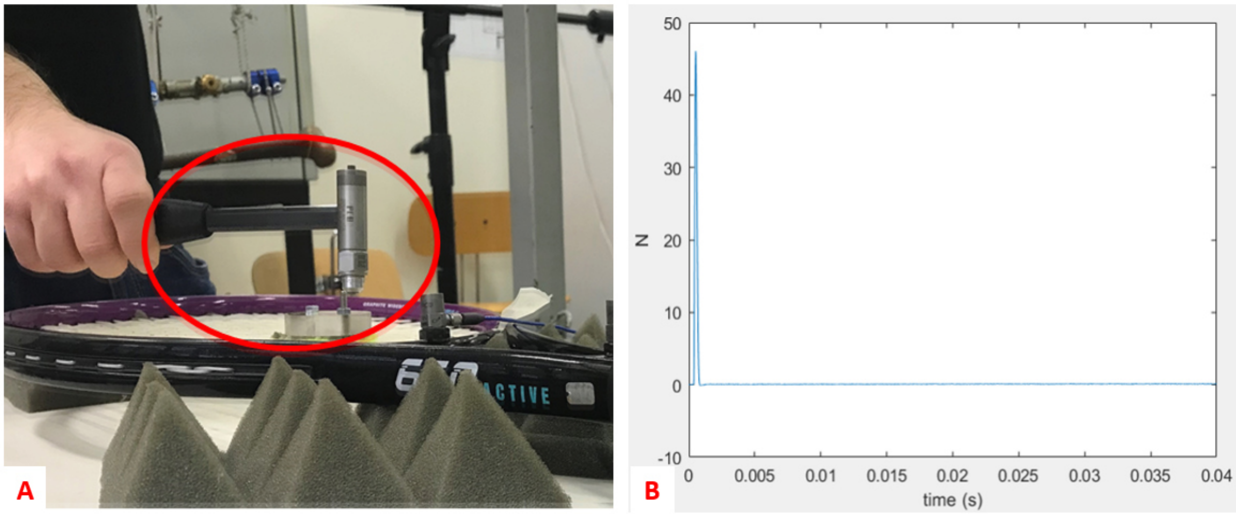


Figure. 3.1: Instrumented hammer and generated impulse. Image A shows the instrument during its use; the impact occurs in a direction perpendicular to the plane on which the structure to be excited is positioned. Image B shows the signal recorded by the load cell positioned on the tip of the instrument in the time domain; its time duration is of the order of 1ms.

ACCELEROMETERS and FIXING SYSTEMS

The measure of the vibrations was executed by three monoaxial piezoelectric accelerometers, schematically represented in Fig. 3.2. From the study of the literature, it was evidenced that triaxial accelerometers weren't needed, since most vibrational phenomena happened on one axis, the motion axis (orthogonal to the string bed).

The oscillations of the structure determine consecutive compressions of the piezoelectric crystals present in the base of the sensors, in contact with the vibrant structure. These compressions generate electrostatic charge loads on the two faces of the quartz crystals, that through an integrated circuit are acquired as tensions. Considering the sensibility of the sensors, the signals acquired in Volt, are transduced in m/s^2. Finally, the Fourier analysis allowed to pass from the time domain to the frequency domain and consequently to obtain the information of interest.

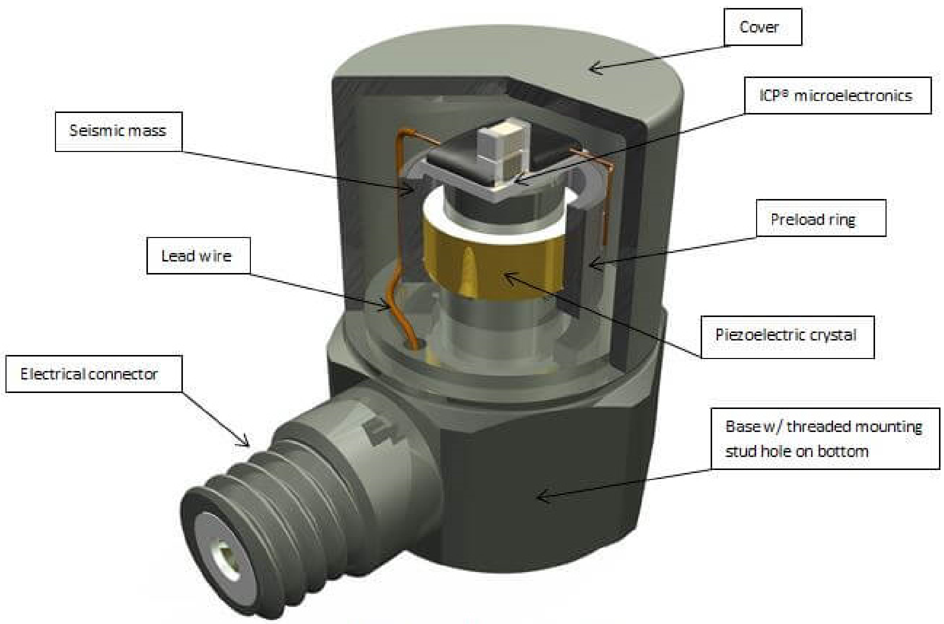


Fig. 3.2: Schematic representation of an accelerometer with view on the internal components

This type of sensor is connected to a motherboard through cables and needs to be fastened solidly to the vibrant structure, but still guaranteeing freedom of vibration. In the first experiment the fixing was easy, since the racket is simply resting on foam rubber supports. It was achieved by fixing the cables and with the application of a thin layer of beeswax on the accelerometers-racket interface. The beeswax is very sticky and allowed to firmly connect the sensor to the structure without impacting on the transmission of vibrations.

The second and third trials required a more elaborated fixture of the sensors since the racket was held in hand by the subject and the set up included the placement of one accelerometer on the wrist and one on the elbow.

The second experiment consisted in simply holding the racket suspended and still in front of the cannon, while the third experiment included the possibility to move the racket freely. Most attention was paid to guaranteeing adhesion of the accelerometers to the racket frame and to the subject’s arm. The beeswax was sufficiently sticky to maintain the sensor firmly attached to the racket frame even when the racket is in motion. However, it was not adhesive enough to guarantee the placement of the accelerometers on the subject’s arm.

To accomplish this task a set of particular elastic bands were employed. These bands were made of a silicon polymer, which provides great elasticity and resistance to mechanical stresses. A hole was applied on the band to insert the accelerometers in between. Once ready, it was sufficient to wear the elastic band like a bracelet. The bands were fastened very tightly around the wrist and the elbow of the subject, ensuring freedom to vibrate and guaranteeing the maintenance of the positioning by the accelerometers (Fig. 3.3).

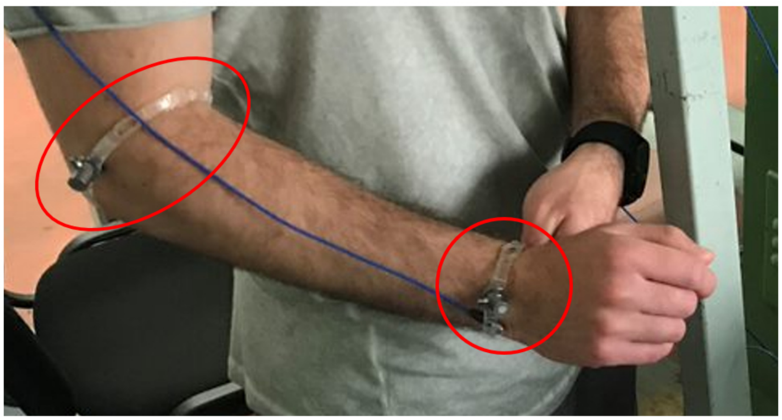


Fig 3.3: Application of the accelerometers on the arm of the subject with elastic bands for test 2 and 3.

Tennis ball cannon

The second experiment required a shooting system for the tennis balls, which had to be able to maintain the same speed and direction of the ball, over multiple launches.

For this reason, a tennis ball's cannon was designed, realized and employed. The cannon was designed to exploit the elastic potential energy of an elastic string, which was anchored to the top of a PVC tube. The PVC tube, that constituted the body of the cannon, was 1m long and had a diameter of 75mm. Two engraves were applied on both sides of the tube's bottom to provide a path for the string to correctly slide during the loading and shooting phases.

To enable the elastic string to correctly slide and to properly remain in contact with the ball, a loading device was realized. This device, of approximately 20 grams, was composed of two cylinders of different diameters, attached to one another and directly connected to the elastic string. The difference in the diameters create a cup shape, which guaranteed the correct resting position of the tennis ball inside the cannon during the loading phase and a correct push of the ball during the shooting phase.

By orienting upwards the defined cannon, it was possible to simplify the loading process. In fact, for inclinations higher than 10° it was sufficient to insert, from the top of the tube, the ball, which fell into the cup-shaped device only by the action of gravity. In this way, gravity also guaranteed that the ball and the launching system remained in contact during the push and so allowing to produce "clean shoots".

After the first stage of calibration, a tube of 30cm and 2cm of engraves were added to the previous configuration to increase the shooting power. The final shooting speed was assessed around 22km/h with a medium variation of maximum ±2-3 km/h, with a final orientation for the experiment set at 20°.

The final result and the set up in the second experiment are shown in the figure 3.4.

Fig 3.4: Lateral view of the realized cannon with an upward inclination and an elastic string to fix it to a base.

3.1.2 PROGRAMS

LABVIEW

All the data produced by the accelerometers and the instrumented hammer were acquired through a motherboard that sends the data to a self-written and predefined LABVIEW program.

The LABVIEW program that was used allowed to record and immediately show, in different windows, the measured signal and their FFT.

In the first experiment, the program was triggered by the signal coming from the instrumented hammer, when this last overtook a predefined threshold.

In the second and third experiments the hammer was not present, for this reason the trigger was associated to the accelerometer attached to the racket's frame, and so the one closer to the point of excitation. The threshold was defined at different values for the two trials, being the solicitation in the last much higher with respect to the previous one.

So, after a calibration phase to set the best values for the threshold and for the acquisition of the data, the program was ready. This program allowed not only the acquisition of the data for further numerical analysis, performed in the MATLAB environment, but also an immediate evaluation of the tests and therefore enabling us to decide if to discard or accept the data obtained from the test. This functionality was of particular importance in all three experiments, but especially in the first trial where the number of discarded tests were much higher than the number of accepted tests, due to the great difficulty in obtaining a "clean hit" with the instrumented hammer.

MATLAB

From the LABVIEW program, we were able to extract the values of tension measured by the accelerometers. To analyze these variables and extract the main information, we employed a self-written MATLAB program, which included few but essential functions.

The main purpose of the code was to perform the FFT and extract the PSD to obtain the energy distribution in the domain of frequency. From the PSD, we defined two different variables to evaluate. The first was the energy contained inside the 200 Hz band (for all the experiments) and 500Hz band (only for the first experiment). The second variable to extract was the energy contained in bands of 25 Hz, starting from the 25-50 Hz to the 225-250 Hz band, in order to define the vibrational behavior of the tennis racket.

3.2 TEST 1: laboratory test

This experiment represented the first step of our analysis. The conditions implemented are very far from the real ones, however they were meant to allow a first fast and easy to apply measuring bench, to any typology of racket.

3.2.1 Experimental set up of test 1

In this experiment, it was tried to reduce as much as possible the instrumentation applied to the racket, to analyze the intrinsic behavior of the tool. In fact, it was considered that no static constraint can be representative of the various playing conditions, due to the direction, orientation and intensity of the grip force, which changes continuously during the different phases of the game, corresponding also in the variation of the resonant frequencies and the way of vibrating of the different points of the racket. For this reason, in this experiment the racket was left free to vibrate without any type of constraint ("free free" measurement conditions). In order to isolate the racket from the rigid support on which it rested for the execution of the tests; a system of supports, made of foam rubber wedges, was used (Fig. 3.5). This support system has a minimum impact on the vibrations, guaranteeing low damping effect and good propagation of the oscillations up to the points of application of the accelerometers (Fig. 3.7). The input force was produced using an instrumented hammer which allowed to precisely locate the points of application and quantify the transmitted impulse. With this technique, it was possible to excite with an almost flat band until 500Hz.

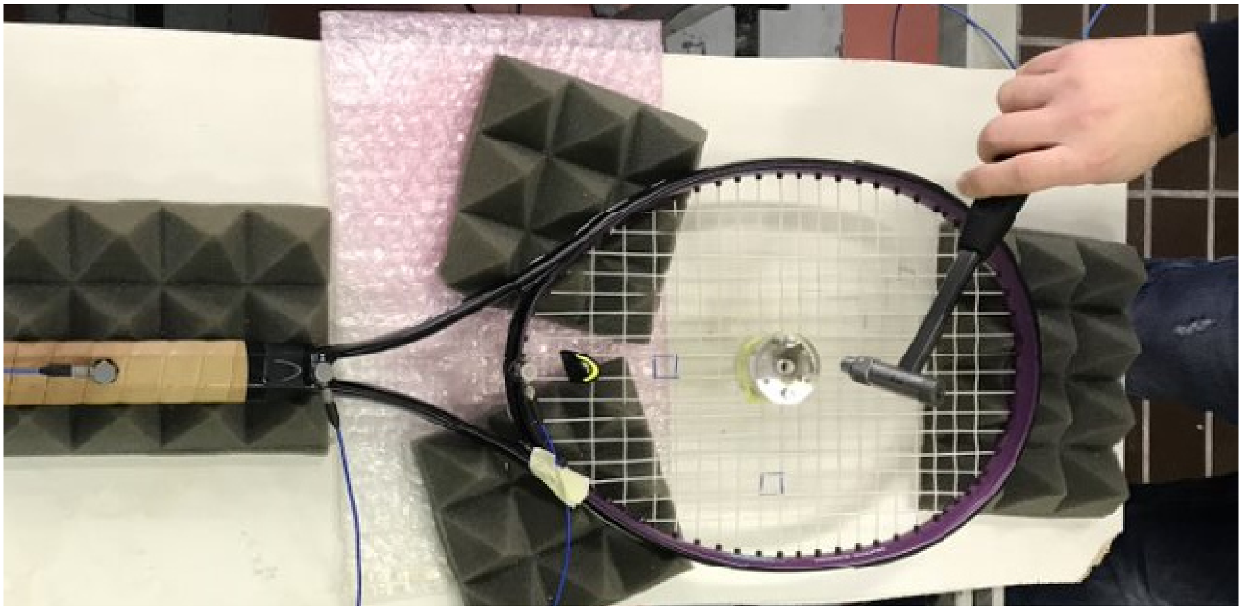


Fig. 3.5: Measurement set up with "free free" method. In this image it's possible appreciate the entire set up of measures with the racket suspended above the foam supports, the instrumented hammer, the impact device fixed in a central position. Furthermore, the sensors attached to the racket in the selected positions are clearly visible together with the excitation points (blue squares) and the damper applied to the string bed.

The excited modes that have significant energy are at a frequency lower than 300Hz with contributions reaching up to 500Hz, for this reason the parameters that were obtained neglect all the parts of signals at frequencies higher than 500Hz. Furthermore, it should be considered that if the applied force is punctual, any impact at the node would not excite the vibrations of the fundamental mode. Therefore, an impact support system was adopted to guarantee the distribution of the load on a surface. This system was composed by a couple of thin and rigid plastic plates interspersed by sections of the tennis ball and tightened with two screws that orthogonally passed the string bed. This system allowed to extend the impact region to an area of the same size as the typical impact area of the string plate with a common tennis ball.

This structure made the entity (intensity and frequency) of the vibrations of the string bed independent of the point where the support is struck with the instrumented hammer. Furthermore, it should be noted that if the head had been hit directly with the tip of the hammer, since this is more rigid than the outer wall of the ball, there would have been a considerable variation in the bandwidth of the system (increase in stiffness of the hammer-string system means an increase in the resonant frequency of the structure). Lastly, a punctual shot, in addition to not representing the real ball-racket impact, would only excite one or at most two strings (if the shot occurred in an intersection), causing a vibration of the system that is too far from the real playing condition to be considered as a case study.

Fig. 3.6: Support used to simulate the impact of the ball on an extended surface with.

A test of this type is the most suitable method for simulating the inertial and vibrational properties of the racket for an impact with the ball.

This approach, for the transfer of impulsive forces to the string bed, allowed to neglect the role of the materials on the interpretation of the frequency spectrum of the measurement signals.

For the selection of impact points on the string plate, five reference areas were individuated to simulate impacts in real playing conditions.

- Point A: central area of the string bed, associated with the fundamental vibrational mode. This area is the most used and consequently solicited during the game.
- Point B: top area of the string bed, exploited during the service. This area is linked with the second resonance frequency of the system.
- Point C: lower area of the string bed with maximum COR.
- Point D: lateral area of the string plate. This area is unrelated to particular shots but can be frequently associated with inaccurate strokes by the athletes.

- Point E: top area of the frame. This area was selected to further evaluate the vibrational response of the racket when solicited outside of the string bed.

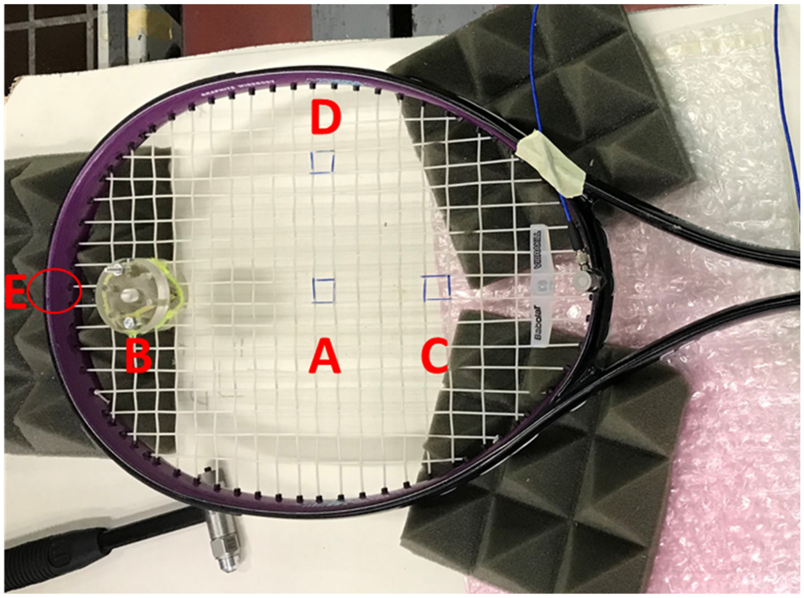


Fig. 3.7: Solicitation points. Distance from the closest edge to the frame: DT = 5cm, AT = 12.5cm, BT = 7.5cm, CT = 10cm (T indicates the internal edge of the frame).

To measure the vibrations, three monoaxial piezoelectric accelerometers were positioned as follows: (fig. 3.4 A):

- Sensor 1: placed at the interlocking point of the strings in the frame. The vibrations measured here have maximum amplitude, as this area is closer to the stimulation point.

- Sensor 2: located at the throat of the racket. This is where the connection between the outer frame and the handle of the racket takes place. The analysis of vibrations in this area could be essential for the design of dampers that could dissipate the vibrational energy just before it is transferred to the player's hand.

- Sensor 3: located at the level of the handle. Here the oscillations that reach the hand-arm system are measured after having propagated through the various structures of the racket.

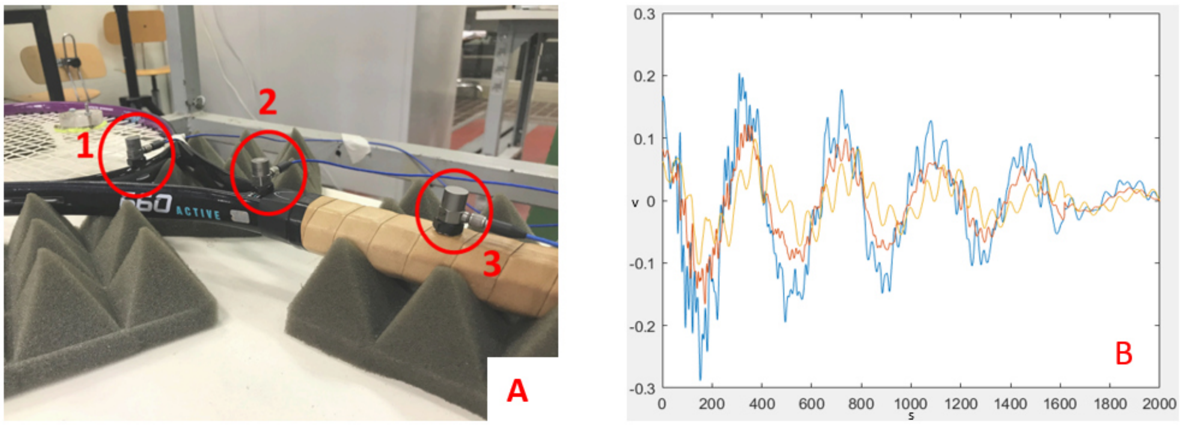
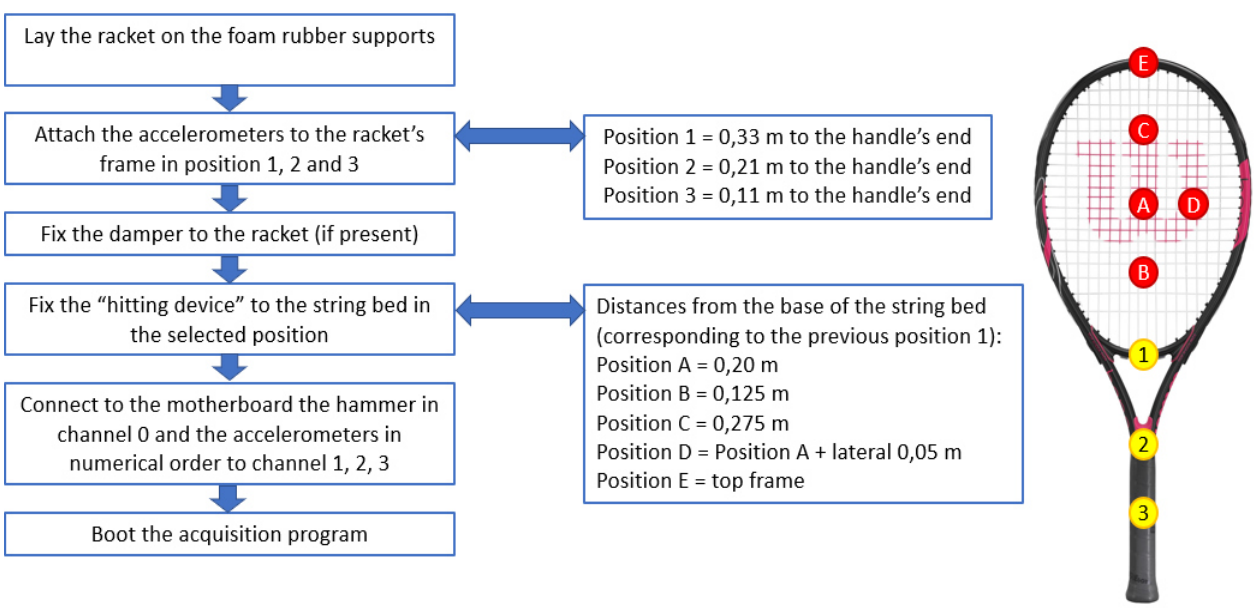


Fig. 3.8: A- sensor arrangement; B- Acceleration trend, measured in volts, over the samples, sensed by the 3 sensors for a shot in the lateral position. In blue the accelerometer in position 1, in red the one in position 2 while the orange is associated with the accelerometer in position 3. Moving away from the point of impact, the structural damping imposed by the racket reduces the vibrational energy transferred is visible.

The final set up of the laboratory test is summarized in the following block diagram (scheme 1).



Scheme 1: Block diagram of the test bench for the laboratory test with indicated the positions of solicitation and the points of application of the accelerometers.

3.2.2 Working protocol of test 1

The following procedure was adopted as working protocol for this experiment:

1- Set the racket free to vibrate on the foam rubber support.
2- Fix the hitting device and the selected damper (if needed) on the string bed.
3- Position the accelerometers in the predefined locations.
4- Boot the LabView acquisition program.
5- Hit with the instrumented hammer's tip the plastic device on the string bed.
6- If the shot was not "clean", repeat the hit

The instrumented hammer is very sensible and not of easy utilization. The hits had to be done with a particular technique and with very rapid movements, in particular, in the retraction of the hammer tip from the surface of contact. Otherwise, the sensors detect secondary touches which generate a lot of noise, making the signal unreadable and requiring the results to be discarded. In particular, the shoots were not considered "clean" if they presented more than one clear peak or an insufficient input force (at least 5N). In all these cases, the signals were discarded and the test repeated.

The test was repeated ten times in every position and for every typology of dampers for a total of 420 tests, excluding all the discarded trials. In this way, it was possible to statistically analyzed and compare all the proposed configurations.

Except for the difficulty in obtaining a clean shot with the hammer, the experiment resulted fast, easy to reproduce and able to provide meaningful information, which was the ultimate goal of this test.

3.3 TEST 2: racket handling test

The second experiment design aimed to reproduce a real tennis ball-racket impact in a controlled environment. To achieve this result a particular set up was proposed and realized.

3.3.1 Experimental set up test 2

In this experiment, the solicitation of the racket was produced by the tennis ball cannon previously described. The shooting device was fixed inside a “cage-like structure”, where it was tight to a base with an upward angle of 20°. The cannon was aimed to the centre of the string plate, with the racket held orthogonally with respect to the ball’s trajectory. By fixing the cannon it was ensured that the trajectory of the ball remained constant.

The cage structure helped to solve the issue of holding in hand the racket by maintaining it always in the exact same position. In particular, a thin nylon thread was tied around the racket’s frame, on the lateral side of the string bed, to the external structure. This tread allowed to maintain and eventually recover the correct position and, due to its limited mass and dimensions, its impact on the racket’s behaviour was neglected.

During the test, the subject was invited to hold firmly but without excessive force the handle of the racket with his right hand, without perturbing the previous configuration. In this way, it was possible to have the racket with the correct position and orientation for the entire duration of the test.

The accelerometers were applied with great attention to strategic points selected from the study of the literature. In particular, accelerometer 1 was attached with the application of a thin layer of beeswax to the racket a little over the handle (Fig. 3.5.A). Accelerometer 2 was applied to the wrist in the protruding part of the radius, while accelerometer 3 was applied to the lateral epicondyle on the elbow. Accelerometers 2 and 3 were fixed to the arm with elastic bands while their cables were attached with adhesive tape around the subject shoulders, ending on the forearm (Fig. 3.5.B)

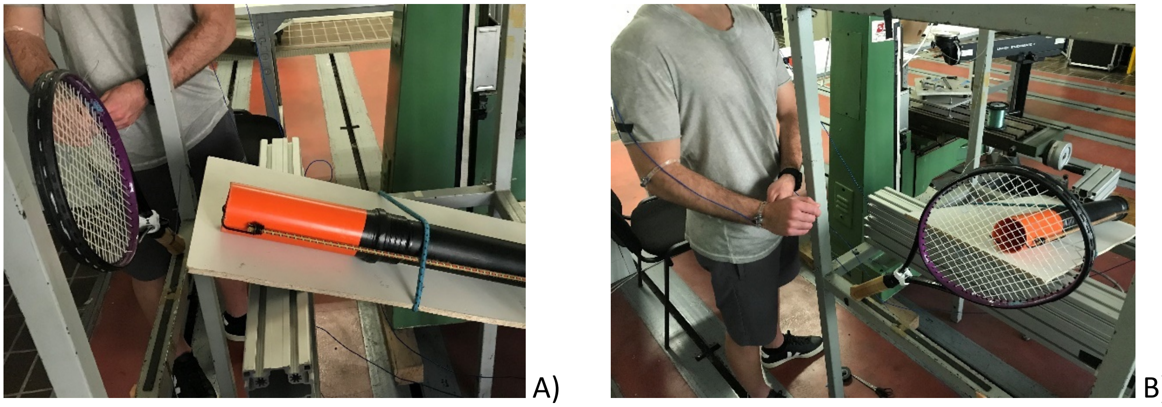
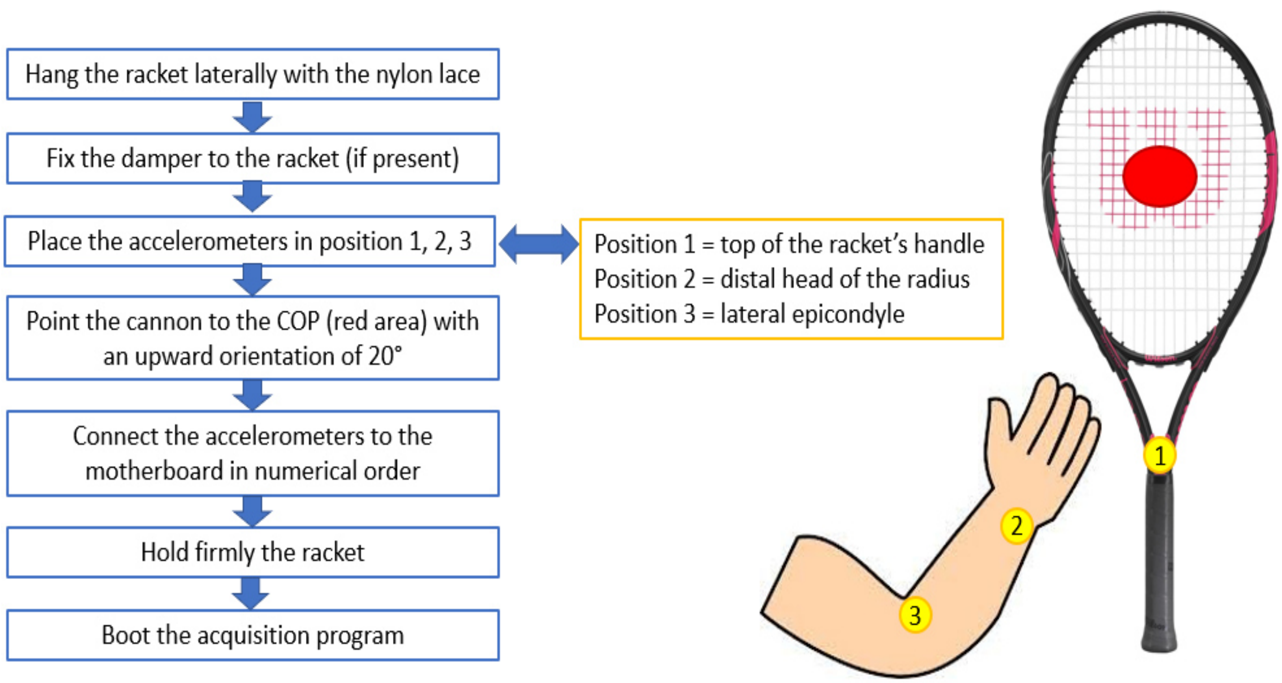


Fig. 3.9: two points of views of the experiment 2 set up. fig A, shows the upward orientation of the cannon and the position of accelerometer 1. In fig B, it's possible to see the position of the accelerometers 2 and 3 on the arm of the subject. In both images it can be noticed the little impact of the nylon thread, barely visible. In this test the Fluendo damper were applied to the racket.

Scheme 2 reports the summary of the experimental set up adopted during test 2.



Scheme 2: Block diagram of the test bench for the racket handling test with shown and indicated the target hitting area and the points of application of the accelerometers.

3.3.2 Working protocol of test 2

The working protocol in this test was of very easy application once the initial set up was completed. The process is very similar to the one of the previous test. The only difference is related to the presence of the cannon instead of the instrumented hammer.

More specifically, once the initial configuration was completed, it was sufficient to load the cannon by simply inserting the ball from the top and booting the LabView acquisition program.

The shooting procedure was easy to perform and to reproduce. The cup base of the cannon which held the ball, was attached to a string. By pulling this string always at the same distance, it was possible to obtain the same extension of the elastic strings linked to the cup base. In this way, the shooting velocity of the ball remained almost constant in all the tests that were performed at around 22Km/h.

Considering the set up and the results obtained from the previous test, it was decided to focus the study only on the centre of the string plate, which was where the cannon pointed to.

The only considered handle was the one to perform a straight shoot.

As in the previous test, it was decided to perform the test ten times for each applied damping system and without any, for a total of 60 trials.

3.4 TEST 3: operative condition test

Test 3 was thought to analyse the amount of vibrations transmitted to the arm of the tennis player and the efficiency of the damping systems in almost real playing conditions.

3.4.1 Experimental set up of test 3

The set up of this test was particularly simple. In fact, the only difficulty was the necessity to provide to the subject handling the racket the possibility to move freely.

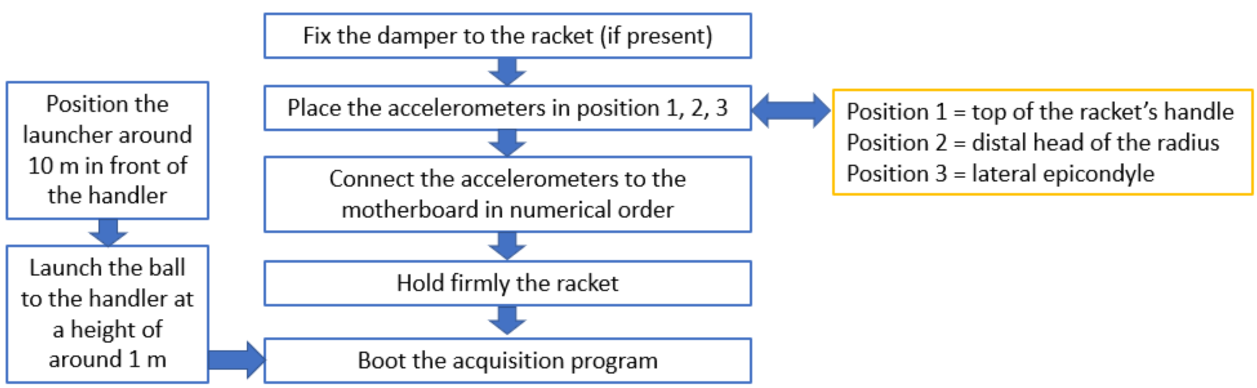
The disposition and the fixing of the accelerometers was the exact same of the previous test. So, accelerometer 1 a little over the handle, accelerometer 2 on the wrist and accelerometer 3 on the elbow, in the same anatomical points. Accelerometer 1 was attached with the beeswax, which demonstrated ability to maintain the sensor attached to the racket frame even with fast movements and impacts of great entity.

Particular attention in this test was paid to the fixing of the accelerometer's cables. In fact, to avoid troubling movements, they were strictly attached with adhesive tape to the arm and to the back of the subject. In this way, it was possible to guarantee to the subject the capacity to move with complete freedom the upper part of the body and allow him to perform a correct and complete swing of the arm.

The only limitation was the short length of the accelerometer's cables that required to maintain the motherboard and the computer in proximity to the subject. Despite this restriction, the subject was able to do a few shots, which were sufficient to adjust his position for the incoming balls.

The balls were manually thrown at approximate velocity that ranged between 20 km/h and 40 km/h, from a distance of 8 m up to 12 m.

In scheme 3, the experimental set up of the operative condition test is summarized.



Scheme 3: Block diagram of the test bench for the operative condition test with indicated the points of application of the accelerometers.

3.4.2 Working protocol of test 3

The procedure of test 3 also was very easy. In fact, once the accelerometers were placed in position, it was sufficient to boot the LabView acquisition program and throw the ball.

It was requested of the subject to hold firmly but without excessive force the racket and to perform straight shots (frontal swing of the arm and a slight rotation of the torso), hitting the ball with the centre of the string plate. Furthermore, the subject was invited to hit the ball always in the same way and with the same velocity. Low power responses were considered representative of the general phenomena.

As before, ten tests were reproduced for each applied damping system and without any, for a total of 60 trials.

Fig. 3.10: Example of one shot, where the wide and progressive swing movement of the arm is visible.

3.5 DATA ANALYSIS

To follow, there is the description of the programs developed for the analysis of the acceleration's data, obtained in the time domain and then converted into the frequency domain.

All the signals were measured with a sampling frequency of 50000Hz in the first test for 0.04s (2000 samples), while in the second and third test for a duration of 0.2s (10000 sample). These time durations, although limited, were sufficient to cover the entire phenomenon. In fact, the energy decay was very quick, in particular, for the first test, which was performed with very limited input forces.

The signals were recorded by a capture motherboard at four channels. In the first experiment, the first channel was used for the instrumented hammer and the other three were associated with the accelerometers. In the second and third experiments the first channel was unused, in order to maintain the previous program's design.

The first pre-processing step included the application of two third order Butterworth filters; one high-pass filter with cut-off frequency of 5Hz and one low-pass filter with cut-off imposed at 550Hz. Together these two filters produce a passing band, which broadly includes the band of interest for the harmful vibrational phenomena of the tennis racket.

In the following phase, it was executed the FFT of the signal to identify the frequencies associated with the fundamental modes of the vibrating system. At this point, with the signal in the frequency domain, the PSD was computed to obtain the energy of the signal normalized for the considered frequency range.

The values of the acceleration were recorded by the sensor in volts. The conversion from volt to a direct measure of acceleration was not performed given that, for the purpose of the study the absolute values of acceleration did not carry significant information.

3.5.1 Data analysis of laboratory test

In the first experiment, the higher frequency which we were able to stimulate with the instrumented hammer was of around 500Hz. Bands with higher frequencies would not carry significant information. For this reason, we decided to select two different bands, the first from 0 to 200Hz and the second from 0 to 500Hz. The first low-frequency band was the core of our study, due to a band that includes only the range of harmful frequencies. We also selected a second frequency range a little wider, in order to perform a comparison between the vibrations in the first 200Hz and 500Hz and evaluate the effect of the dampers on non-harmful vibrations.

From the selected bands we computed our parameters; the energy contained in the first 200Hz (E200) and 500Hz (E500) and the ration between the energies in the two bands (E200/E500).

After obtaining the absolute value of energy contained in the selected bands, the result of the test with the racket in free free conditions was taken as reference while all other tests were expressed as percentage of the reference value. The comparison between the reference value and the energy obtained with applied the dampers, represented the main parameter evaluated in this study, indicating the amount of energy absorbed by the damper in ideal conditions.

Furthermore, in order to have an idea of the energy distribution at low frequencies, nine bands of 25Hz were defined in the interval 25-250Hz. This allowed to observe more in detail the racket's vibrational characteristics variations after the introduction of the dampers.

3.5.2 Data analysis of test 2 and 3

Due to their similarities, the analysis of the data obtained from test 2 and 3 was exactly the same. Furthermore, test 1 data analysis showed redundant results in the two different frequency ranges considered. For this reason, it was decided to study in both cases the energy contained only in the frequency range associated with vibrations dangerous for the health of the athletes, and so from 0 to 200Hz and so ratio E200/E500 was not calculated.

However, in these tests, it was impossible to measure the input forces related to the ball-racket impact. This was due to the limitations of the tools at our disposal, making it impossible to measure the ball velocity and, in the third test, also the racket velocity with accuracy. For this reason, it was decided not to study the absolute value of energy content of the selected frequency band, as for test 1, but to compute the difference of energy contents provided by the different accelerometers. This parameter was thought to represent the residual energy that was transmitted between the sensors, and so of the damping of the initial energy. The parameter, signed as AXY (where x and y are the accelerometer's identification number) was calculated as one minus the difference between the energy contained inside the 0-200Hz band by each accelerometer, as reported in the formula:

$$\mathrm{AXY} = \left(1 - \frac{AX - AY}{AX}\right) * 100$$

All three possible combinations A12, A13 and A23 were calculated. A23 can be related only to the intrinsic damping effect of the forearm, but nonetheless, it was calculated for completeness providing also an additional parameter to confirm the validity of the proposed test.

As before, to completely evaluate the vibrational behaviour of the racket, nine frequency bands between 25Hz and 250Hz were defined and then the energy contained in them was calculated and expressed as percentage.

4. RESULTS

In this chapter the main results of the three experimental tests will be presented.

This work was not focused on the evaluation of the amount of vibrational energy on the racket and on the arm, but it aimed to quantitatively evaluate the portion of energy transmitted. For this reason, the absolute value of accelerations is not reported, and the results are expressed only as ratios or differences with a reference value.

The vibrational beahviour at low frequency of the racket was studied dividing the first 250Hz of the spectrum in nine bands and calculating the energy content. The three tests produced very similar results, with the main difference recorded in test 2 for the accelerometers applied to the arm of the subject.

More in detail, the obtained energy distribution always presented a peak in the bands from 125hz to 175Hz, highlighting the presence of a first vibrational mode in this range.

Different energy distribution was found in the trials of test 2 with the accelerometers applied on the arm of the subject. In these cases, the recorded energy distribution was shifted towards lower frequencies. However, even if the main peak is at lower frequencies, a reduced local peak of energy was always visible around 150Hz (Fig. 4.1)

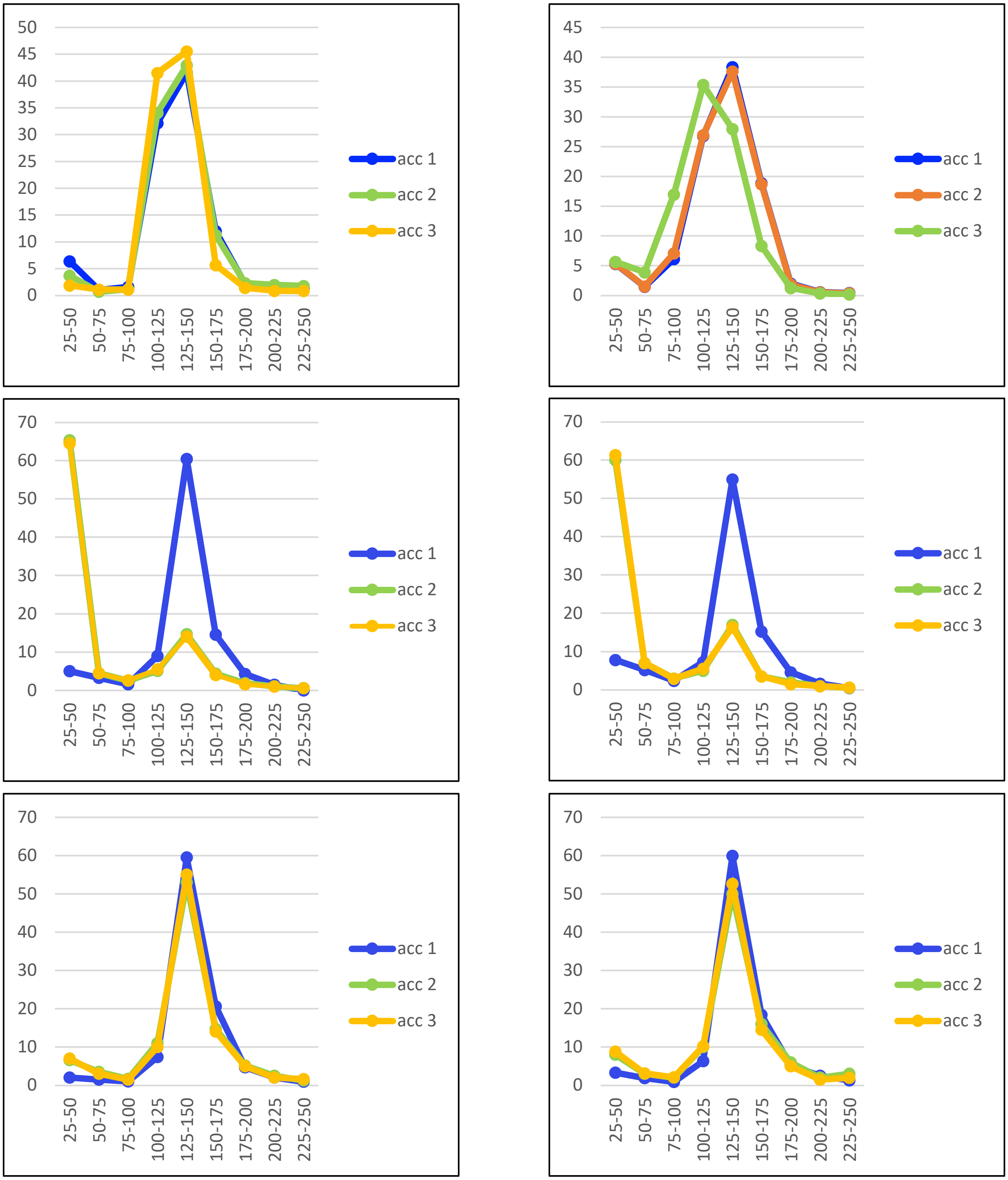


Fig. 4.1.1-6: Results of the analysis of the energy distribution inside the 25-250Hz band. Fig. 4.1.1: from test 1, with no damping system applied and hit in position 1. Fig.4.1.2: from test 1, damper type 1 and solicitation produced in position C with the handle gripped by hand. Fig.4.1.3: from test 2 and without any damping systems. Fig.4.1.4: from test 2 and type of damper 4. Fig.4.1.5: from test 3 and no damper applied. Fig.4.1.6: from the results of test 3 and with damping system 5.

4.1 Results of the laboratory test

Table 2 and table 3 show the energy content inside the 200Hz and 500Hz expressed as a percentage of the value obtained from the trial with the racket devoid of any damping system. Fig. 4.2 and 4.3 visually show the results reported in the tables.

The + sign indicates the trials that are not in “free free” conditions, but those in which the handle was gripped by hand. The * sign indicates the trials with a p-value lower than the threshold.

Table 2: reports the value of E200 of each damper and for each position, compared to the E200 without damping systems.

E200					
Position of solicitation	**Damper 1**	**Damper 2**	**Damper 3**	**Damper 4**	**Damper 5**
A+	162%	22%*	169%	47%	25%
A	14%	2%*	13%*	8%*	3%*
B	4%*	5%*	34%	9%*	8%*
C+	160%	87%	73%	25%	6%*
C	74%	3%*	57%	92%	5%*
D	30%	1%*	12%*	37%	2%*
E	59%	3%*	17%*	12%*	2%*

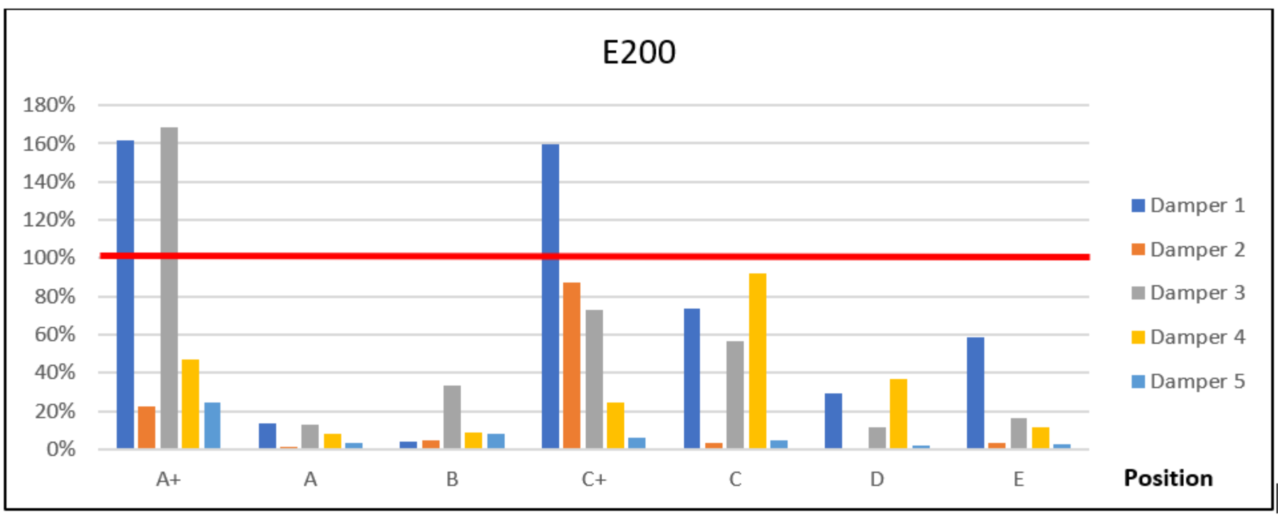


Fig. 4.2: histogram of the E200 obtained from test one and expressed as percentage of the E200 in free free condition, represented as the red line (100%).

Table 3: reports the value of E500 of each damper and for each position, compared to the E500 without damping systems.

E500					
Position of solicitation	**Damper 1**	**Damper 2**	**Damper 3**	**Damper 4**	**Damper 5**
A+	25%	3%*	175%	7%*	3%*
A	14%*	2%*	29%	8%*	3%*
B	4%*	5%*	113%	9%*	9%*
C+	136%	76%	62%	50%	6%*
C	74%	5%*	152%	93%	5%*
D	31%	1%*	42%	37%	2%*
E	52%	18%*	102%	10%*	3%*

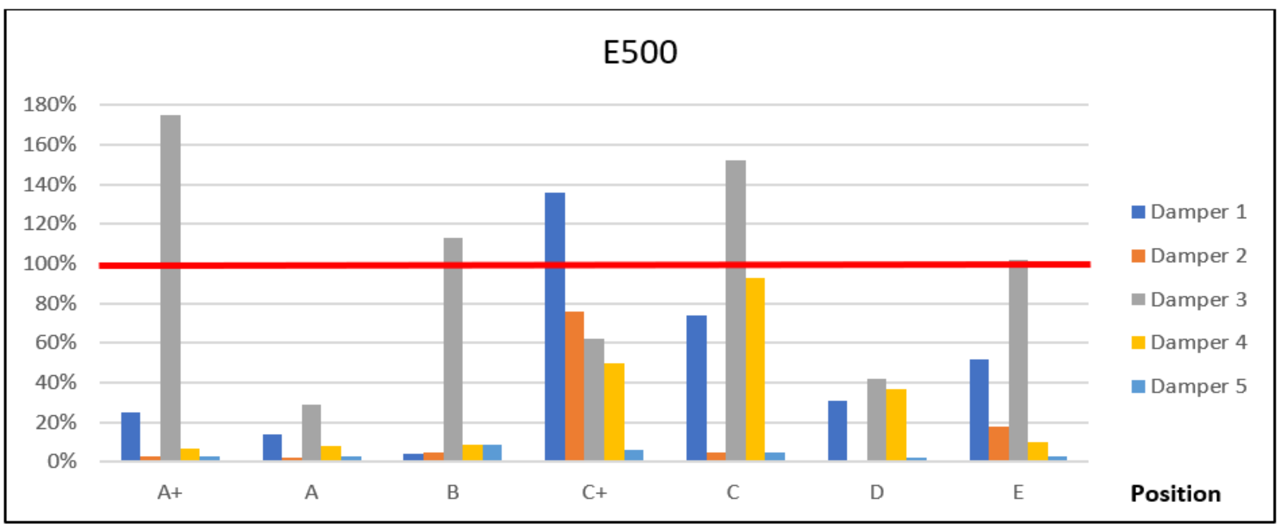


Fig. 4.3: histogram of the E500 obtained from test one and expressed as percentage of the E200 in free free condition, represented as the red line (100%).

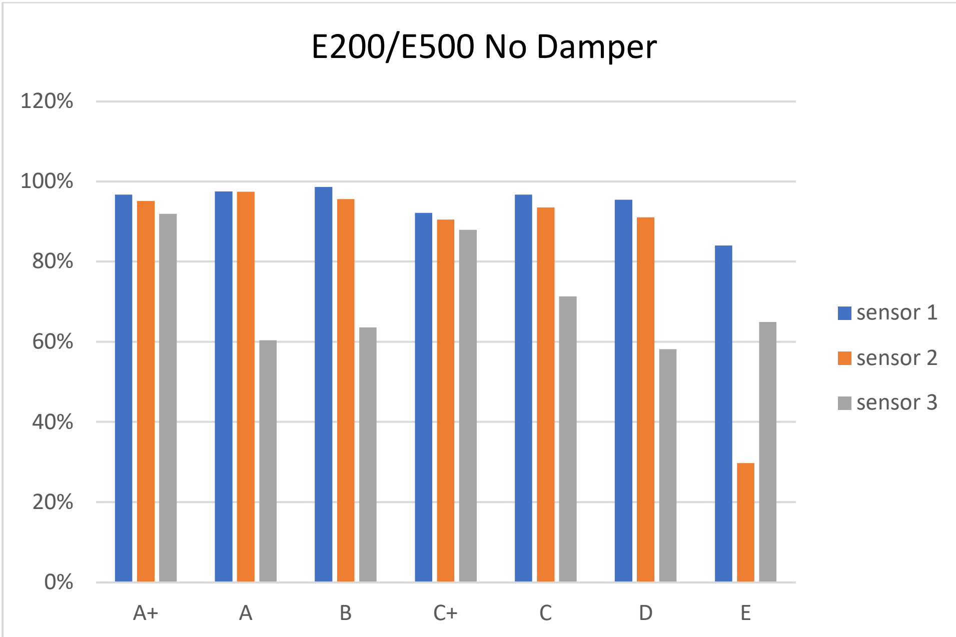
Results of E200/E500

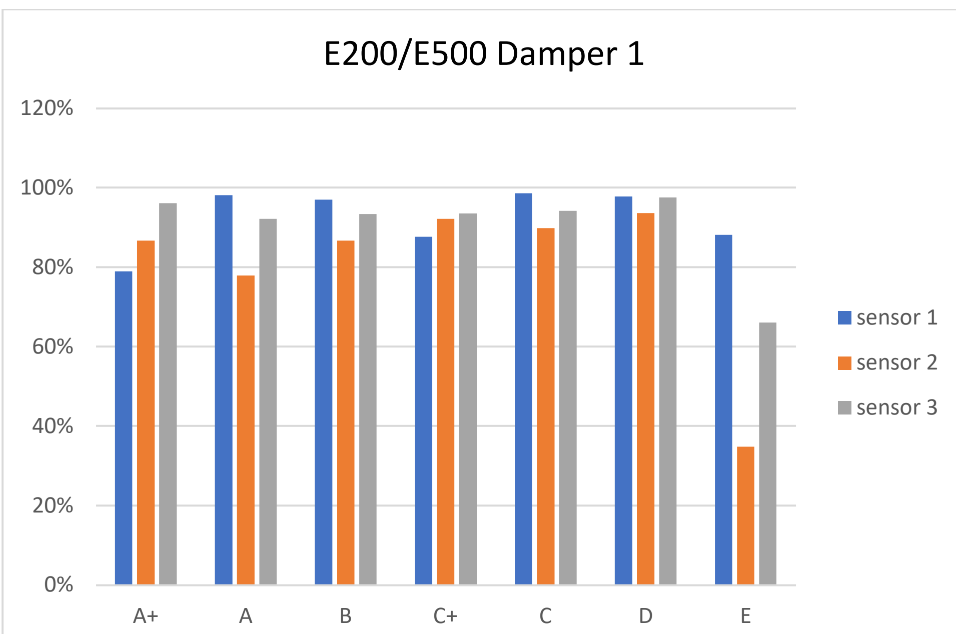
In this test, the ratio E200/E500 was also calculated for all the accelerometers and in every input condition (Fig. 4.4.A-F).

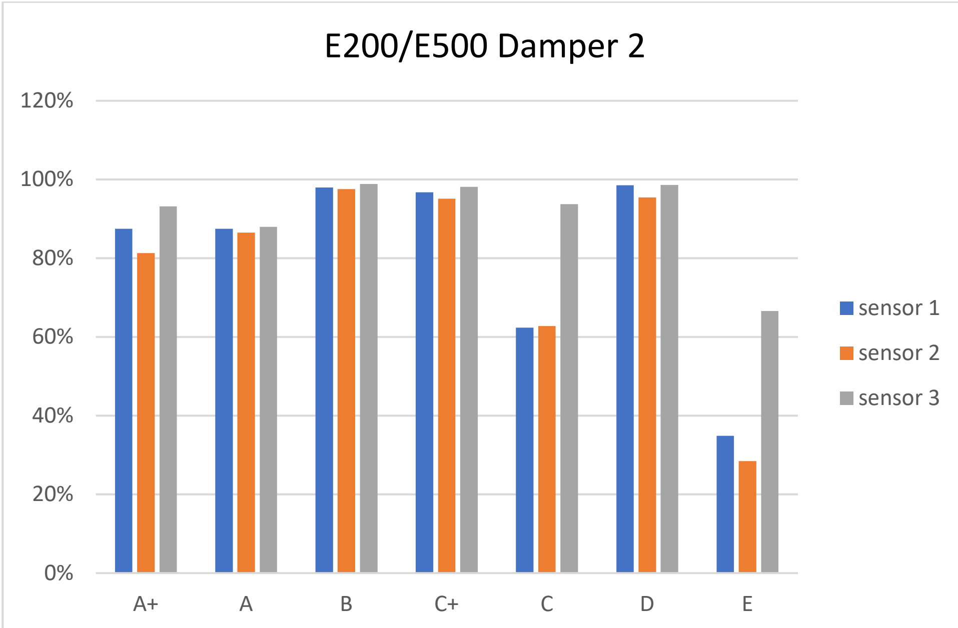
In the undamped case, it’s possible to observe a significant and progressive reduction of this parameter from sensor 1 to sensor 3, for all the excited positions on the string plate.

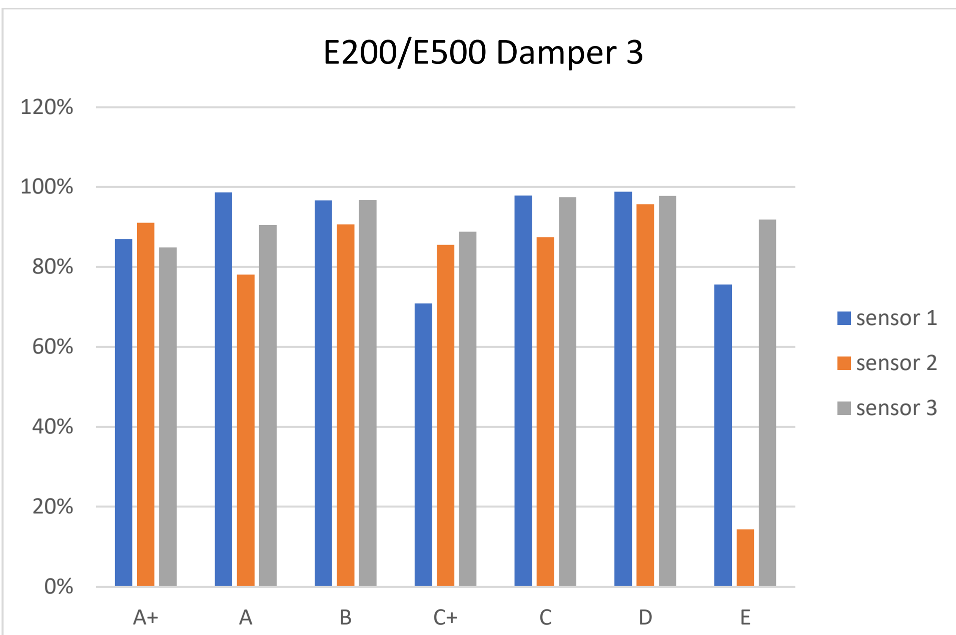
The trials with the racket in undamped conditions and the input force applied on the string bed present the same trend, with an important reduction passing from accelerometer 2 to 3. Instead, when the racket was held in hand, the energy ratio reduction between accelerometer 2 and 3 was of lower entity. The only case that produced a different result was the trial performed by hitting the top of the racket’s frame. In this case, the entire trend was different with the lowest energy ration in the position of the accelerometer 2.

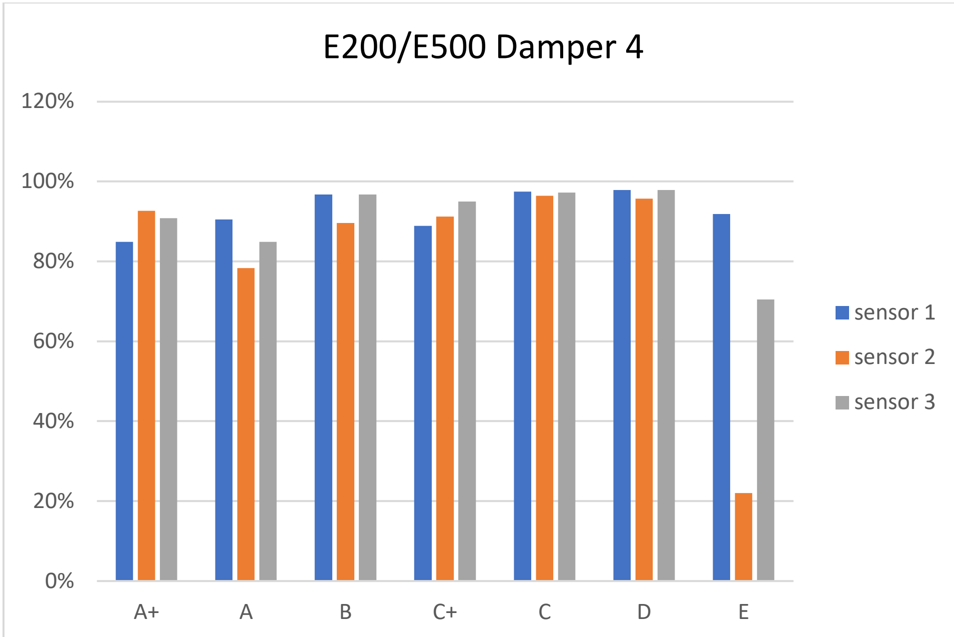
Instead, in all the tests performed with a damper applied on to the racket, it is not possible to see a clear pattern of the results. As shown in Fig 4.4.B-F, the solicitation applied in different positions generates results unrelated between the multiple tests.


E200/E500 No Damper
120%
100%
80%
60%
40%
20%
0%
A+
A
B
C+
C
D
E
sensor 1
sensor 2
sensor 3


E200/E500 Damper 1
120%
100%
80%
60%
40%
20%
0%
A+
A
B
C+
C
D
E
sensor 1
sensor 2
sensor 3


E200/E500 Damper 2
120%
100%
80%
60%
40%
20%
0%
A+
A
B
C+
C
D
E
sensor 1
sensor 2
sensor 3


E200/E500 Damper 3
120%
100%
80%
60%
40%
20%
0%
A+
A
B
C+
C
D
E
sensor 1
sensor 2
sensor 3



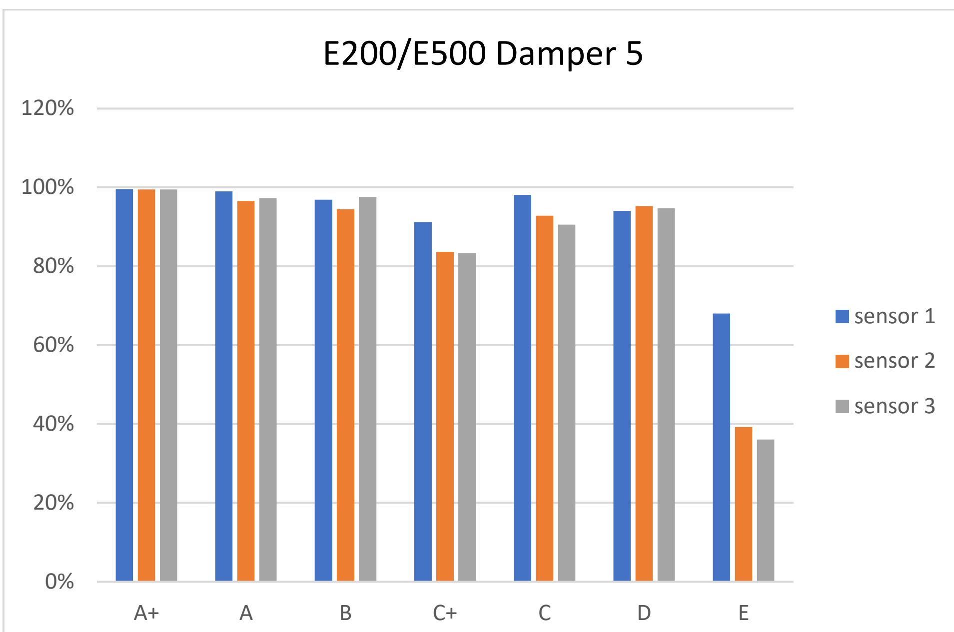


Fig. 4.4.A-B-C-D-E-F: these historgrams reports the values of E200/E500 for every test. These graphs allow to see the limited differences between each test.

4.2 Results of racket handling test and operative condition test

The aim of test 2 and 3 was to quantify the portion of the vibrational energy produced by the ball-racket impact that was reached the wrist and then the elbow.

Table 4 and 5 show the residual energy obtained from respectively test 2 and test 3 with the standard deviations, always expressed in percentage.

In no cases a statistically significant difference was found between damped and undamped racket, with any of the applied dampers (p-value always higher than 0.05).

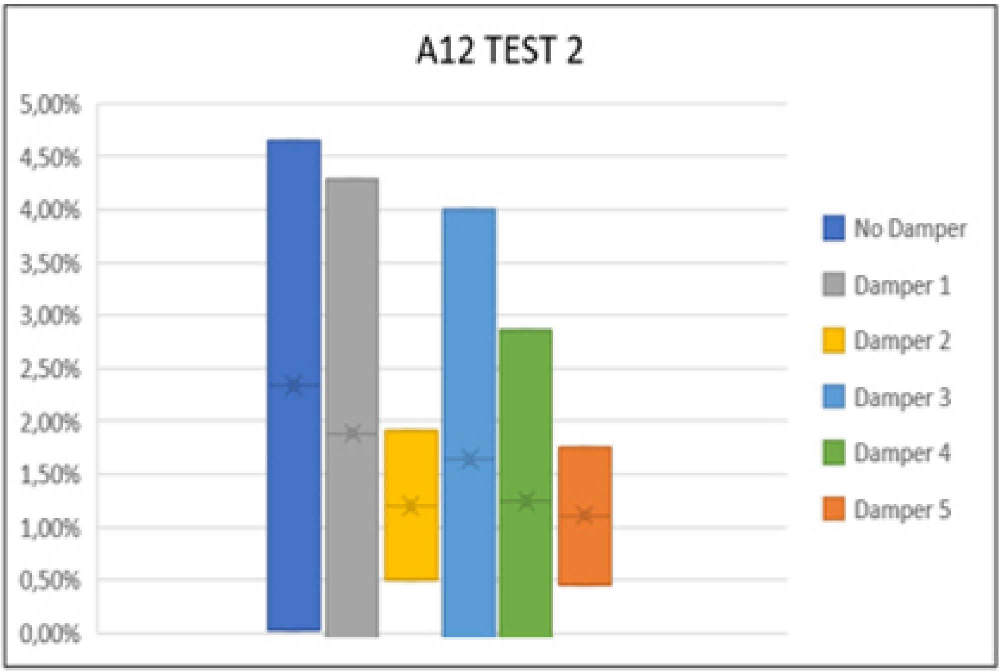
Table 4: reports the values and the standard deviations of the transmitted energy in test 2 for every damper applied.

	Transmitted Energy TEST 2					
	E200			**Standard Deviation**		
Damper	**A12**	**A13**	**A23**	**SD12**	**SD13**	**SD23**
No Damper	**2.34%**	**0.53%**	**22.88%**	**2,31%**	**0,357%**	**15,288%**
Damper 1	1.88%	0.43%	23.07%	0,65%	0,142%	12,810%
Damper 2	1.20%	0.43%	36.04%	2,41%	0,460%	24,445%
Damper 3	1.64%	0.46%	27.86%	0,70%	0,154%	12,807%
Damper 4	1.24%	0.39%	31.10%	2,36%	0,725%	44,218%
Damper 5	1.11%	0.29%	26.53%	1,62%	0,362%	29,068%

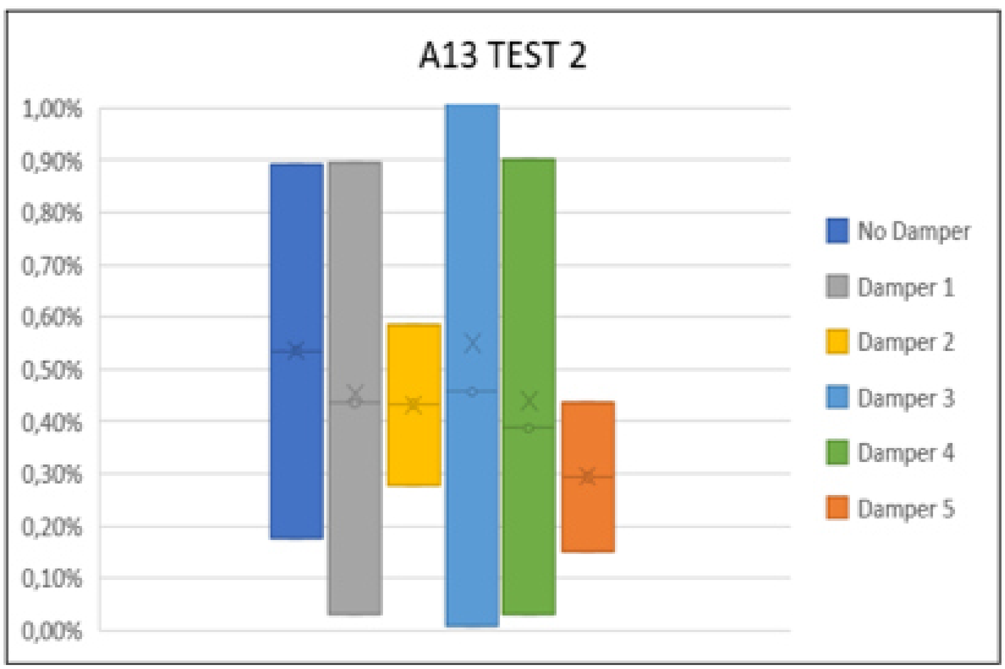
	Transmitted energy TEST 3					
	E200			Standard Deviation		
Damper	**A12**	**A13**	**A23**	**SD12**	**SD13**	**SD23**
No Damper	**5.81%**	**0.68%**	**11.65%**	**2,58%**	**0,499%**	**8,580%**
Damper 1	4.89%	0.43%	8.77%	4,38%	0,588%	13,924%
Damper 2	4.87%	0.21%	4.28%	5,22%	0,419%	8,581%
Damper 3	5.06%	0.64%	12.62%	1,66%	0,353%	7,246%
Damper 4	5.54%	0.53%	9.63%	3,49%	1,018%	20,126%
Damper 5	4.22%	0.42%	9.90%	3,45%	0,325%	5,861%

Table 5: reports the values and the standard deviations of the transmitted energy in test 2 for every damper applied.

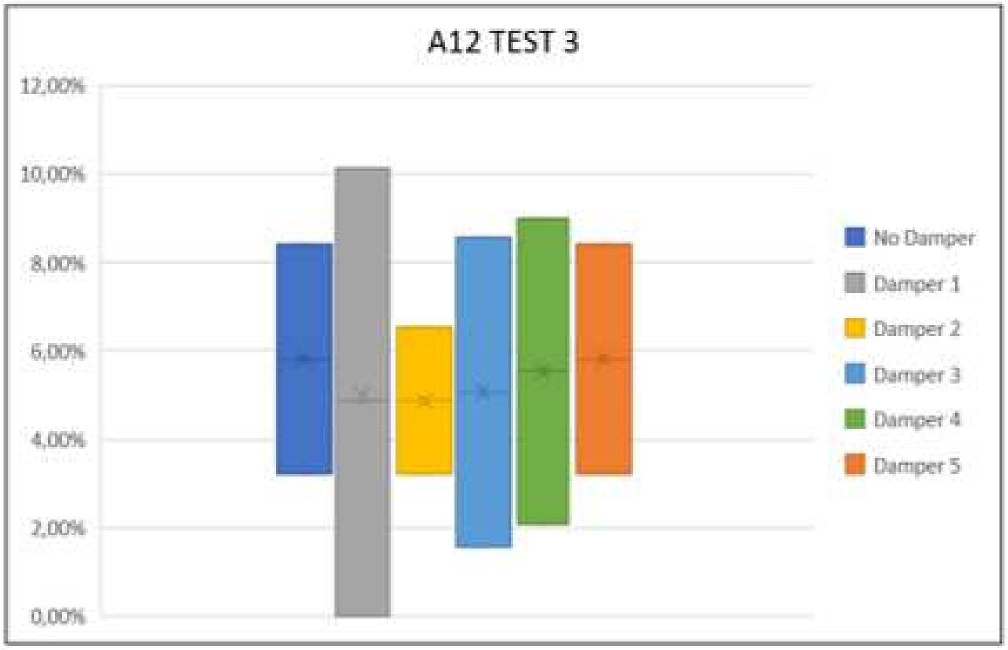
The following boxplots show the values and visually compare the results reported in the table 4 and 5, showing also the standard deviation.


A12 TEST 2
5,00%
4,50%
4,00%
3,50%
3,00%
2,50%
2,00%
1,50%
1,00%
0,50%
0,00%
No Damper
Damper 1
Damper 2
Damper 3
Damper 4
Damper 5

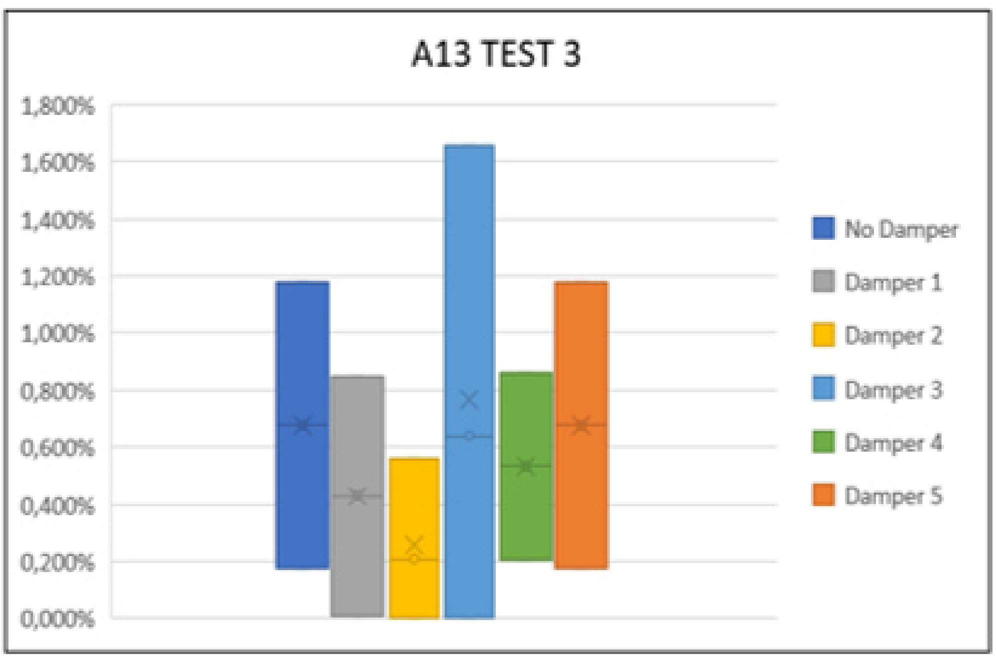
A


A13 TEST 2
1,00%
0,90%
0,80%
0,70%
0,60%
0,50%
0,40%
0,30%
0,20%
0,10%
0,00%
No Damper
Damper 1
Damper 2
Damper 3
Damper 4
Damper 5

B



C



D

Fig. 4.5.A-B-C-D: the boxplots show the data reported respectively by the tables 4 and 5.

5. DISCUSSION

Despite the three tests were proposed with different properties and with a wide range of assumptions, the results obtained were mostly in agreement with each other and with the results obtained from the study of the literature.

In these tests a comparative approach was adopted to extract from processed data the information of interest. For each accelerometer and for each condition, the parameters extracted were compared to fully evaluate the racket behaviour and to quantify the efficacy of the different damping systems.

Moreover, the different conditions in which the three tests were performed allow to make various consideration about the obtained results.

5.1 Energy Distribution

The first result analysed in this study was the vibrational energy distribution inside the bands between 25-250 Hz. This parameter was defined from the study of the literature to understand the general energy distribution at low frequency and, in particular, to verify the presence of a vibrational mode inside the range of frequencies harmful for the player's health.

The results showed the presence of a first resonance frequency in the band 125-150 Hz, confirming our hypothesis.

In every condition, it was always possible to appreciate an energy peak around 150 Hz. Only test 2 produced different energy distributions, exclusively with the accelerometers applied on the arm of the subject. In these trials the racket was held in hand in static conditions and stricken with the ball shot by the cannon. In our opinion, the energy distribution presents a different pattern due to the particular conditions in which the tests were performed. More in detail, we attribute the specific energy pattern to the impossibility for the subject that held the racket to adapt the grip force with the typical preparation before the hit. In fact, in test 3 the subject could prepare for the hit, being able to see the incoming ball, while in the trials performed with the hand on the handle in test 1, all accelerometers were applied directly on the racket. In test 2, the subject could not see the ball coming and so he had to hold the racket firmly and stable the racket without a proper preparation. This caused a change in the grip force pattern, that could be related to the different energy distribution sensed in test 3 on the wrist and on the elbow.

The main way to verify these hypotheses would be to repeat the three tests and measure the grip force to see if there is a significant difference in the force pattern. Alternatively, test 1 could be repeated with the accelerometers applied on the wrist and with the subject unable to properly prepare for the in-put force.

The impossibility to see significant differences between the results obtained from the same set of tests performed with different damping systems, testify the little impact of these devices on the general vibrational behaviour of the racket.

Despite this, the results obtained from this analysis gave a first confirmation of the goodness and righteousness of our experimental set ups and working protocols.

5.2 Energy Content Analysis

The analysis of the energy content has been proposed as the main parameter for evaluating the ability of the selected damping systems to reduce the low frequency vibrations transmitted to the handle and therefore to the player's arm.

As previously stated, this study was not focused on the quantification of the solicitations that reach the wrist and the elbow, but on the comparison of the vibrational energy between the different trials. The decision not to consider the absolute value of solicitations measured during the test was mainly related to the limitations of the tools at our disposal that in test 2 and 3 did not allow to measure with precision the velocity of the ball. In test 1 the configuration was too far from real playing conditions to produce a quantitative analysis. In fact, the input forces introduced in test 1 were at maximum of 30N, when in a real impact they can not be under a few hundreds of newtons, even if at low velocities. This was also appreciable by the fact that 0.04 s were enough to observe the entire vibrational phenomenon, while in real condition this time is not even sufficient for the ball to leave the string bed.

In test 1, three different parameters were proposed and computed: E200, E500 and E200/E500.

E200 was thought as the principal parameter of this entire work from the beginning. In fact, the band was selected to include the whole range of harmful vibrations. However, 200 Hz is only an approximate and representative threshold of the vibrations that can be defined as harmful. For this reason, the parameter E500 was proposed to consider a slightly wider range of frequencies and to spot possible differences with respect to the previous band. However, since the results of the first test showed no significant difference between the values of E200 and E500, this prompted us to not consider this parameter for tests 2 and 3.

The third parameter, E200/E500 was calculated to evaluate possible effects produced by the intrinsic damping process that occurs inside the racket frame.

The values of E200 obtained in almost ideal conditions (listed in table 2) showed that the dampers have an impact also on the vibrations at low frequencies and, in few cases, this impact has a not so low entity. In fact, there are multiple cases in which the statistical difference is significant. More in detail, damper 5 showed to have the higher damping impact but also damper 2 and 4 presented more than one statistically significant value. The general trends, that are visible in Fig. X.X, show how the vibration energy is damped in practically every condition and by all the dampers. Instead, in the trials with the hand on the handle of the racket the results are not so encouraging. In these cases, it was possible to see energy values even higher than those of the undamped one. The values of E500 present almost the same pattern of E200, even if, in general, a lower difference between the undamped trial and damped ones was found.

We believe that the conditions imposed in test 1 allow to show the ability of these device to damper the vibrations in these two frequency bands. However, it must be considered that the low entity of the introduced solicitations could make the damping effect of the vibration's absorbers more evident.

The trials performed with the hand on the handle produced unreliable results. In our opinion, the inclusion of the handling of the racket, adds instability to the system, which is no longer still. In fact, the little motions introduced, in real conditions would be absolutely neglectable but in this test the solicitations are very limited and this gives great importance to this small instability.

No clear pattern or trend is visible from the values of E200/E500 as shown in Fig 4.4. This indicates the low informative value of this parameter that was proposed to verify if the intrinsic structural damping, imposed by the racket frame in the region next to the hand, is able to produce a partial reduction of the oscillations in the frequency range that is most risky for the player's health (below the 200Hz). For this reason, we decided not to calculate this parameter for the subsequent tests.

6. CONCLUSION

The different tests performed in this study allowed us to cover many aspects linked to the transmission of vibrations from the tennis racket to the arm of a player. Moreover, the distinct properties of each test allowed to make a series of conclusions on the racket's vibrational behaviour and on the efficacy of the latest damping devices.

The first result that was analysed was the energy distribution inside the first 250 Hz. Concordance of the results of this parameter is found in every condition of every test. This allowed us to conclude that the structural properties of the racket give to it its typical vibrational characteristics, which are not influenced significantly by the presence of any type of damper.

The analysis of the energy content in the low frequencies, obtained in this work, seems to confirm the results found in the literature on the efficacy of the dampers. In particular, these devices produce a damping effect that clearly changes the sound produced by the racket at the moment of impact and also the athlete's sensing of the tool. However, this damping effect seems to not significantly impact the vibrational modes at the most dangerous and harmful frequencies for the athlete's health.

In general, it seems that the benefits produced by the use of these devices are very limited, considering also that usually the change in the perception of sportive tools is not appreciated or welcomed by the athletes.

In our opinion, the limited impact of these vibration absorbers is related to their reduced masses, which is at maximum around 5% of the total mass of the system. In support of this hypothesis, the damper 5 that has the greatest mass in this study, seems in general to present the greatest vibration reduction effect. Furthermore, the second damping system studied, which was composed by the simultaneous application of two damper 1, compared with this last produce better results in almost every test and condition.

The damper's reduced mass is a design choice that is meant to reduce as much as possible the impact of the damper on the intrinsic properties of the racket, such as the mass or the balancing.

To obtain the perfect trade-off between the damping of the harmful vibrations and to avoid impacting the design properties of the racket, we think that the future tennis rackets should be designed with damping systems already integrated in its frame. In this way, it would be possible to increase the damper mass and maintain the desired racket properties. A first attempt is represented by the Prokinetic racket from Prokennex, that embed in the frame around the string plate a series of granular dampers and today is considered the best racket for those prone to develop epicondylitis.

For future studies it would be useful to compare different rackets and dampers with the Prokinetic racket to evaluate the real efficacy of this solution.

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