UNIVERSITA' POLITECNICA DELLE MARCHE

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PROVE DI VITA ACCELERATA PER IMPIANTI INDUSTRIALI

ACCELERATED LIFE TESTING FOR INDUSTRIAL PLANTS

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INTRODUZIONE

Il test accelerato consiste in una varietà di metodi di prova per abbreviare la vita dei prodotti o accelerare il degrado delle loro prestazioni. Lo scopo di tali test è quello di ottenere rapidamente dati che, opportunamente modellati ed analizzati, forniscano le informazioni desiderate. Tali test consentono ovviamente di risparmiare molto tempo e soldi.

1. MATERIALI

I materiali più comunemente utilizzati per questo tipo di prove sono metalli, plastiche, dielettrici e isolanti, ceramiche, adesivi, gomma ed elastici, alimenti e farmaci, lubrificanti, rivestimenti protettivi e vernici, calcestruzzo e cemento, materiali da costruzione.

- **Metalli**: Le prestazioni comprendono la durata a fatica, lo scorrimento, la rottura da scorrimento, l'innesco e la propagazione delle cricche, l'usura, la corrosione, l'ossidazione e la ruggine. Le prove accelerate includono lo stress meccanico, la temperatura, la geometria del provino e la finitura superficiale. I fattori di accelerazione chimica includono l'umidità, il sale, i corrosivi e acidi.
- **Plastiche**: I test accelerati sono utilizzati con molte materie plastiche, tra cui materiali per edilizia, isolanti (elettrico e termico), componenti meccanici e rivestimenti. I materiali includono polimeri, cloruro di polivinile (PVC), uretano, schiume, e poliesteri, Le prestazioni includono la durata a fatica, l'usura, le proprietà meccaniche. Le prove accelerate includono carico meccanico (incluse vibrazioni e urti), temperatura (inclusi ciclismo e urti), e gli agenti atmosferici (radiazioni ultraviolette e umidità).
- **Dielettrici** ed **isolanti**: I test accelerati sono utilizzati con molti dielettrici e isolamenti elettrici, compresi i solidi (polietilene, epossidici), liquidi (olio per trasformatori), gas e compositi (carta oleata). I test accelerati comprendono la temperatura, lo stress da tensione, i cicli termici ed elettrici e urti, vibrazioni, sollecitazioni meccaniche, radiazioni e umidità.
- **Ceramiche**: Le applicazioni riguardano la durata a fatica, l'usura e la degradazione delle proprietà meccaniche ed elettriche.
- **Gomme** ed **elastici**: i prodotti includono pneumatici e nastri industriali. Le prestazioni comprendono la durata a fatica e l'usura. Le prove di accelerazione includono carico meccanico, temperatura, struttura della pavimentazione e agenti atmosferici (radiazione solare, umidità e ozono).
- **Lubrificanti**: i materiali usati sono solidi (grafite, disolfuro di molibdeno e teflon), olio, grasso e altri lubrificanti. Le prestazioni includono l'ossidazione, l'evaporazione e la contaminazione. Le prove accelerate includono velocità, temperatura e contaminanti (acqua, rame, acciaio e sporcizia).
- **Materiali da costruzione**: i materiali utilizzati sono legno, particelle, cartone, plastica, materiali compositi, vetro e altri materiali da costruzione. Le prestazioni includono resistenza all'abrasione, solidità del colore, resistenza e altre caratteristiche meccaniche proprietà. Le prove accelerate comprendono il carico e gli agenti atmosferici (radiazione solare, temperatura, umidità).
- **Condensatori**: i test accelerati sono utilizzati con la maggior parte dei tipi di condensatori, compresi quelli elettrolitici, in polipropilene, a film sottile e al tantalio. Le prestazioni sono di solito la vita. Le variabili di accelerazione includono temperatura, tensione e le vibrazioni.
- **Resistori**: i test accelerati sono utilizzati con resistenze a film sottile e spesso, ossido di metallo, pirolitici e a film di carbonio. Le prestazioni sono a vita. Le variabili includono la temperatura, la corrente, la tensione, la potenza, la vibrazione, l'attacco elettrochimico (umidità) e la radiazione nucleare.
- **Celle e batterie**: i test accelerati sono utilizzati con celle ricaricabili, non ricaricabili e solari. Le prestazioni includono la durata, l'autoscarica, la corrente e la profondità di scarica. Le variabili di accelerazione includono la temperatura, densità di corrente e velocità di carica e scarica.

1.1. MECCANISMI DI DEGRADAZIONE

- **Fatica**: la fatica è un fenomeno meccanico di progressiva degradazione di un materiale sottoposto a carichi più o meno variabili nel tempo che può portare alla sua rottura anche se è rimasto nel suo limite di elasticità (intensità massima dei carichi inferiore rispetto alla tensione di rottura o snervamento del materiale stesso). La principale sollecitazione riguardo il tipo di carico, marginali sono la temperatura e i fattori chimici.
- **Usura**: nelle applicazioni, molti materiali sono soggetti ad attrito che rimuove il materiale. Ad esempio, i pneumatici in gomma perdono il battistrada, le vernici per la casa si lavano gli ingranaggi, i cuscinetti e le macchine utensili si consumano. Le prove accelerate comprendono la velocità, il carico (entità e tipo), la temperatura, la lubrificazione e le sostanze chimiche (umidità).
- **Corrosione/ossidazione**: la maggior parte dei metalli e molti alimenti, prodotti farmaceutici, ecc., si deteriorano per reazione chimica con l'ossigeno (ossidazione e ruggine), fluoro, cloro, zolfo, acidi, alcali, sale, perossido di idrogeno e acqua. Le prove accelerate includono la concentrazione del prodotto chimico, gli attivatori, la temperatura, la tensione e il carico meccanico (stress-corrosione).
- **Condizioni meteorologiche**: questo riguarda gli effetti del tempo sui materiali all'aperto. Tali materiali includono metalli, rivestimenti protettivi (vernice, galvanoplastica e anodizzazione), plastica e gomme. Le sollecitazioni di accelerazione includono la radiazione solare (lunghezza d'onda e intensità) e le sostanze chimiche (umidità), sale, zolfo e ozono). La degradazione generalmente comporta corrosione, ossidazione (ruggine), appannamento o altre reazioni chimiche.

2. TIPI DI DATI

I dati ottenuti dalle prove accelerate si possono dividere in due gruppi:

1) **Vita utile**

2) **Altre tipologie di misura delle prestazioni** comprendono i dati sulle prestazioni: si può essere interessati a come le prestazioni del prodotto si degradino con l'età. In tali prove, i campioni sono testati sotto un alto stress, e le loro prestazioni misurate in periodi diverse. Tali dati sulle prestazioni vengono analizzati applicando un modello di degradazione per stimare la relazione tra prestazioni, vita utile stress.

3. TIPI DI ACCELERAZIONE E CARICHI DI SOLLECITAZIONE

Questa sezione descrive i comuni tipi di accelerazione delle prove (alto tasso di utilizzo, sovraccarico, censura, degradazione e progettazione del provino) e sollecitazione di carico.

Alto tasso di utilizzo. Un modo semplice per accelerare la vita di molti prodotti è quello di far funzionare il prodotto a un tasso di utilizzo più elevato, o semplicemente più velocemente. Per esempio, in molte prove di durata, i cuscinetti volventi funzionano a circa tre volte la loro normale velocità. Questo metodo può essere utilizzato anche in combinazione con test di sovrastress.

Tempo di riposo ridotto. Molti prodotti sono fuori servizio per la maggior parte del tempo di utilizzo effettivo. In tale ambito le prove si effettuano facendoli funzionare una frazione maggiore del tempo. Ad esempio, nella maggior parte delle case, un grande apparecchio (ad esempio, lavatrice o asciugatrice) funziona un'ora o due al giorno; in prova funziona 24 ore al giorno. Un piccolo apparecchio (ad esempio, tostapane o caffettiera) esegue alcuni cicli al giorno; in prova cicli molte volte al giorno.

Test di sovrastress. Il test di sovrastress è la forma più comune di test accelerato, consiste nel far funzionare un prodotto ad un livello più alto del normale livello di alcuni stress accelerati per abbreviare la vita del prodotto o per degradarlo prestazioni del prodotto più velocemente. Le sollecitazioni tipiche di accelerazione sono la temperatura, la tensione, il carico meccanico, i cicli termici, l'umidità e le vibrazioni.

Scopo. Lo scopo di tali test è quello di stimare la distribuzione della vita del prodotto a tassi di utilizzo normali. Si presume che il numero di cicli, giri, ore, ecc., che porta a rottura in fase di test è lo stesso che si osserverebbe durante il normale tasso di utilizzo. Per esempio, si suppone che un cuscinetto che gira 6,2 milioni di volte prima di arrivare a guasto (ad alto numero di giri) rimangono ugualmente 6,2 milioni a regime normale. I dati sono trattati come un campione proveniente dall'uso effettivo. Poi le analisi standard dei dati sulla vita utile forniscono una stima della percentuale di guasto sulla garanzia, la vita media. Non è automaticamente vero che il numero di cicli per arrivare a rottura a tassi di utilizzo elevati e normali è lo stesso. Di solito il test deve funzionare con particolare attenzione per assicurare che il funzionamento e la sollecitazione del prodotto rimangano normali in tutti gli aspetti, ad eccezione del tasso di utilizzo. Ad esempio, l'utilizzo ad alto tasso di utilizzo di solito aumenta la temperatura del prodotto. Questo di solito si traduce in un minor numero di cicli che il prodotto può reggere prima di arrivare a guasto. Così molti di questi test prevedono il raffreddamento del prodotto per mantenere la temperatura ad un livello normale. Al contrario, i prodotti sensibili ai cicli termici possono durare più a lungo se eseguiti in modo continuo senza cicli termici. Per questa ragione, i tostapane in prova sono raffreddati a forza da un ventilatore tra un ciclo e l'altro.

La vita di alcuni prodotti può essere accelerata grazie alle dimensioni, alla geometria e alla finitura dei campioni.

- **Dimensioni**. Generalmente i campioni grandi falliscono prima di quelli piccoli. Per esempio, I condensatori ad alta capacità si guastano prima di quelli a bassa capacità dello stesso design. Allo stesso modo, i cavi lunghi si guastano prima di quelli corti.
- **Geometria e finitura superficiale**. La geometria del campione può influire sulla sua durata. Per esempio, alcuni campioni metallici sollecitati a fatica sono dentellati. Tali dentellature producono un forte stress locale e una precoce rottura. Inoltre, anche la finitura superficiale (rugosità) e le tensioni residue dei provini metallici influiscono sulla durata a fatica.

Il carico di sollecitazione in una prova accelerata può essere applicato in vari modi.

- Carico costante.

La sollecitazione più comune è la sollecitazione costante. Ogni provino viene eseguito ad un livello di sollecitazione costante. La Figura 3.1 mostra tre diversi livelli di carico costante. Qui è rappresentata la storia di un provino come lo spostamento lungo una linea orizzontale fino a quando non arriva a rottura (indicato con una X). Al livello più alto, tutti quattro campioni sono arrivati a rottura; al livello intermedio, solo uno dei cinque provini ha superato la prova; al livello più basso invece, quattro sono arrivati a rottura, quattro no. Nella pratica, la maggior parte dei prodotti funziona a stress costante. Le prove di vita verranno quindi effettuate a sollecitazione costante, imitando il modello reale.

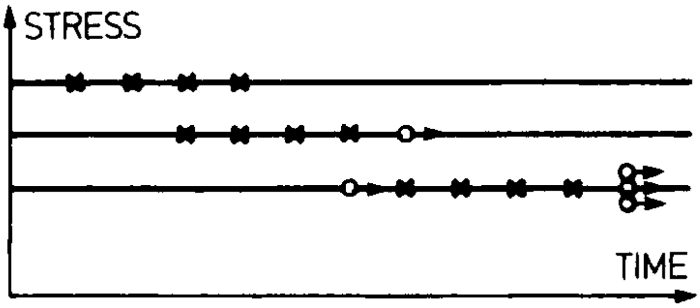

Figure 3.1. Constant stress test (× failure, ○→ runout).

- Stress a gradino.

Nelle prove a gradino, un provino viene sottoposto in successione a livelli di stress più elevati. In primis viene sottoposto ad una determinata livello di sollecitazione per un determinato periodo di tempo: se non fallisce, viene sottoposto ad un livello più elevato per un ulteriore periodo. La sollecitazione su un campione è quindi

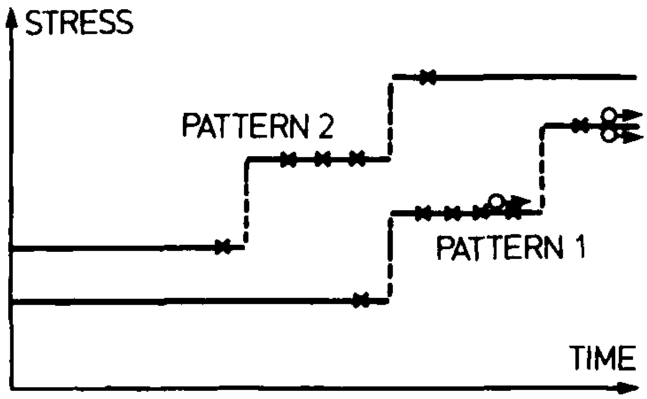

Figure 3.2. Step-stress test (× failure, ○→ runout).

aumentata passo dopo passo fino a quando non fallisce. Di solito tutti gli esemplari passano attraverso lo stesso modello specificato dei livelli di stress e dei tempi di prova. A volte modelli diversi sono applicati a diversi campioni. La Figura 3.2 mostra due di questi modelli.

Il vantaggio principale di una prova di stress a gradino è che produce velocemente dei fallimenti, a causa dei crescenti livelli di stress. Gli statistici sono contenti di avere dei fallimenti, perché producono stime del modello e di vita del prodotto. Gli ingegneri sono più felici quando non ci sono guasti, il che suggerisce (forse in modo errato) che il prodotto è affidabile. Una prova di sollecitazione costante con pochi campioni guasti di solito produce una maggiore precisione rispetto a una prova di stress a passi più brevi in cui tutti i campioni falliscono. In linea di massima, il tempo totale della prova (sommato su tutti i provini) determina l'accuratezza, non il numero di guasti. In parole povere, il tempo totale di prova (sommato su tutti i campioni) determina la precisione - non il numero di guasti.

C'è di contro uno svantaggio importante delle prove di stress a gradino per la stima dell'affidabilità. La maggior parte dei prodotti funziona a sollecitazione costante, non a passi. Pertanto, il modello deve tenere adeguatamente conto dell'effetto cumulativo dell'esposizione a sollecitazioni successive. Inoltre, il modello deve anche fornire una stima della vita sotto costante stress. Un tale modello è più complesso di uno per una prova a sollecitazione costante. Un secondo svantaggio di un test di stress a gradini è che le modalità di guasto che si verificano a livelli di stress elevati (in fasi successive) possono differire da quelli delle condizioni d'uso.

- Stress progressivo.

Nel carico di sollecitazione progressiva, un provino viene sottoposto a un livello di stress in continuo aumento. Diversi gruppi di campioni possono subire diversi modelli di stress progressivo. La Figura 3.3 illustra tale prova con tre modelli, ciascuno con una sollecitazione linearmente crescente. Come mostrato in figura 3.3, con un basso tasso di aumento dello stress, gli esemplari tendono a vivere più a lungo e

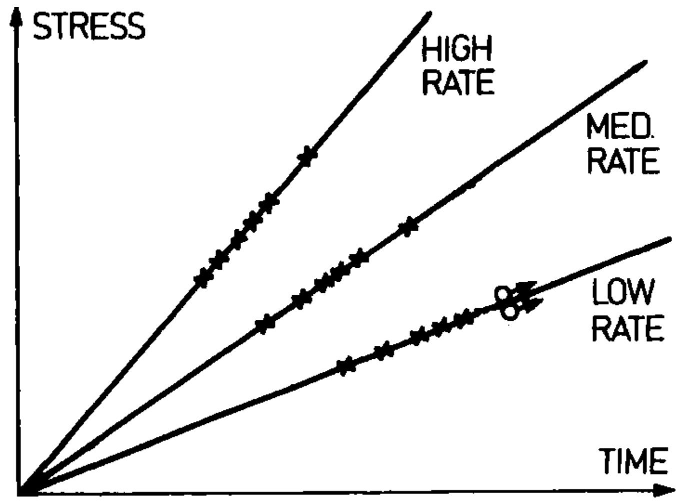

Figure 3.3. Progressive stress test (× failure, O→ runout).

falliscono con uno stress minore.

Le prove di sollecitazione progressiva hanno gli stessi svantaggi dei test di stress a gradino. Inoltre, può essere difficile controllare il progressivo stress in modo abbastanza accurato. Per questo motivo sono generalmente consigliate prove di stress costante su prove di stress progressivo per la stima dell'affidabilità.

- Stress ciclico.

Durante l'uso, alcuni prodotti subiscono ripetutamente una sollecitazione ciclica. Ad esempio, l'isolante sotto tensione alternata vede una sollecitazione sinusoidale.

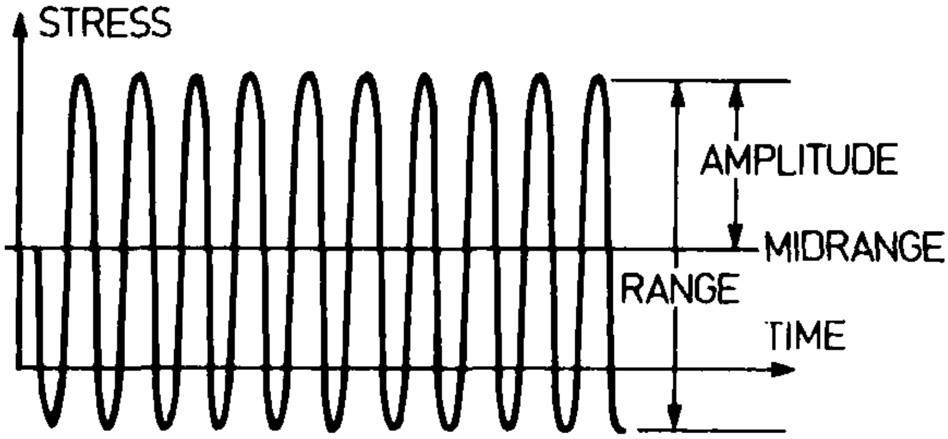

Figure 3.4. Cyclic-stress loading.

La figura 3.4 mostra una prova con sollecitazione ciclica. Per molti prodotti, la curva è di tipo sinusoidale.

Nella maggior parte di tali prove, la frequenza e la durata di un ciclo sono gli stessi che nell'uso effettivo del prodotto. Per alcuni prodotti sono diversi, ma si suppone che abbiano un effetto trascurabile sulla vita, e non vengono presi in considerazione. Per altri prodotti, la frequenza e la lunghezza di un ciclo influenzano la vita; quindi sono inclusi nel modello come variabili di stress.

4. CONSIDERAZIONI INGEGNERISTICHE

Molte considerazioni influiscono sulla validità e l'accuratezza delle informazioni ricavate da una prova accelerata. Questa sezione prende in esame alcune considerazioni di ingegneria e di gestione coinvolte nella pianificazione scientifica e nella realizzazione di una prova. Si tenga presente che la maggior parte delle considerazioni si applicano alle prove di degrado delle prestazioni e agli esperimenti di ingegneria in generale. Generalmente tali decisioni implicano la collaborazione di manager, progettisti, ingegneri di produzione e di prova, statistici e altri. I seguenti argomenti evidenziano alcune decisioni di gestione e di ingegneria su un test:

- SCOPO DEL TEST

Gli scopi delle prove di vita accelerate sono molteplici:

1)eliminare difetti di progettazione, o ridurli attraverso la ridondanza

2)scegliere tra design, componenti, fornitori

3) identificare difetti di fabbricazione, eliminarli attraverso una migliore produzione

4)eliminare guasti precoci (rodaggio)

5)controllo qualità

6) misurare l'affidabilità, che comprende anche la stima dei costi di garanzia e produzione

7)convalidare il test: dimostrare che il test è coerente con se stesso e con altri test nel tempo
8) decidere quando ispezionare, riparare o sostituire e quanti ricambi e sostituzioni per la produzione e lo stoccaggio

- PRESTAZIONI DEL PRODOTTO

Nelle prove di vita accelerate il tempo necessario al “fallimento” è la variabile principale (ma non l’unica) da tenere in considerazione. Proprio per tale motivo è necessario in primis chiarire il significato di “fallimento”.

1)Fallimento catastrofico: all’improvviso i prodotti smettono di lavorare. Ne sono un esempio le lampadine ad incandescenza.

2)Fallimento definito: per alcuni prodotti le prestazioni si degradano lentamente e non si può stabilire una chiara fine della vita utile.

3)Fallimento definito dal cliente: il prodotto “fallisce” quando è il cliente a dirlo.

Quale di queste definizioni di “fallimento” si deve utilizzare?

Non c’è in realtà una definizione più giusta delle altre. Ognuna ha un suo valore, ognuna fornisce delle informazioni che andranno esaminate separatamente.
Il tempo necessario al fallimento è solo una possibile misura dell'uso o dell'esposizione. Per alcuni prodotti vengono utilizzate altre misure: per i cuscinetti a sfera è il numero di giri, per molti prodotti è il numero di cicli di funzionamento (tostapane, lavastoviglie, celle di batteria ricaricabili, interruttori e interruttori automatici). Molti di questi prodotti "ciclati" hanno una garanzia del cliente per un tempo di calendario specificato (per esempio, un anno), sebbene l'utilizzo del ciclo differisca molto da cliente a cliente. Inoltre, se il prodotto funziona solo per una parte del tempo, si deve decidere se misurare il tempo di funzionamento effettivo o semplicemente utilizzare i giorni di calendario in servizio. I giorni di calendario sono di solito più facili e meno costosi da determinare ma, in ogni caso, la scelta dell'esposizione e il modo di misurarla sono decisioni ingegneristiche e gestionali.

- CAMPIONI DI PROVA REALISTICI

Molti campioni di prova differiscono dall'effettivo prodotto finale. Gli ingegneri si rendono conto che la vita del campione può essere diversa da quella del prodotto, quindi presumono che la vita del campione sia paragonabile o inferiore alla vita del prodotto.

- Fatica nei metalli.

I campioni metallici di fatica sono di solito piccoli cilindri, le parti sono generalmente più grandi, hanno una geometria complessa con spigoli vivi che sollevano localmente le sollecitazioni, hanno un trattamento superficiale diverso, ecc.

- Campione rappresentativo

Un campione rappresentativo è il migliore per stimare l'affidabilità di una popolazione di prodotti. Tale campione è eterogeneo e comprende campioni provenienti da molti periodi di produzione o lotti. Ad esempio, per confrontare due metodi di produzione, un campione omogeneo viene diviso in modo casuale tra i due metodi. A causa della minore variabilità tra campioni omogenei, il confronto rileverà minori differenze tra i due metodi. Allo stesso modo, le variabili di prova come l'umidità, la temperatura, ecc. possono essere mantenute costanti su tutti i campioni per ridurre al minimo la variabilità in un confronto.

- CONDIZIONI DI PROVA REALISTICHE

Le prove di via accelerate dovrebbero, idealmente, riprodurre esattamente le stesse condizioni d’uso del prodotto finito, ad eccezione dell’elevato livello della variabile di sovrasollecitazione. Tuttavia, molte prove differiscono molto dall'uso effettivo, ma possono comunque essere utili. Gli ingegneri presumono (sulla base dell'esperienza) che un prodotto che si comporta bene su un tale test si comporterà bene nell'uso effettivo. Inoltre, si assume che un design, materiale, fornitore, o metodo di produzione che si comporta meglio di un altro (progettazione, materiale, ecc.) in prova si comporterà meglio nell'uso effettivo.

- PROVE DI VITA ACCELERATE

Ma come effettuare una prova accelerata? Bisogna usare l'alta temperatura, carichi meccanici, vibrazioni o cos'altro? Quale tipo di sollecitazione, tra i vari tipi, dobbiamo scegliere?
Sollecitazioni standard. Per molti prodotti esistono metodi di prova standard. Ad esempio, le alte temperature e la tensione sono di solito usate per accelerare le prove di vita degli isolanti elettrici e dell'elettronica.
Sollecitazioni non standard. Per altri prodotti potrebbero non esserci metodi standard. Saranno gli ingegneri, attraverso lavori sperimentali, a determinare i giusti metodi. Tali sollecitazioni dovrebbero accelerare le modalità di guasto più probabili, evitando di accelerare le modalità di guasto che non si verificano alle condizioni di progetto.
Sollecitazioni multiple. Si può sottoporre il prodotto a più di una sollecitazione. Questo viene fatto per due principali motivi: sapere come varia la sua vita utile quando il prodotto è soggetto a più sollecitazioni simultanee; potrebbe non essere possibile aumentare una prima sollecitazione ad un valore tale da portare a rottura il prodotto, quindi in molti casi è necessario utilizzare una seconda sollecitazione. In questi casi di fondamentale importanza è capire quale delle due (o più) sollecitazioni sia la principale responsabile del degrado o della rottura del prodotto.
Sollecitazioni costanti. Nella pratica molti prodotti funzionano a sollecitazioni costanti. Così è nel caso degli isolanti termici, che lavorano a temperature e tensioni costanti; così è nel caso dei cuscinetti che lavorano a temperature e tensioni pressoché costanti. Nelle prove accelerate, quindi, è bene ricreare le stesse condizioni, di conseguenza carichi costanti sono sempre preferibili.

- ALTRE VARIABILI

Altri fattori che influenzano la vita utile del prodotto sono la progettazione del prodotto stesso, il materiale, la produzione, il funzionamento e le variabili ambientali.
Variabili continue e categoriche. Una variabile continua può assumere qualsiasi valore numerico in un intervallo continuo. Ad esempio, il contenuto di legante di un isolante può (teoricamente) avere un valore compreso tra 0 e 100%. Una variabile categoriale può invece assumere solo valori distinti (discreti). Per esempio, una variabile categorica è il turno di produzione che fa un campione; ha tre valori, giorno, sera e notte.
Variabili sperimentali. Diverse prove vengono effettuate variando il valore di determinati parametri. Si pensi ad un isolante sotto forma di nastro adesivo. Le variabili sono due: la quantità di nastro che si sovrappone a se stesso mentre viene avvolto attorno al conduttore e la quantità di nastro che si stacca dallo strato precedente.
Variabili costanti. Alcune variabili, in fase di test, vengono invece mantenute costanti. Nell'esempio precedente si faceva in modo che il nastro venisse tutto da un unico lotto omogeneo di produzione.
Variabili incontrollate osservate. Sono variabili che per l'appunto variano in modo incontrollato ma possono essere osservate e misurate sperimentalmente (per ogni campione). Nel caso di sopra questo tipo di variabile può essere rappresentato dal contenuto del legante o dal fattore di dissipazione. Se tali variabili avranno un effetto sulla vita utile del prodotto ovviamente andranno incluse nei dati, altrimenti possono essere trascurate.
Variabili incontrollate non osservate. Gli ingegneri sono consapevoli di questa tipologia di variabili (umidità dell'ambiente, temperatura durante la prova). Spesso il loro effetto si presume trascurabile.

- ERRORI DI MISURA

Nella misurazione delle variabili si commetteranno quasi sempre degli errori di misura. Questi, per quanto possibile, dovranno essere ridotti al minimo, utilizzando strumenti di misura quanto più precisi possibile.

5. TEST ACCELERATI COMUNI

Questa sezione ha lo scopo di descrivere brevemente alcuni comuni accelerati test: test sugli elefanti, test con una sola condizione o con più condizioni e burn in.

Elephant tests

Gli elephant tests si chiamano con molti nomi, tra cui killer test, test sui limiti di progettazione, prove di margine di progettazione, prove di qualificazione della progettazione. Se il prodotto supera il test gli ingegneri responsabili si sentono più fiduciosi. Se il prodotto fallisce, gli ingegneri prendono le misure appropriate e di solito lo riprogettano o migliorare la produzione per eliminare la causa del guasto.

Procedura di prova.

Una prova di elefante di questo tipo comporta generalmente uno o più campioni. Il campione può essere sottoposto ad un unico livello grave di stress (per esempio, la temperatura) o a più sollecitazioni, contemporaneamente o in successione.

- Pentole. Ad esempio, un produttore di pentole in ceramica utilizza un test dell'elefante per monitorare la qualità della produzione. La produzione viene campionata regolarmente. Ogni articolo viene riscaldato ad una determinata temperatura e immerso in acqua ghiacciata. Il ciclo di riscaldamento e di shock termico viene ripetuto fino a quando l'articolo non fallisce.

Scopi. Nel lavoro di progettazione e sviluppo della produzione, questi test hanno il compito di rivelare le modalità di guasto. Gli ingegneri cambiano poi il prodotto o la produzione processo per superare tali fallimenti. Questo importante uso dei test sugli elefanti è una pratica ingegneristica standard che spesso migliora un prodotto. Per il controllo qualità, i test sugli elefanti di campioni provenienti dalla produzione possono rivelare cambiamenti nel prodotto. I fallimenti dei test indicano un cambiamento di processo che ha degradato il prodotto. Per esempio, i compressori sono stati campionati dalla produzione e sottoposti ad una elevata pressione per la rapida individuazione di un guasto del lubrificante. Il test non può essere utilizzato su un nuovo progetto le cui parti metalliche non potrebbero resistere al test pressione. I test sugli elefanti sono anche utilizzati per confrontare qualitativamente diversi disegni, fornitori, metodi di fabbricazione, ecc.

Quando un test dell'elefante si definisce buono?

La risposta è facile: quando produce gli stessi guasti e nelle stesse proporzioni che si verificheranno durante il servizio. La domanda difficile è: come si fa a concepire un tale test, soprattutto per un nuovo progetto?

Che tipo di elefante (africano o asiatico)? Di che colore (grigio, bianco o rosa)? Di che sesso ed età? L'elefante deve calpestare il prodotto? Queste sono domande più difficili. Può essere usato più di un elefante? Se sì, simultaneamente o in sequenza?

Limitazioni. I test sugli elefanti forniscono solo informazioni qualitative riguardo la bontà di un prodotto. Per le applicazioni di cui sopra, tali informazioni sono sufficienti. Inoltre, il test dell'elefante si applica a prodotti complessi che possono essere assemblaggi di molti componenti diversi. Molte applicazioni di affidabilità richiedono una stima del tasso di guasto, percentuale di mancato funzionamento in garanzia, vita tipica, ecc. I test sugli elefanti non forniscono informazioni quantitative sull'affidabilità.

Una sola condizione di prova

Alcune prove di sovrasollecitazione per la stima dell'affidabilità comportano una singola condizione di prova. Poi la stima della distribuzione della vita del prodotto in una condizione d'uso dipende da alcuni presupposti. Spesso le ipotesi sono scarsamente soddisfatte, e le stime potrebbero essere piuttosto crude. Si descrivono brevemente i due modelli caratteristici di tale test: il fattore di accelerazione e la dipendenza parzialmente nota della vita dalla sollecitazione. Inoltre, tali test sono utilizzati per prove dimostrative e per confrontare progetti, materiali, metodi di produzione, ecc.

Fattore di accelerazione

Definizione. Un certo test fa girare un motore diesel al 102% della potenza nominale e si presume che abbia un fattore di accelerazione pari a 3. In parole povere, questo significa che se un motore ha funzionato per 400 ore fino al guasto durante il test, si presume che avrebbe funzionato 3 x 400 = 1200 ore al guasto in servizio. Ipotizziamo che il fattore di accelerazione in un secondo test sia di 5. Allo stesso modo, se un motore funziona 300 ore senza fallire nella seconda prova, si suppone che avrebbe funzionamento 5 x 300 = 1500 ore senza guasti in servizio.

Questo metodo comporta varie ipotesi matematiche che nella pratica possono essere parzialmente valide.

I) Fattore noto. Di solito si assume che il fattore di accelerazione sia "conosciuto". A volte viene stimato dai dati di prova e dai dati sul campo di un precedente design del prodotto. La conversione del tempo di prova in tempo di servizio e la successiva analisi dei dati (in particolare gli intervalli di confidenza) non tengono conto delle incertezze nel fattore. Tale incertezza è dovuta alla casualità del fattore campioni statistici del test precedente e dei dati di servizio. Ancora più importante, il fattore si basa su progetti precedenti. Quindi questo metodo presuppone (spesso in errore) che il nuovo progetto ha lo stesso fattore di accelerazione.

2) Stessa forma. C'è una sottile supposizione nel moltiplicare i dati di prova per il fattore. Vale a dire, questo implica che la distribuzione della vita utile è semplicemente un multiplo della distribuzione della vita di prova. Di conseguenza è come se si assumesse che le due distribuzioni abbiano la stessa forma, ma la distribuzione della vita utile è allungata verso una vita più elevata di una quantità pari al fattore. In altre parole, il parametro della scala della distribuzione della vita utile è un multiplo di quello della distribuzione dei test. Questo potrebbe non essere così.

3) Modalità di guasto. Spesso ci sono più di una modalità di guasto nei dati. Quindi, se si ignorano le diverse modalità, si suppone che tutte abbiano lo stesso fattore di accelerazione. Tipicamente, i diversi modi di guasto hanno differenti fattori di accelerazione. Ciò è particolarmente vero per i prodotti complessi costituito da assemblaggi di diversi componenti.

Burn-in

Il burn-in consiste nel far funzionare le unità in condizioni di progetto o accelerate per un periodo di tempo adeguato. Il burn-in è un'operazione di produzione che tende a far fallire unità di breve durata (difetti, a volte chiamati freaks). Se il burn- in funziona, le unità sopravvissute che entrano in servizio hanno pochi guasti precoci. Le unità che si guastano precocemente hanno tipicamente difetti di fabbricazione. Il burn-in è principalmente utilizzato per i componenti elettronici e gli assemblaggi.

6. CONSIDERAZIONI STATISTICHE

Qui di seguito sono descritte alcune considerazioni che saranno utili nel proseguo.

Modelli statistici. Unità nominalmente identiche, realizzate e utilizzate sotto le stesse condizioni, di solito hanno valori diversi di prestazioni, dimensioni, vita, ecc. Tale variabilità è insita in tutti i prodotti e può essere descritta da un modello statistico o da una distribuzione.

Popolazione e campione. Generalmente per analizzare le informazioni su di una popolazione di oggetti si fa sempre riferimenti ad un elemento, definito campione, che ne racchiude tutte le caratteristiche. Si tenga a mente che i dati calcolati in precedenza (ad esempio l'anno prima) potrebbero non essere corretti nell'immediato, i dati raccolti in laboratorio possono differire da quelli raccolti sul campo.

Natura dell'analisi dei dati. La soluzione statistica di un problema reale che coinvolge l'analisi dei dati ha sette passi fondamentali.

- Dichiarare chiaramente il vero problema e lo scopo dell'analisi dei dati. In particolare, specificare le informazioni numeriche necessarie per disegnare conclusioni e prendere decisioni.
- Formulare il problema in termini di modello.

- Pianificare sia la raccolta che l'analisi dei dati che produrranno i dati numerici desiderati.
- Ottenere dati appropriati per la stima dei parametri del modello.
- Adattare il modello ai dati e ottenere le informazioni necessarie dal modello montato.
- Verificare la validità del modello e dei dati. Se necessario, cambiare il modello, omettere alcuni dati, o raccoglierne altri (spesso trascurati dagli statistici), e rifare i passi 5 e 6.
- Interpretare le informazioni fornite dal modello montato per facilitare il disegno conclusioni e prendere decisioni per il problema ingegneristico.

MODELS FOR LIFE TESTS WITH CONSTANT STRESS

1. INTRODUCTION

Purpose. This chapter presents mathematical models for accelerated life tests with constant stress. These models are essential background for subsequent chapters. All planning and data analyses for accelerated tests are based on such models. A model depends on the product, the test method, the accelerating stress, the form of the specimen, and other factors.

Model. A statistical model for an accelerated life test consists of

- a life distribution that represents the scatter in product life;
- a relationship between "life" and stress.

Usually the mean (and sometimes the standard deviation) of the life distribution is expressed as a function of the accelerating stress. Such relationships express a distribution parameter (such as a mean, percentile, or standard deviation) as a function of the accelerating stress and possibly of other variables. The most widely used basic relationships are

- the Arrhenius relationship for temperature-accelerated tests;
- the inverse power relationship

Standard Models. For many products, there are standard accelerating variables and models. For example, life testing of motor insulation is usually accelerated with high temperature, and the data are analyzed with the Arrhenius-lognormal model. For some products with a standard accelerating variable, the form of the life distribution or the life-stress relationship may be in question. For example, for certain voltage-endurance testing of an insulation, the Weibull and lognormal distributions and the inverse power and other relationships were fitted to the data ; the various distributions and relationships were compared to assess which fitted significantly better.

New models. For still other products, one may need to choose an accelerating variable and to develop and verify an appropriate model. Such work involves long-term effort of product experts, perhaps abetted by a skilled statistician.

Single failure cause. The models here are best suited to products that have only one cause of failure. However, they also adequately describe many products that have a number of causes.

Multiples tests. For some products there are two or more accelerated tests with different accelerating variables. Each test is run on a different set of units to accelerate different failure modes. For example, certain failure modes may be accelerated by high temperature, while others are accelerated by high voltage or vibration.

2. BASIC CONCEPTS AND THE EXPONENTIAL DISTRIBUTION

This section presents basic concepts for product life distributions.

2.1. Cumulative Distribution Function

Definition. A cumulative distribution function F(t) represents the population fraction failing by age t. Any such continuous F(t) has the mathematical properties:

- it is a continuous function for all t,
- $\lim_{t\to-\infty} F(t) = 0 \;\; and \;\; \lim_{t\to\infty} F(t) = 1$
- F(t) $\leq F(t')$ for all $t < t'$.

The range of t for most life distributions is from 0 to ∞, but some useful distributions have a range from −∞ to ∞.

Exponential cumulative distribution function. The population fraction failing by age t is

$$F(t) = 1 - e^{-t/\theta} \quad t \geq 0$$

$\theta > 0$ is the mean time to failure. θ is in the same measurement units as t, for example, hours, months, cycles, etc. Figure 2.1 shows this cumulative distribution function. Its failure rate is defined as

$$\lambda \equiv 1/\theta$$

and is a constant. This relationship between the constant failure rate λ and the θ holds only for the exponential distribution. λ is expressed in failures per million hours, percent per month, and percent per thousand hours. In terms of λ,

$$F(t) = 1 - e^{-\lambda t}$$

The exponential distribution describes the life of insulating oils and fluids (dielectrics) and certain materials and products. It is often badly misused for products better described with the Weibull or another distribution. Some elementary reliability books mistakenly suggest that the exponential distribution describes many products. In the author's experience, it adequately describes only 10 to 15% of products in the lower tail of the distribution.

2.2. Reliability Function

Definition. The reliability function R(t) for a life distribution is the probability of survival beyond age t, namely,

$$R(t) \equiv 1 - F(t)$$

This is also called the survivor or survivorship function.

2.3. Exponential reliability

The population fraction surviving age t is

$$R(t) = e^{-t/\theta}$$

Figure 2.2 shows this reliability function. It is the cumulative distribution function (Figure 2.1) "turned over."

2.4. Percentile

Definition. The 100Pth percentile of a distribution F ( ) is the age τ_p by which a proportion P of the population fails. It is the solution of

$$P = F(\tau_p)$$

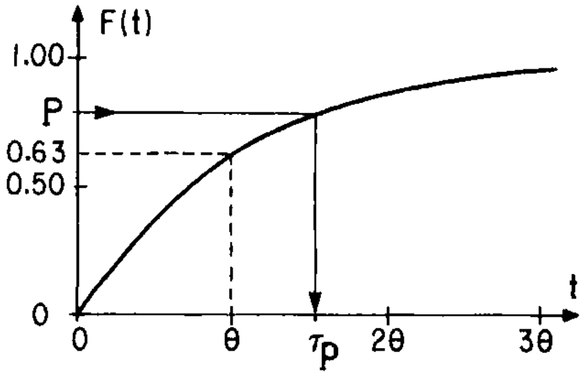

Figure 2.1. **Exponential cumulative distribution.**

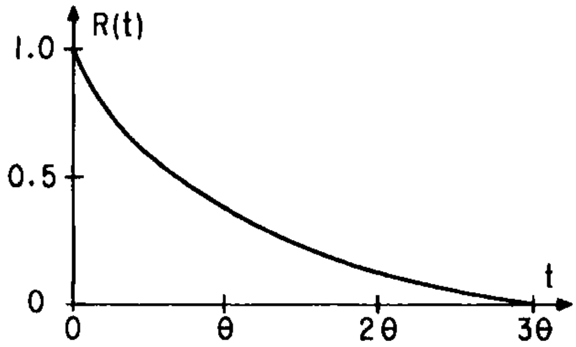

Figure 2.2. **Exponential reliability function.**

In life data work, one often wants to know low percentiles such as the 1% and 10% points, which correspond to early failure. The 50% point is called the median and is commonly used as a "typical" life. τ_p can be obtained as shown in Figure 2.1. Enter the figure on the vertical axis at the value P, go horizontally to the curve for F(t), and go down to the time axis to read τ_p.

2.5. Probability Density

Definition. The probability density is

$$f(t) = \frac{dF(t)}{dt}$$

which must exist mathematically. It corresponds to a histogram of the population life times. Equivalently, the population fraction failing by age t is the integral of the previously equation, namely

$$F(t) = \int_{-\infty}^{t} f(u)\, du$$

If the lower limit of a distribution range is 0, the integral ranges from 0 to t. Similarly,

$$R(t) = \int_{t}^{\infty} f(t) dt$$

2.6. Mean

Definition. The mean or expectation E (T) of a distribution for random time to failure T with probability density f(t) is the value of the integral

$$E(T) \equiv \int_{-\infty}^{\infty} tf(t)dt$$

The integral runs over the range of the distribution (usually 0 to ∞ or -∞ to ∞). The mean is also called the average or expected life. It correspond to the arithmetic average of the lives of all units in a population. Like the median, it is used as still another "typical" life.

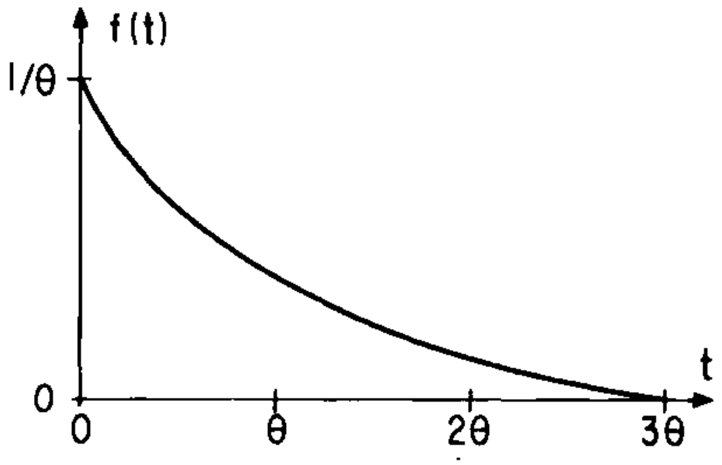

Figure 2.3. Exponential probability density.

2.7. Exponential mean.

The mean is

$$E(T) = \int_{0}^{\infty} t\,({}^{1}\!/_{\theta})\, e^{-t/\theta}\, dt = \theta$$

This shows why θ is called the mean time to failure (MTTF).

2.8. Variance and Standard Deviation

Definition. The variance of a distribution with a probability density f(t) is

$$Var(T) = \int_{-\infty}^{\infty} [t - E(T)]^2\, f(t)dt$$

The integral runs over the range of the distribution. The variance is a measure of the spread of the distribution. Var(T) has the units of time squared, for example, hours squared. Used by statisticians, variance is equivalent to standard deviation below, which is easier to interpret.

2.9. Exponential standard deviation.

For an exponential distribution

$$\sigma(T) = (\theta^2)^{1/2}$$

3. NORMAL DISTRIBUTION

This section presents the normal (or Gaussian) distribution. Its hazard function increases without limit. Thus it may describe products with wear-out failure. It has been used to describe the life of incandescent lamp (light bulb) filaments and of electrical insulations. It is also used as the distribution for product properties such as strength (electrical or mechanical), elongation, and impact resistance in accelerated tests. It is important to understanding and using the lognormal distribution, which is widely used to interpret accelerated test data. Also, the sampling distribution of many estimators is approximately normal.

3.1. Normal cumulative distribution function

The population fraction failing by age y is

$$F(y) = \int_{-\infty}^{y} (2\pi\sigma^2)^{-1/2}\ exp\left[-\frac{1}{2}(\frac{x-\mu}{\sigma})^2\right]dx \qquad -\infty < y < \infty$$

Figure 3.1 depicts this function. μ is the population mean and may have any value. σ a is the population standard deviation and must be positive. μ and σ are in the same measurement units as *y*, for example, hours, months, cycles, etc. (3.1) can be expressed in terms of the standard normal cumulative distribution function $\Phi()$ as

$$F(y) = \Phi[(y-\mu)/\sigma] \qquad -\infty < y < \infty$$

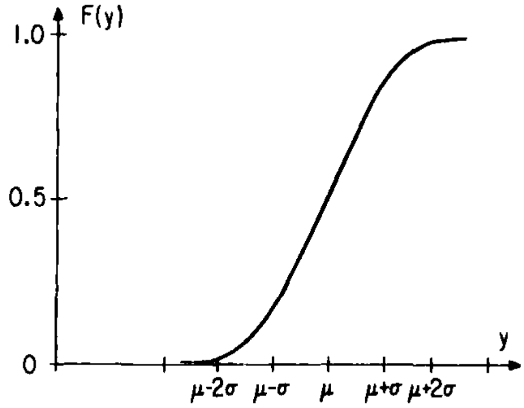

Figure 3.1. Normal cumulative distribution.

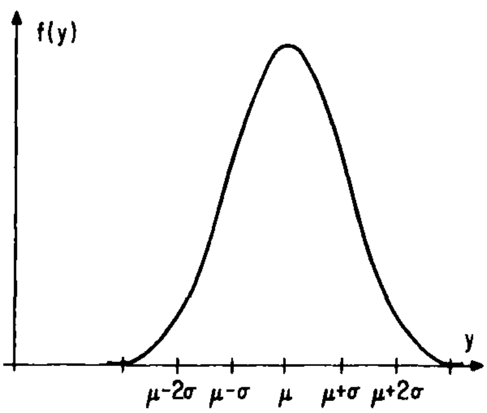

Figure 3.2. Normal probability density.

3.2. Normal probability density

The probability density is

$$f(y) = (2\pi\sigma^2)^{-1/2} \exp\left[-\frac{(y-\mu)^2}{2\sigma^2}\right] \qquad -\infty < y < \infty$$

Figure 3.2 depicts this probability density, which is symmetric about the mean μ. The figure shows that μ is the median and σ determines the spread.

3.3. Normal percentile

The 100Pth percentile is

$$\eta_p = \mu + z_p\sigma$$

z_p is the 100Pth standard normal percentile. The median (50th percentile) of the normal distribution is $\eta_{.50} = \mu\,, since\ z_{.50} = 0$

3.4. Normal mean and standard deviation

For the normal distribution,

$$E(Y) = \mu \quad and \quad \sigma(Y) = \sigma$$

4. LOGNORMAL DISTRIBUTION

The lognormal distribution is widely used for life data, including metal fatigue, solid state components (semiconductors, diodes, etc.), and electrical insulation. The lognormal and normal distributions are related; this fact is used to analyze lognormal data with methods for normal data.

4.1. Lognormal cumulative distribution

The population fraction failing by age t is

$$F(t) = \Phi\{[\log(t) - \mu]/\sigma\} \qquad t > 0$$

Figure 4.1 shows lognormal cumulative distribution functions. μ is the mean of the log of life. μ is called the log mean and may have any value from $-\infty$ to ∞.

σ is the standard deviation of the log of life. σ is called the log standard deviation and must be positive. μ and σ are not "times" like t; instead they are unitless pure numbers. The cumulative distribution can also be written as

$$F(t) = \Phi\{[\log(t/\tau_{.50})]/\sigma\} = \Phi\{\log(t/\tau_{.50})\}^{1/\sigma}$$

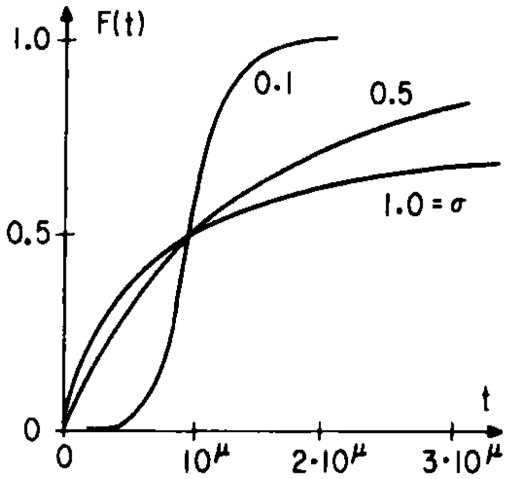

Figure 4.1. Lognormal cumulative distribution.

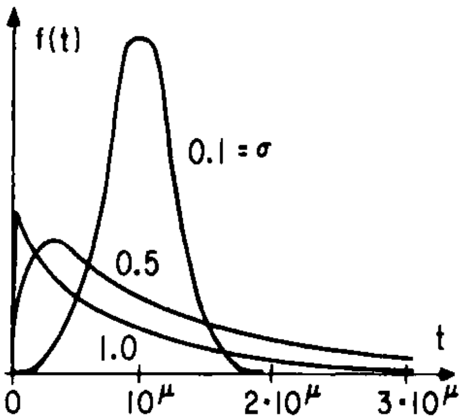

Figure 4.2. Lognormal probability densities.

4.2. Lognormal probability density

For a lognormal distribution,

$$f(t) = \left\{\frac{0.4343}{\left[(2\pi)^{1/2} t\sigma\right]}\right\} \exp\left\{-\frac{[\log(t) - \mu]^2}{2\sigma^2}\right\} \qquad t > 0$$

Figure 4.2 shows probability densities, which have a variety of shapes. The value of a determines the shape of the distribution, and the value of determines the 50% point and the spread in life t.

4.3. Percentile

The 100Pth lognormal percentile is

$$\tau_p = antilog\left[\mu + z_p\sigma\right] = 10^{\mu+z_p\sigma}$$

here z_p is the 100Pth standard normal percentile. The median (50th percentile) is $\tau_{.50} = antilog[\mu]$.

4.4. Lognormal reliability function

The population fraction surviving age t is

$$R(t) = 1 - \Phi\left\{\frac{[\log(t) - \mu]}{\sigma}\right\} = \Phi\{-[\log(t) - \mu/\sigma]\}$$

Relationship with the normal distribution. Suppose life t has a lognormal distribution with parameters μ and σ. Then the (base 10) log of life y = log(t) has a normal distribution with mean μ and standard deviation σ. Thus the analysis methods for normal data can be used for the logarithms of lognormal data.

4.5. Base e lognormal

Much engineering work now employs the base e lognormal distribution. The base e lognormal cumulative distribution is

$$F(t) = \Phi\{[\ln(t) - \mu_e]/\sigma_e\}$$

5. WEIBULL DISTRIBUTION

The Weibull distribution is often used for product life, because it models either increasing or decreasing failure rates simply. It is also used as the distribution for product properties such as strength (electrical or mechanical), elongation, resistance, etc., in accelerated tests. It is used to describe the life of roller bearings, electronic components, ceramics, capacitors, and dielectrics in accelerated tests. According to extreme value theory, it may describe a "weakest link" product. Such a product consists of many parts from the same life distribution, and the product fails with the first part failure.

5.1. Weibull cumulative distribution.

The population fraction failing by age t

$$F(t) = 1 - exp\left[-(t/\alpha)^{\beta}\right]$$

The shape parameter β (“slope” parameter) and the scale parameter α (characteristic life) are positive. It is always the 63.2th percentile. α has the same units as t, for example, hours, months, cycles, etc. β is a unitless pure number. For most products and materials, p is in the range 0.5 to 5. Figure 5.1 shows Weibull cumulative distribution functions.

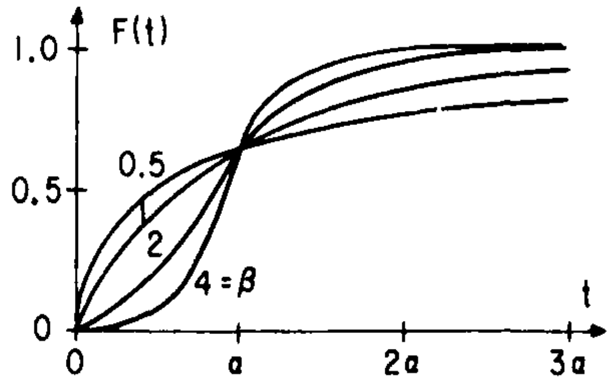

Figure 5.1. Weibull cumulative distributions.

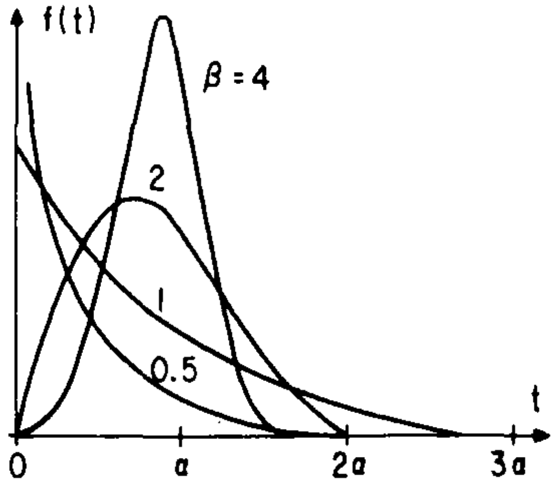

Figure 5.2. Weibull probability densities.

5.2. Weibull probability density

For a Weibull distribution,

$$f(t) = \left(\frac{\beta}{\alpha^{\beta}}\right) t^{\beta-1}\, exp\left[-(t/\alpha)^{\beta}\right], \quad t > 0$$

The Weibull probability densities in Figure 5.2 show that β determines the shape of the distribution and α determines the spread. β determines the spread in log life; high (low) β corresponds to small (great) spread. For β = 1, the Weibull distribution is the exponential distribution. For much life data, the Weibull distribution fits better than the exponential, normal, and lognormal distributions.

5.3. Weibull reliability function

The population fraction surviving age t is

$$R(t) = exp\left[-(t/\alpha^{\beta})\right], \ \ t > 0$$

5.4. Weibull percentile

The 100Pth percentile of a Weibull distribution is

$$\tau_p = \ \alpha[-\ln(1-P)]^{1/\beta}$$

5.5. Relationship to the Exponential Distribution

The following relation is used later to analyze Weibull data in terms of the simpler exponential distribution. Suppose time T to failure has a Weibull distribution with parameters α and β. Then T' = T^{β} has an exponential distribution with mean $\theta = \ \alpha^{\beta}$. In such analyses, one assumes β is known, and α and other quantities are estimated from the data.

5.6. Weibull versus lognormal

In many applications, the Weibull and lognormal distributions (and others) may fit a set of data equally well, especially over the middle of the distribution. When both are fitted to a data set, the Weibull distribution has an earlier lower tail than the corresponding lognormal distribution. That is, a low Weibull percentile is below the corresponding lognormal one. Then the Weibull distribution is more pessimistic.

6. EXTREME VALUE DISTRIBUTION

The (smallest) extreme value distribution is needed background for analytic methods for Weibull data. Indeed the (base e) log of time to failure for a Weibull distribution has an extreme value distribution. The extreme value distribution also describes certain extreme phenomena; these include electrical strength of materials and certain types of life data. Like the Weibull distribution, the smallest extreme value distribution may be suitable for a "weakest link" product. In other words, suppose a product consists of many nominally identical parts from the same strength (life) distribution (unbounded below) and the product strength (life) is that of the weakest (first) part to fail. Then the smallest extreme value distribution may describe the strength (life) of units. An example is the life or electrical strength of cable insulation. That is, a cable may be regarded as consisting of many segments, and the cable fails when the first segment fails.

6.1. Extreme value cumulative distribution

The population fraction below y is

$$F(y) = 1 - exp\left\{- exp\left[\frac{y-\xi}{\delta}\right]\right\}, \qquad -\infty < y < \infty$$

The location parameter may have any value from $-\infty\ to\ \infty$. The scale parameter ξ is positive, and it determines the spread of the distribution. $\xi$and δ are in the same units as y, for example, hours, cycles, etc. Figure 6.1 depicts this function. The distribution range is $-\infty\ to\ \infty$. Lifetimes must, of course, be positive. Thus the fraction below zero must be small for this to be a satisfactory life distribution.

6.2. Extreme value density

For an extreme value distribution,

$$f(y) = \left(\frac{1}{\delta}\right) exp[(y-\xi)] exp\left\{-exp\left[\frac{y-\xi}{\delta}\right]\right\}, \qquad -\infty < y < \infty$$

Figure 6.2 shows the probability density, which is asymmetric.

6.3. Extreme value mean and standard deviation

For an extreme value distribution,

$$E(Y) = \ \xi - 0.5772\delta,\ \ \sigma(Y) = 1.283\delta$$

here 0.5772 is Euler's constant and 1.283 is $\pi/\sqrt{6}$.

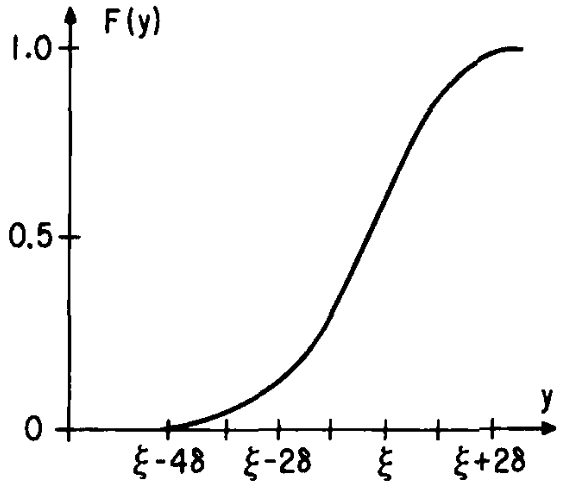

Figure 6.1. Extreme value cumulative distribution.

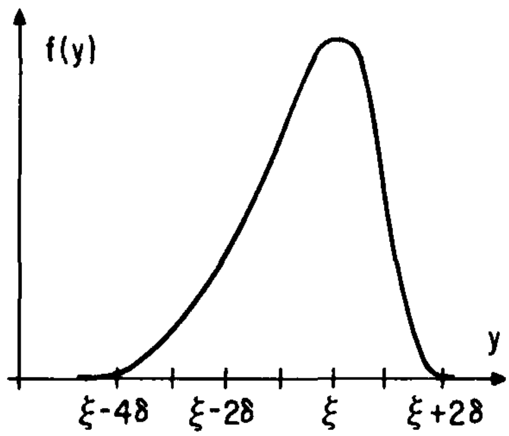

Figure 6.2. Extreme value density.

6.4. Relationship to the Weibull distribution

The extreme value distribution is used to analyze Weibull data. The following relationships are used to analyze the base e logs of Weibull data. The log data are easier to analyze with the simpler extreme value distribution, because it has a single shape and simple location and scale parameters, similar to the normal distribution. Suppose a Weibull life distribution has shape and scale parameters $\beta$and α. The (base e) logarithm $y = \ln t$ of life $t$has an extreme value distribution with

$$\xi = \ln \alpha , \quad \delta = 1/\beta$$

The last equation shows that the spread in In life is the reciprocal of β. Thus low corresponds to high spread of log life, and high β to low spread. The Weibull parameters can be expressed as

$$\alpha = \exp(\xi) , \quad \beta = 1/\delta$$

Similarly, the Weibull parameters in terms of the standard deviation $\sigma(Y)$ and mean $E(Y)$ of the extreme value distribution are

$$\beta = \frac{1.283}{\sigma(Y)}, \quad \alpha = exp[E(Y) + 0.4501\sigma(Y)]$$

7. OTHER DISTRIBUTIONS

The basic distributions above are commonly used for accelerated tests. The following other distributions and ideas may be useful. They include failure at time zero, eternal survivors, a mixture of distributions, the generalized gamma distribution, and nonparametric analysis.

- *Distributions with failure at time zero.* A fraction of a population may already be failed at time zero or fail soon after. For example, consumers may purchase a product that does not work when installed. The model for this consists of the proportion p failed at time zero and a continuous life distribution for the rest. Such a cumulative distribution appears in Figure 7.1. The sample proportion failed at time zero is used to estimate p, and the other sample failure times are used to estimate the continuous distribution.

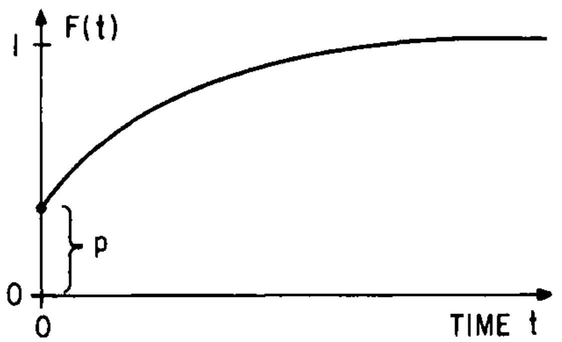

Figure 7.1. A cumulative distribution with failures at time zero.

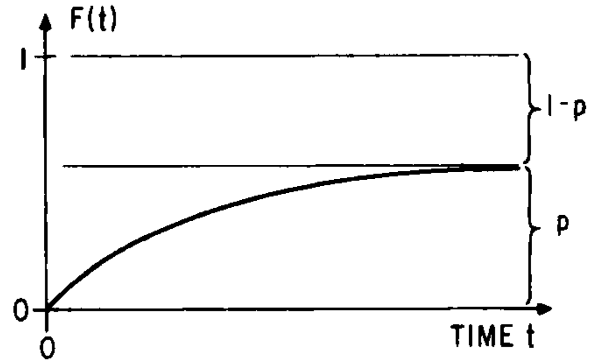

Figure 7.2. A cumulative distribution with eternal survivors.

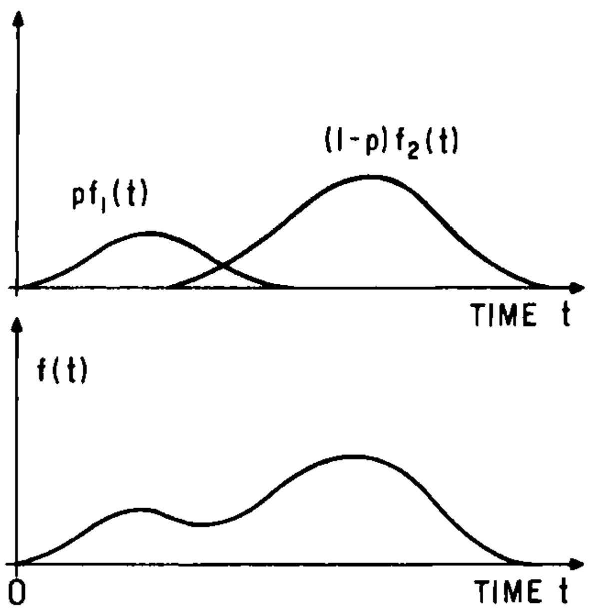

Figure 7.3. A mixture of distributions.

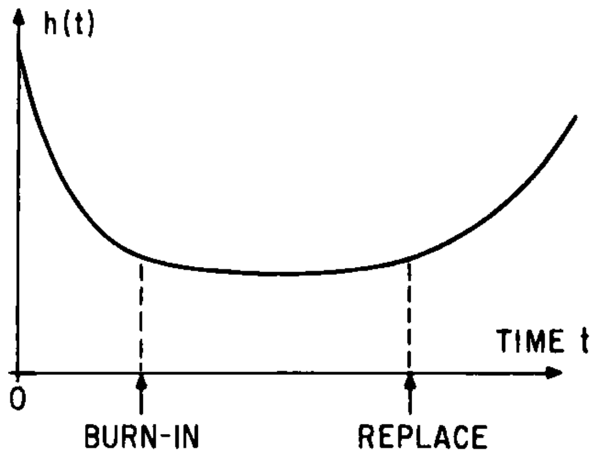

Figure 7.4. "Bathtub curve" hazard function.

- Distributions with eternal survivors. Some units may never fail. This applies to (1) the time to death from a disease when some individuals are immune, (2) the time to redemption of trading stamps (some stamps are lost and never redeemed), (3) the time to product failure from a particular defect when some units lack that defect, and (4) time to warranty claim on a product whose warranty applies only to original owners, some of which sell the product before failure. Figure 7.2 depicts such a cumulative distribution. Meeker (1985,1987) presents an application to integrated circuits.

- Mixtures of distributions. A population may consist of two or more subpopulations. Figure 7.3 depicts two subpopulations comprising proportions p and $1-p$ of the population. Units from different production periods may have different life distributions due to differences in design, raw materials, environment, usage, etc. It is often important to identify such a situation and the production period, customers, environment, etc., with poor units. Then suitable action may be taken on that portion of the population. If two subpopulations have cumulative distribution functions $F_1(t)$ and $F_2(t)$, then the population has the cumulative distribution function

$$F(t) = pF_1(t) + (1-p)F_2(t)$$

Such a mixture should be distinguished from competing failure modes. Everitt and Hand (198.1), McLachlan and Basford (1987), and Titterington, Smith, and Makor (1986) treat mixture distributions in detail. Hahn and Meeker (1982) offer practical advice on mixtures in analyzing product life data. Peck and Trapp (1978) divide certain semiconductors into two subpopulations; they call the early failures "freaks," which may be as much as 20 or 30% of a product starting development and typically 1 to 2% of a mature product. Vaupel and Yashin (1985) show how one may badly misinterpret life data when unaware that the population is a mixture.

- *The bathtub curve*. Some products have a decreasing failure rate in the early life and an increasing failure rate in later life. Figure 7.4 shows such a hazard function, called a "bathtub curve." However, most products have a failure rate that just decreases throughout their observed life or else just increases. Thus, for many products, the bathtub curve does not hold water. It commonly appears in reliability books but describes only 10 to 15% of the author's applications, usually products with competing failure modes (Chapter 7). Hahn and Meeker (1982) carefully distinguish between models for a mixture of subpopulations and those for competing failure modes. Both situations can have a bathtub failure rate for the population.

- *Burn-in*. Some products, such as high-reliability capacitors and semiconductor devices have a decreasing failure rate and are subjected to a burn-in. This weeds out early failures before units are put into service. Such burn-in is most effective if the population is a mixture of a small subpopulation of defectives (from manufacturing problems) that all fail early and a main population with satisfactory life. Peck and Trapp (1978) and Jensen and Petersen (1982) comprehensively treat planning and analysis of burn-in procedures, including the economics and accelerated burn-in. Such burn-in to weed out early failures is one of the purposes of environmental stress screening (ESS), also called shake and bake. Tustin (1986) surveys ESS. Also, some other products may be removed from sersice before wear-out starts. Thus units are in service only in the low failure rate portion of their life. This increases their reliability in service.

- *Generalized gamma distribution*. Farewell and Prentice (1977), Kalbfleisch and Prentice (1980), Cohen and Whitten (1988), and Lawless (1982) present the generalized gamma distribution. It includes the lognormal and Weibull distributions as special cases. The distribution (of log life) has three parameters (location, scale, and shape). When the distribution is fitted to data, the estimate of the shape parameter is used to compare the Weibull and lognormal fits. Such a comparison is useful if experience does not suggest either distribution. Farewell and Prentice have a computer program that fits this distribution to censored data from an accelerated test where the location parameter is a linear function of (possibly transformed) stress. Bowman and Shenton (1987) survey the simpler gamma distribution.

- Nonparametric analysis. Nonparametric analysis of data does not involve an assumed (parametric) form of the distribution; that is, fitting is distribution free. Widely used for biomedical life data, nonparametric estimates are rarely used for engineering data. First, nonparametric estimates are not as accurate as parametric ones, provided the assumed parametric distribution is adequate. Second, nonparametric estimates of percentiles or fraction failed outside the range of the sample data do not exist; that is, one cannot extrapolate nonparametrically into the lower or upper tail of the distribution. Nonparametric fitting of distributions and regression models to censored life data appears in various biomedical books. These include (from basic to advanced) Lee (1980), Miller (1981), Kalbfleisch and Prentice (1980), Cox and Oakes (1984), Viertl (1988), and Lawless (1982). They all present the widely used Cox model, also called the proportional hazards model. All such regression models employ parametric relationships between life and stress or other variables; only the life distribution does not have an assumed parametric form.

- *Birnbaum-Saunders distribution*. Birnbaum and Saunders (1969) proposed this distribution to describe metal fatigue. They mathematically derive their distribution from a model for crack propagation. Its cumulative distribution function is

$$F(t) = \Phi\left\{\left[(t/\beta)^{1/2} - (\beta/t)^{1/2}\right]/\alpha\right\} \quad t > 0$$

here $\Phi()$ is the standard normal cumulative distribution function, is the median, and determines the distribution shape. It is an alternative to the lognormal distribution, which is widely used to describe metal fatigue life. It is comparable to the lognormal distribution in important respects. This cumulative distribution is close to a lognormal one for small a (usually found in practice), say, $\alpha < 0.3$. Then the corresponding (base e) lognormal parameters are approximately $\mu_e = \ln\beta \ \ and \ \sigma_e \approx \alpha$. Its hazard function is zero at $t = 0$, increases to a maximum with age, and then finally decreases to a constant value; the lognormal hazard function is similar but finally decreases to zero. The distribution has a shorter lower tail than the corresponding lognormal one. That is, its 0.1% point is above the corresponding lognormal 0.1% point. This assumes that the distributions are matched by equating two percentiles above 0.1% or matched by some other reasonable means.

8. LIFE-STRESS RELATIONSHIPS

This section motivates the life-stress relationships and models in following sections. These relationships are for constant-stress tests. Indeed, many products run at nominally constant stress in actual use and at constant stress in accelerated tests.

8.1. Relationships

Typical life data from a constant-stress test are plotted as x's against stress in Figure 8.1A. The figure has linear scales for life and for stress. Lifetimes at low stress tend to be longer than those at high stress. Also, the scatter in life is greater at low stress than at high stress. The smooth curve through the data represents "life" as a function of stress. Engineering theory for some curves does not specify exactly what "life" means; it is some "nominal" life, which is not made precise.

Data can be conceptually simpler when plotted on paper with logarithmic or other suitable scales. On suitable paper the plotted points tend to follow a straight line, as in Figure 8.1B. Then a straight line through the data represents the life-stress relationship between product "life" and stress. A straight line is easier to fit to the data than a curve. Moreover, it is mathematically easy to extrapolate the straight line to a low stress to estimate the nominal life there, assuming the straight line is adequate. On the other hand, it is difficult to extrapolate a curve like that in Figure 8.1A. Use of a straight line on a special plotting paper is equivalent to using a particular equation to represent life versus stress.

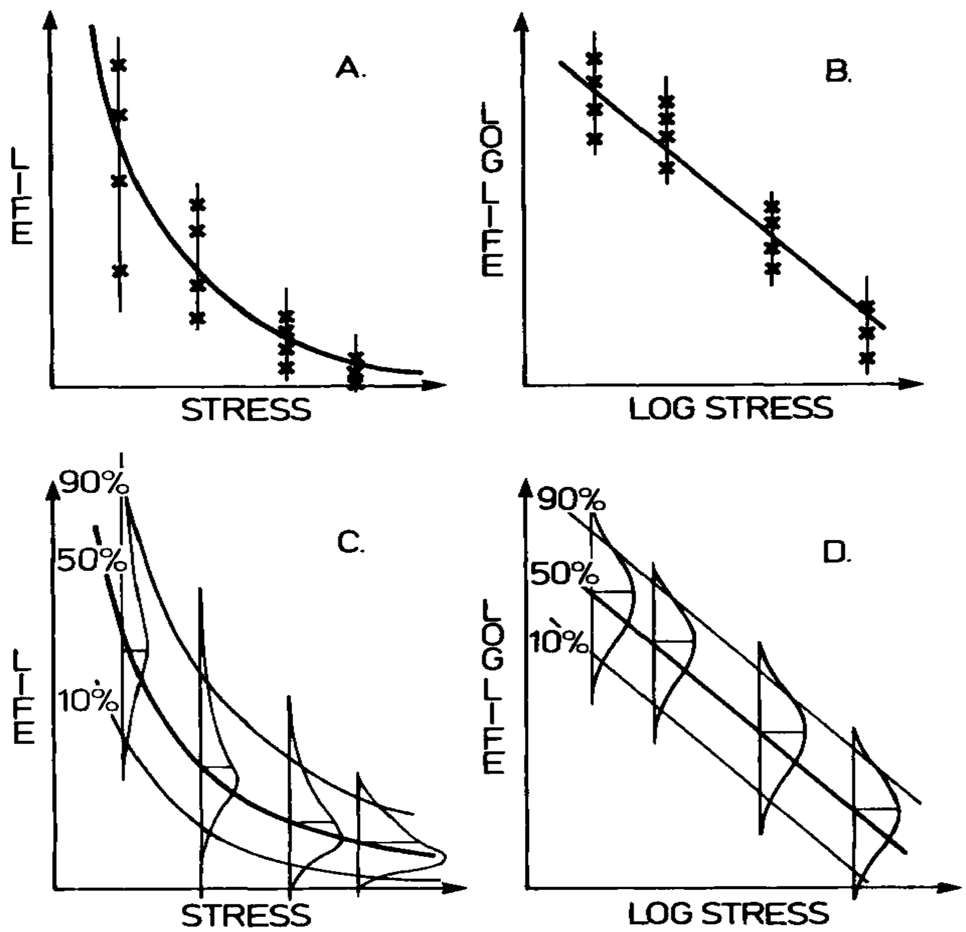

Figure 8.1. Failure time versus stress.

8.2. Models with Distributions and Relationships

A simple relationship does not describe the scatter in the life of the test units. For each stress level, the units have some statistical distribution of life. A more refined model employs a statistical distribution to describe the scatter in life. Figure 8.1C depicts such statistical distributions. The curve for the probability density (histogram) of life at a stress would be perpendicular to the page, but it has been drawn flat on the figure. A heavy curve passes through the 50 percent point of the distribution at each stress. Lighter curves pass through the 10 and 90 percent points. Such a curve can be imagined for any percentile. Thus the model here consists of a combination of a life distribution and a life-stress relationship. The percentile curves depict the model.

Many such models are simpler on plotting paper (with logarithmic or other suitable scales) where the relationship between life and stress is a straight line. The model in Figure 8.1C is depicted in Figure 8.1D on paper on which the relationship is a straight line. The relationships for other percentiles of the life distributions plot as parallel straight lines for many models, as shown in the figure.

8.3. Distribution Plots

In Figure 8.1, life data are plotted against test stress. Another plot of such data is useful. Figure 8.2A shows a plot of the cumulative percentage of the sample that has failed as a function of time. The plotted points are

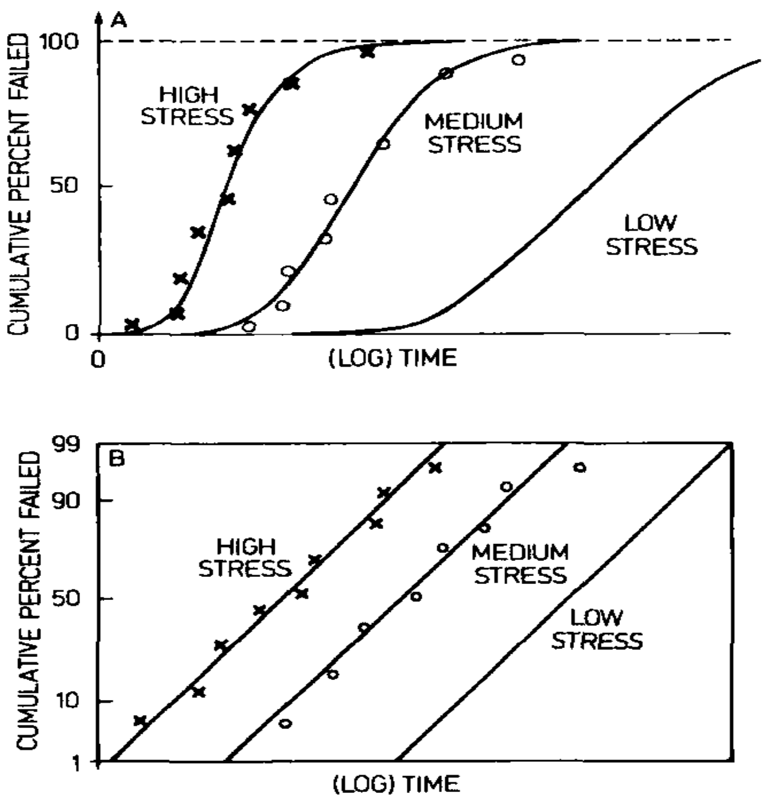

Figure 8.2. **Cumulative percentage failed versus time.**

The sample failure times, and the smooth curve depicts the population cumulative percentage failing as a function of time. The plot shows sample data for two stresses and the corresponding population distributions. Figure 8.2A also shows the population cumulative distribution for a low design stress.

Such a plot is simpler on probability paper. Such paper has a suitable data scale (usually logarithmic) for time and a probability scale for the cumulative percentage failed. One uses a paper on which the plotted points tend to follow a straight line, as in Figure 8.2B. Then a straight line represents the population cumulative percentage failing as a function of time at that stress. There are probability plotting papers for the exponential, normal, lognormal, Weibull, extreme value, and other distributions. A straight line on a probability paper is a cumulative distribution function for its distribution. The straight lines for the various stresses are parallel in Figure 8.2B. The Arrhenius model (Section 9) and the inverse power law model (Section 10) have distributions that are parallel lines on suitable probability paper. More general models need not have distributions that plot as parallel straight lines on such paper. Instead the distributions plot as curves that do not cross.

9. ARRHEMUS LIFE-TEMPERATURE RELATIONSHIP

Applications. The Arrhenius life relationship is widely used to model product life as a function of temperature. Applications include

- electrical insulations and dielectrics;
- solid state and semiconductor devices;
- battery cells;
- lubricants and greases;
- plastics;
- incandescent lamp filaments.

Based on the Arrhenius Law for simple chemical-reaction rates, the relationship is used to describe many products that fail as a result of degradation due to chemical reactions or metal diffusion. The relationship is adequate over some range of temperature.

9.1. The Relationship

Arrhenius law. According to the Arrhenius rate law, the rate of a simple chemical reaction depends on temperature as follows

$$rate = A' exp[-E/(kT)],$$

where

E is the activation energy of the reaction, usually in electron-volts;

k is Boltzmann's constant, 0.000086171 electron-volts per ˚C;

T is the absolute Kelvin temperature; it equals the Centigrade temperature plus 273.16 degrees; the absolute Rankine temperature equals the Fahrenheit temperature plus 459.7 Fahrenheit degrees;

A' is a constant that is characteristic of the product failure mechanism and test conditions.

The rate of metal diffusion is described by the same equation. Thus the following Arrhenius life relationship may describe failures due to diffusion in solid state devices and certain other products made of metal, if geometry of the distinct metals is not an important factor.

Motivation. The following relationship is based on a simple view of failure due to such a chemical reaction (or diffusion). The product is assumed to fail when some critical amount of the chemical has reacted (or diffused); a simple view of this is

(critical amount) = (rate) x (time to failure).

Equivalently,

(time to failure) = (critical amount) / (rate).

While naive, this suggests that nominal time τ to failure ("life") is inversely proportional to the rate ($rate = A' exp[-E/(kT)]$). This yields the Arrhenius life relationship

$$\tau = A\ exp[E/(kT)]$$

A is a constant that depends on product geometry, specimen size and fabrication, test method, and other factors. Products with more than one failure mode have different A and E values for each mode.

Linearized relationship. The (base 10) logarithm of last equation is

$$\log(\tau) = \gamma_0 + (\frac{\gamma_1}{t}),$$

where $\gamma_1 = \log(e)(E/k) \cong 0.4343E/k$

Thus the log of "nominal life," $\log\tau$, is a linear function of inverse absolute temperature x = 1/ T. "Life" τ is usually taken to be a specified percentile or the mean of the (log) life distribution. Common choices are the 50th, 63.2th, and 10th percentiles.

For most diodes, transistors, and other solid states devices, E is in the range 0.3 to 1.5 electron-volts. Moreover, E varies from one failure mode to another, even for the same device.

Arrhenius acceleration factor. The Arrhenius acceleration factor between life τ at temperature T and life τ' at reference temperature T" is

$$K = \frac{\tau}{\tau'} = exp\left\{(E/k)\left[\left(\frac{1}{T}\right) - (1/T')\right]\right\}$$

For the Class-H insulation, the acceleration factor between T = 453.16˚K

(180˚9 and T' = 533.16˚K (260˚C) is

$$K = \frac{\tau}{\tau'} = exp\left\{(\frac{0.65}{0.000086171})\left[\left(\frac{1}{453.16}\right) - (\frac{1}{533.16})\right]\right\} = 12$$

Thus specimens run 12 times longer at 180˚C than at 260˚C.

Larsen-Miller relationship. The effect of temperature on time T to creep (to a specified % elongation) or to rupture of metals under load is discussed, for example, by Dieter (1961). The Larsen-Miller relationship for an absolute temperature T is written as

$$T[-\gamma_0 + \log(\tau)] = \gamma_1$$

where γ_1 called the Larsen-Miller parameter; it depends only on load (stress in psi) and not on τ or T. Dieter presents other such relationships which are fitted to creep-rupture data at high temperatures to estimate life at lower design temperatures. Usually τ is taken to be the median life, and the scatter in life is often ignored in metallurgical studies. Of course, in high reliability applications where failure is to be avoided, the lower tail of the life distribution must be modeled with a life distribution.

Arrhenius paper. Figure 9.1 shows Arrhenius paper which has a log scale for life and a nonlinear (Centigrade) temperature scale, which is linear in inverse absolute temperature. The linear scale for inverse absolute temperature was added to the figure to show its relation to the temperature scale. On such paper, the Arrhenius (life-temperature) relationship plots as a straight line. The value of A (or equivalently γ_0) determines the intercept of the line (at $T = \infty$). The value of E (or equivalently γ_1) determines the slope.

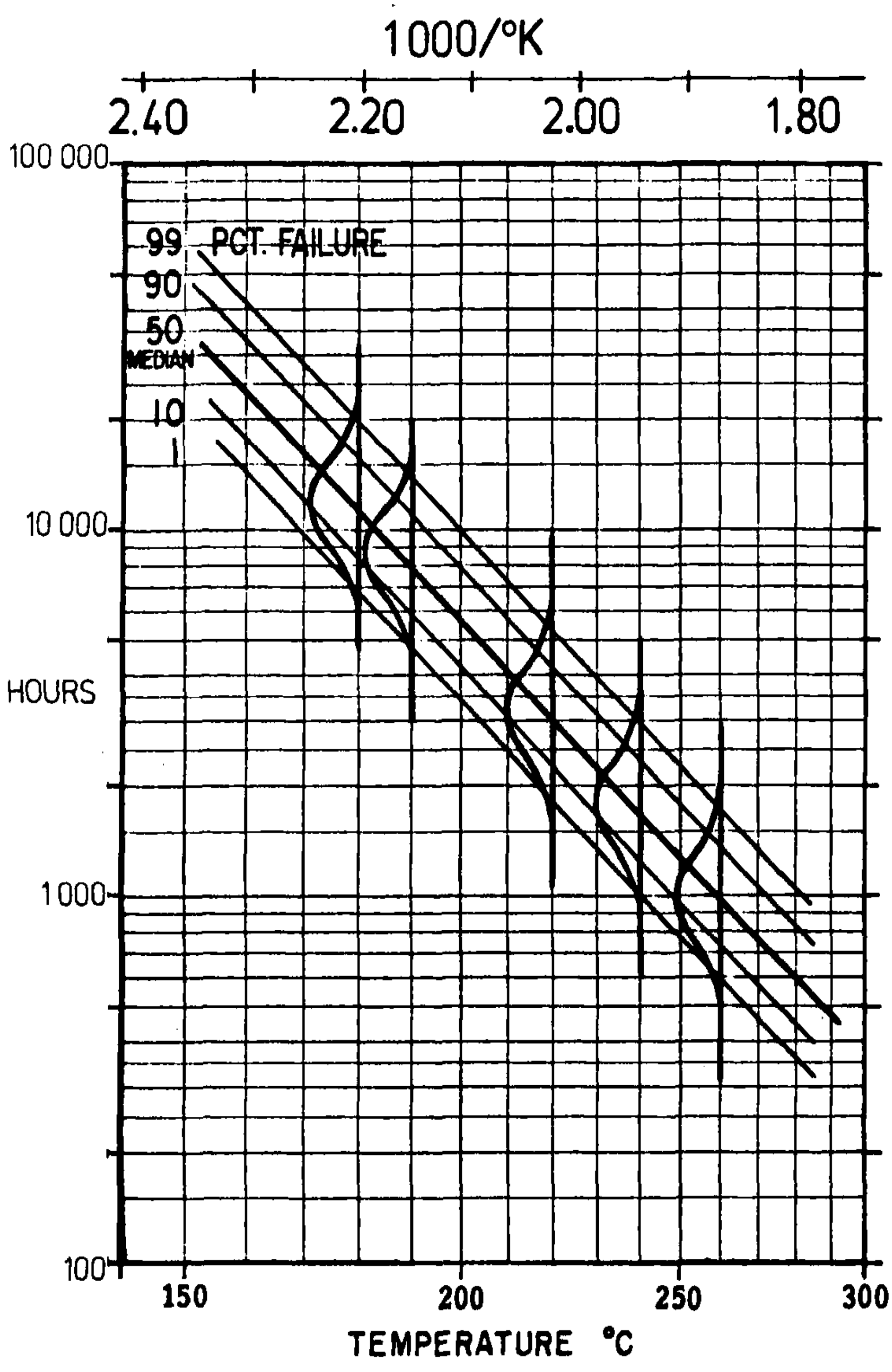

Figure 9.1. Arrhenius relationship (and lognormal percentile lines) on Arrhenius paper.

9.2. Arrhenius-Lognormal Model

The life of many products and materials in a temperature-accelerated test is described with a lognormal distribution. IEEE Standard 101 uses the lognormal distribution for motor insulation. Described below is the Arrhenius-lognormal model. It combines a lognormal life distribution with an Arrhenius dependence of life on temperature.

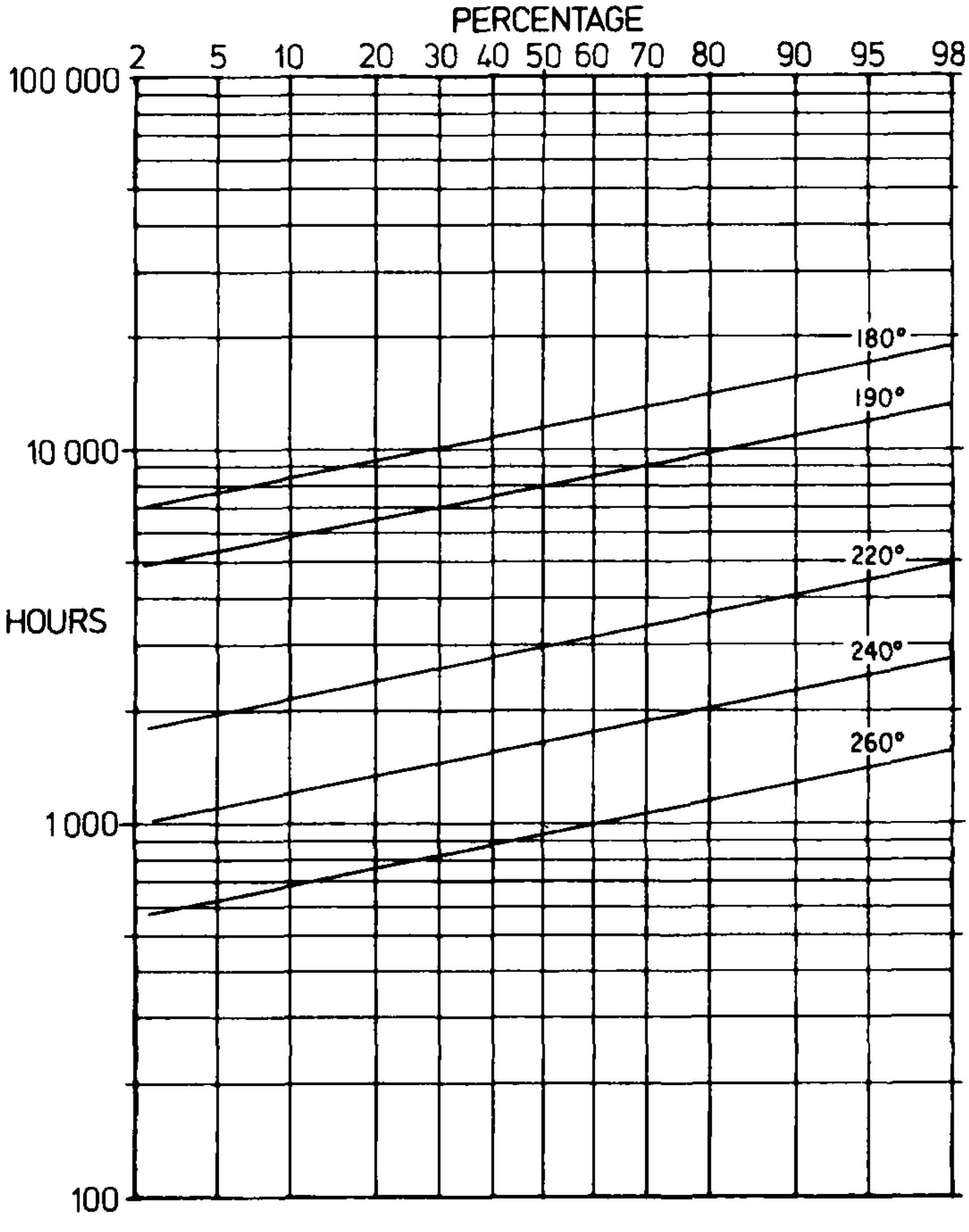

Figure 9.2. Cumulative distributions on lognormal probability paper – Arrhenius-lognormal model.

Assumptions. The assumptions of the Arrhenius-lognormal model are:

- At absolute temperature T, product life has a lognormal distribution. Equivalently, the log (base 10) of life has a normal distribution.
- The standard deviation, σ, of log life is a constant, i.e., independent of temperature.
- The log of median life $\tau_{.50}$ is a linear function of the inverse of the absolute temperature T; that is,

$$\log[\tau_{.50}(T)] = \gamma_0 + (\gamma_1'/T)$$

which is called the Arrhenius life relationship.

Parameters γ_0, γ_1', and σ are characteristic of the product and test method; they are estimated from data.

Equivalently, the mean $\mu(x)$ of log life is a linear function of $x = 1000/T$:

$$\mu(x) = \gamma_0 + \gamma_1 x$$

Here and elsewhere, 1000 is used to scale inverse temperatures, and $\gamma_1 = \gamma_1'/1000$. Resulting numbers are more convenient. These assumptions yield the following cumulative distribution of life and its percentiles.

Fraction failed. At absolute temperature T, the cumulative distribution function (population fraction failed) at age t is

$$F(t,T) = \Phi\{[\log(t) - \mu(x)]/\sigma\}$$

$\Phi()$ is the standard normal cumulative distribution function. This fraction failed plots as a straight line versus t on lognormal probability paper in Figure 9.2. The value of a determines the slope of such lines. A high (low) value corresponds to a high (low) slope and to a wide (narrow) distribution of log life. In Figure 9.2, the distribution lines are parallel. This reflects assumption 2, which is needed for the following reason. Different σ values at different temperatures result in distribution lines with different slopes. Such lines cross, resulting in a lower fraction failed for higher temperature beyond the age where the lines cross. Such crossing is physically implausible. Thus a common (constant) value for a is assumed.

Percentiles. At temperature T, the 100Pth percentile (P fractile) of life is

$$\tau_p(T) = antilog\left[\mu(x) + z_p\sigma\right] = antilog\left[\gamma_0 + \gamma_1\left(\frac{1000}{T}\right) + z_p\sigma\right]$$

where z_p is the standard normal percentile. For a fixed P, $\tau_p(T)$ plotted against Centigrade temperature on Arrhenius paper is a straight line. Figure 9.1 shows such lines for several percentiles; the value z_p determines the vertical position of the corresponding line. The corresponding percentile of log life is

$$\eta_p(x) = \log\left[\tau_p(x)\right] = \mu(x) + z_p\sigma$$

Thus, one can think in terms of the percentile of life or of log life. The median (50th percentile) is a special case; namely,

$$\tau_{.50}(T) = antilog[\mu(x)] = antilog[\gamma_0 + \gamma_1(1000/T)],$$

$$\eta_{.50}(x) = \mu(x) = \gamma_0 + \gamma_1 x$$

For the Class-H insulation at 180°C, the 10th percentile of log life is

$$\eta_{.10}(2.2067) = 4.0611 + (-1.282)0.10533 = 3.9261$$

The 10th percentile of life is

$$\tau_{.10}(453.16°K) = antilog(3.9261) \equiv 8.435 hours$$

This is a point on the 10% line in Figure 9.1. Also, it is a corresponding point on the 180° line in Figure 9.2.

9.3. Arrhenius-Weibull Model

The life of some products and materials in a temperature-accelerated test is described with a Weibull distribution. Described below is the Arrhenius-Weibull model. It combines a Weibull life distribution with an Arrhenius dependence of life on temperature.

Assumptions. The assumptions of the Arrhenius-Weibull model are:

- At absolute temperature T, product life has a Weibull distribution; equivalently, the natural log of life has an extreme value distribution;
- The Weibull shape parameter β is a constant (independent of temperature); equivalently, the extreme value distribution of the natural log of life has a constant scale parameter $\delta = 1/\beta$;
- The natural log of the Weibull characteristic life a is a linear function of the inverse of T:

$$\ln[\alpha(T)] = \gamma_0 + (\gamma_1'/T)$$

- The extreme value location parameter of the distribution of natural log life is a linear function of $x = 1000/T$; that is,

$$\xi(x) = \ln[\alpha(T)] = \gamma_0 + \gamma_1 x$$

These assumptions yield the cumulative distribution of life and its percentiles.

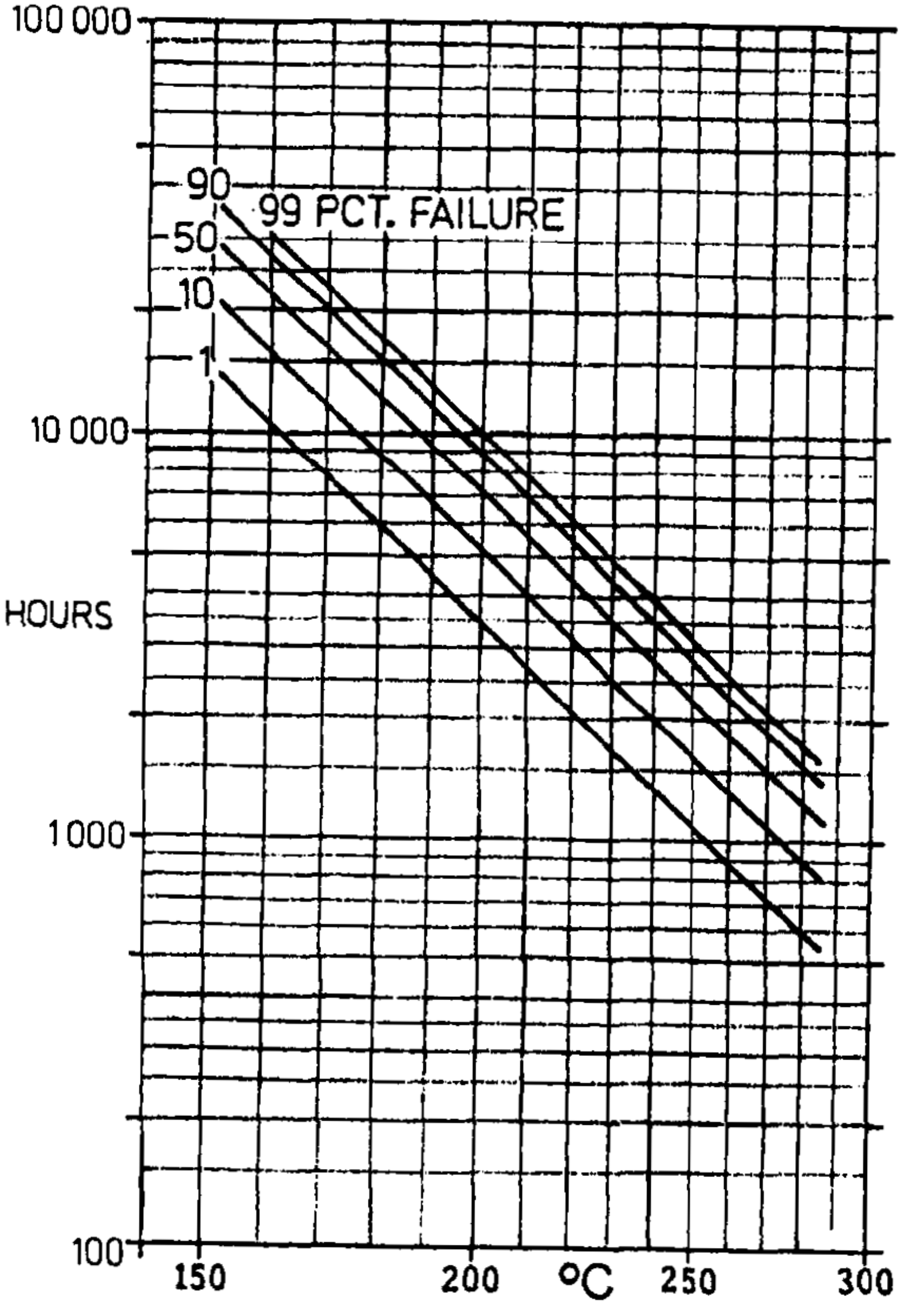

Figure 9.3. Arrhenius relationship and Weibull percentile lines.

The parameters γ_0, γ_1', and β are characteristic of the product and test method; they are estimated from data. $\alpha(T)$ plots as a straight line on Arrhenius paper (Figure 9.3).

Fraction failed. At absolute temperature T, the cumulative distribution function (population fraction failed) at age t is

$$F(t;T) = 1 - exp\{-[t/\alpha(T)]^{\beta}\}$$

For a specific temperature T, this fraction failed plots as a straight line versus t on Weibull probability paper. The value of β determines the slope of such lines on Weibull paper. Thus β is also unfortunately called the slope parameter, not to be confused with the relationship slope γ_1. A high β value corresponds to a narrow distribution of ln life; a low β value corresponds to a wide distribution of ln life. In Figure 10.2, the distribution lines are parallel, a result of assumption 2.

Percentile. At temperature T, the 100Pth percentile is

$$\tau_p(T) = \alpha(T)[-\ln(1-P)]^{1/\beta}$$

For a fixed P, $\tau_p(T)$ plotted against T on Arrhenius paper is a straight line, as in Figure 9.3. However, the spacing of these parallel Weibull percentile lines differs from that of the lognormal percentile lines in Figure 9.1. The corresponding percentile of log life is

$$\eta_p(x) = \xi(x) + u_p\delta$$

where $x = 1000/T$ and $u_p = \ln[-\ln(1-P)]$ is the standard extreme value percentile. Then

$$\tau_p(T) = exp\left[\eta_p(x)\right]$$

Design temperature. Suppose a desired life is specified as a percentile value τ_p^*. For the Arrhenius-Weibull model, the absolute temperature that yields this life is

$$T^* = 1000_{\gamma_1} / \ln\left\{\tau_p^* / [-\ln(1-P)]^{1/\beta}\right\}$$

9.4. Arrhenius-Exponential Model

The life of semiconductor and solid states devices and other electronic components is often (incorrectly) represented with an exponential distribution. The exponential distribution is often a reasonable approximation for the distribution of times between failure for a complex electronic system. However, it is often a poor or misleading approximation to the life distribution of a (nonrepaired) component or material. Moreover, a crude reliability estimate is better than no estimate. Some would use the Weibull distribution, which is better, but lack appropriate test or field data or handbook information. Of course, the Arrhenius-exponential model is a special case of the Arrhenius-Weibull model with $\beta = 1$.

Assumptions. The assumptions of the Arrhenius-exponential model are:

- At any absolute temperature T, life has an exponential distribution.
- The natural log of the mean life θ is a linear function of the inverse of T:

$$\ln[\theta(T)] = \gamma_0 + \frac{\gamma_1'}{T}$$

Model parameters γ_0 and γ_1' are characteristics of the product and test method; they are estimated from data. $\theta(T)$ plots as a straight line on Arrhenius paper. Equivalently,

- The natural log of the (constant) failure rate $\lambda = \frac{1}{\theta}$ is

$$\ln[\lambda(T)] = -\gamma_0 - (\frac{\gamma_1'}{T})$$

Also, $\lambda(T)$ plots as straight line on Arrhenius paper. These assumptions yield the cumulative distribution function of life and percentiles below.

Fraction failed. At absolute temperature T, the cumulative distribution function (population fraction failed by age t) is

$$F(t;T) = 1 - exp\left[-\frac{t}{\theta(T)}\right] = 1 - exp[-t\lambda(T)] = 1 - exp\{-t\, exp[-\gamma_0 - (\gamma_1'/T)]\}$$

For a temperature T, this fraction failed plots as a straight line versus t on Weibull probability paper. Such distribution lines are parallel, as in Figure 9.2, but have different spacings for the exponential distribution.

Percentiles. At absolute temperature T, the 100Pth percentile is

$$\tau_p(T) = exp[\gamma_0 + \gamma_1(1000/T)][-\ln(1-P)]$$

For fixed P, $\tau_p(T)$ plotted against T on Arrhenius paper is a straight line. Such percentile lines for different P are parallel. The 63.2th percentile is, of course, the mean $\theta(T)$.

10. INVERSE POWER RELATIONSHIP

Applications. The inverse power relationship is widely used to model product life as a function of an accelerating stress. Applications include:

- Electrical insulations and dielectrics in voltage-endurance tests;
- Ball and roller bearings;
- Incandescent lamps;
- Flash lamps;
- Simple metal fatigue due to mechanical loading.

The relationship is sometimes called the inverse power law or simply the power law. The term "law" suggests it is universally valid, which it is not. However, while usually not based on theory, the relationship is empirically adequate for many products.

10.1. The Relationship

Definition. Suppose that the accelerating stress variable V is positive. The inverse power relationship (or law) between "nominal" life τ of a product and V is

$$\tau(V) = A/V^{\gamma_1}$$

where A and $\gamma_1$are parameters characteristic of the product, specimen geometry and fabrication, the test method, etc. The parameter is also called the power or exponent.

Coffin-Manson relationship. The inverse power law is used to model fatigue failure of metals subjected to thermal cycling. The "typical" number N of cycles to failure as a function of the temperature range ΔT of the thermal cycle is

$$N = A/(\Delta T)^{\beta}$$

where A and B are constants characteristic of the metal and test method and cycle. This equation is called the Coffin-Manson relationship. The relationship has been used for mechanical and electronic components. In electronics it is used for solder and other connections. For metals, the fatigue life is often modeled with the lognormal distribution, which is combined with the inverse power relationship. For metals, $B \cong 2$. For plastic encapsulants for microelectronics, $B \cong 5$.

Palmgren's equation. Life tests of roller and ball bearings employ high mechanical load. In practice, life (in millions of revolutions) as a function of load is represented with Palmgren's equation for the 10th percentile B_{10} of the life distribution, namely,

$$B_{10} = (C/P)^{p}$$

C is a constant called the bearing capacity and p is the power. B_{10} is called the "B ten" bearing life. P is the (equivalent radial) load in pounds. Bearing life is usually modeled with the Weibull distribution, which is combined with the inverse power relationship. For rolling steel bearings, the Weibull shape parameter β is typically in the range 1.1 to 1.3 in actual use and in the range 1.3 to 1.5 in laboratory tests. For steel ball bearings, the power $p = 3$ is used, and, for steel roller bearings, $p = 10/3$ is used.

Linearized relationship. The natural logarithm of $\tau(V) = A/V^{\gamma_1}$ is

$$\ln(\tau) = \gamma_0 + \gamma_1 [-\ln(V)]$$

Thus the log of "typical life", $\ln(\tau)$, is a linear function of the transformed stress $x = -\ln(V)$. "Life" τ is usually taken to be a specified percentile of the life distribution. Common choices are the 50th, 63.2th, and 10th percentiles.

Log-log paper. Last equation shows that the inverse power relationship $\tau(V) = A/V^{\gamma_1}$ is a straight line on log-log paper. Figure 10.1 shows such a straight linc on such paper. It may sometimes happen that the calculated nominal life is off scale on the line labeled 63.2%. Special log-log papers are needed. Namely, the length of the log cycle for time is much shorter than that for voltage as in Figure 10.1. Ordinary log-log paper

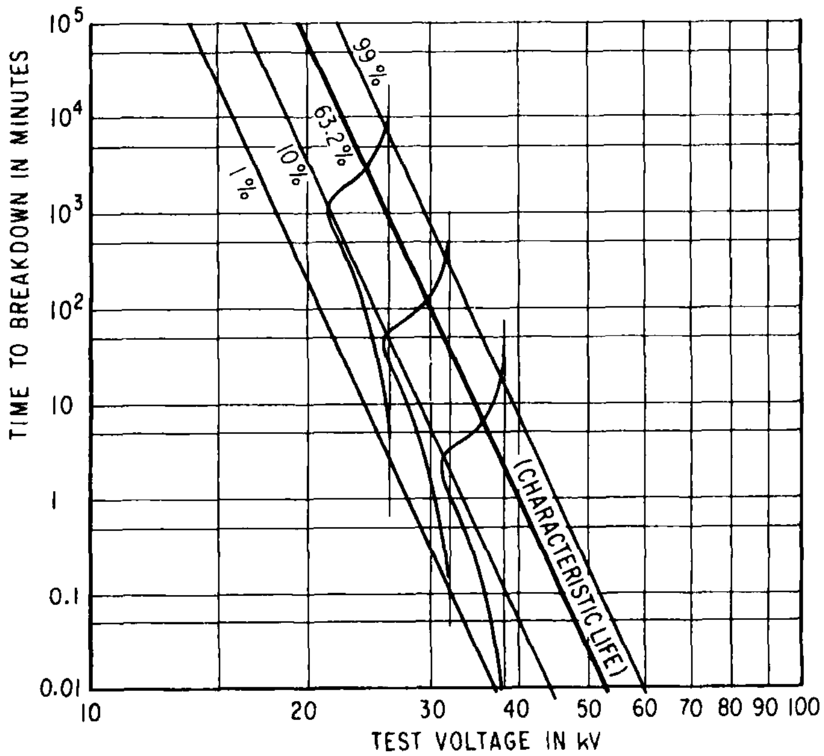

Figure 10.1. **Inverse power relationship (Weibull percentile lines) on log-log paper.**

has the same cycle length on each axis, and it is not suitable for most accelerated testing works. Some organizations have developed their own suitable log-log paper, similar to that in Figure 10.1. Current graphics packages on computers readily custom make such paper with plotters. Figure 10.1 shows time on the vertical axis. Some papers have time on the horizontal axis and stress on the vertical axis; this is common in applications with metal fatigue and insulation endurance.

Power acceleration factor. By $\tau(V) = A/V^{\gamma_1}$, the power acceleration factor between life τ at stress V and life τ' at reference stress V is

$$K = {}^{\tau}/_{\tau'} = (V'/V)^{\gamma_1}$$

10.2. Power-Lognormal Model

The life of certain products is described with a lognormal life distribution whose median is an inverse power function of stress. This is the simplest model used for metal fatigue.

Assumptions. The assumptions of the power-lognormal model are:

- At any stress level V, product life has a lognormal distribution;
- The standard deviation σ, of log life is a constant (independent of V);
- The median life, $\tau_{.50}$, is an inverse power function of stress; that is,

$$\tau_{.50}(V) = \frac{10^{\gamma_0}}{V^{\gamma_1}}$$

Parameters γ_0, γ_1, and σ are characteristics of the product and test method. An example of (10.21) is plotted in Figure 10.1. Equivalently, the mean $\mu(x)$ of (base 10) log life is a linear function of transformed stress $x = -\log(V)$:

$$\mu(x) = \gamma_0 + \gamma_1 x$$

These assumptions yield the following equations for the cumulative distribution of life and its percentiles.

Fraction failed. At stress level V, the cumulative distribution function (population fraction failed by age t) is

$$F(t; V) = \Phi\{[\log(t) - \mu(x)]/\sigma\}$$

where $\Phi()$ is the standard normal cumulative distribution function. This fraction plots as a straight line versus t on lognormal probability paper. For example, see Figure 9.2.

Percentiles. At stress level V, the $100P_{th}$ percentile (P fractile) is

$$\tau_p(V) = antilog\left[\mu(x) + z_p\sigma\right]$$

where z_p is the standard normal percentile. For fixed P, $\tau_p(V)$ plots against V on log-log paper as a straight line. Figure 10.1 shows such lines, but their vertical spacings correspond to a Weibull distribution. The corresponding percentile of log life is

$$\eta_p(x) = log\left[\tau_p(V)\right] = \mu(x) + z_p\sigma$$

The median life or median log life is a special case with $z_{.50} = 0$.

Design stress level. In some applications, one must choose a stress level that gives a desired "life." Such a desired life is usually a specified value τ_p^* of a percentile. For example, at what stress level will the 0.1th percentile of a fatigue life distribution of a metal be 12.000 cycles. For the power-lognormal model, the corresponding stress level V^* is

$$V^* = (1/\gamma_1)antilog\left[\gamma_0 + z_p\sigma - \log(\tau_p^*)\right]$$

In airplane frame and engine design, such a stress level is used, and the part is removed from service at $(\tau_p^*/3)$ cycles. This practice seeks to avoid what the industry terms a "part separation." The safety factor of 3 helps compensate for the uncertainties in estimating fatigue life due to differences between the geometry of specimens and that of actual parts and to differences between test and actual stress, environment, etc.

10.3. Power-Weibull Model

The life of certain products is described with a Weibull life distribution whose characteristic life is a power function of stress. Applications include:

- Electrical insulation and dielectrics. Voltage stress is the accelerating variable;
- Ball and roller bearings. Load is the accelerating variable;
- Metal fatigue. Mechanical stress (pounds per square inch) is the accelerating variable.

The assumptions and properties of the model follow.

Assumptions. The assumptions of the power-Weibull model are:

- At stress level V, product life has a Weibull distribution;
- The Weibull shape parameter β is a constant (independent of V);
- The Weibull characteristic life a is an inverse power function of V:

$$\alpha(V) = e^{\gamma_0}/V^{\gamma_1}$$

The parameters γ_0, γ_1, and β are characteristics of the product and test method. $\alpha(V)$ plots as a straight line versus V on log-log paper as shown in Figure 10.1. Equivalently,

- The natural log of product life has an extreme value distribution.
- The extreme value scale parameter $\alpha = \frac{1}{\beta}$ is a constant.
- The extreme value location parameter $\xi = \ln \alpha$ is a linear function of $x = -\ln(V)$; that is,

$$\xi(x) = \gamma_0 + \gamma_1 x$$

These assumptions yield the cumulative life distribution and its percentiles.

Fraction failed. At stress level V, the cumulative distribution function (population fraction failed by age t) is

$$F(t;V) = 1 - exp\left\{-\left[\frac{t}{\alpha(V)}\right]^{\beta}\right\} = 1 - exp\{-[t\, e^{-\gamma_0} V^{\gamma_1}]^{\beta}\}$$

For a specific stress level V, $F(t;V)$ plots as a straight line versus t on Weibull probability paper. Such distribution lines appear in Figure 10.2. A high (low) β corresponds to a narrow (wide) distribution of ln life.

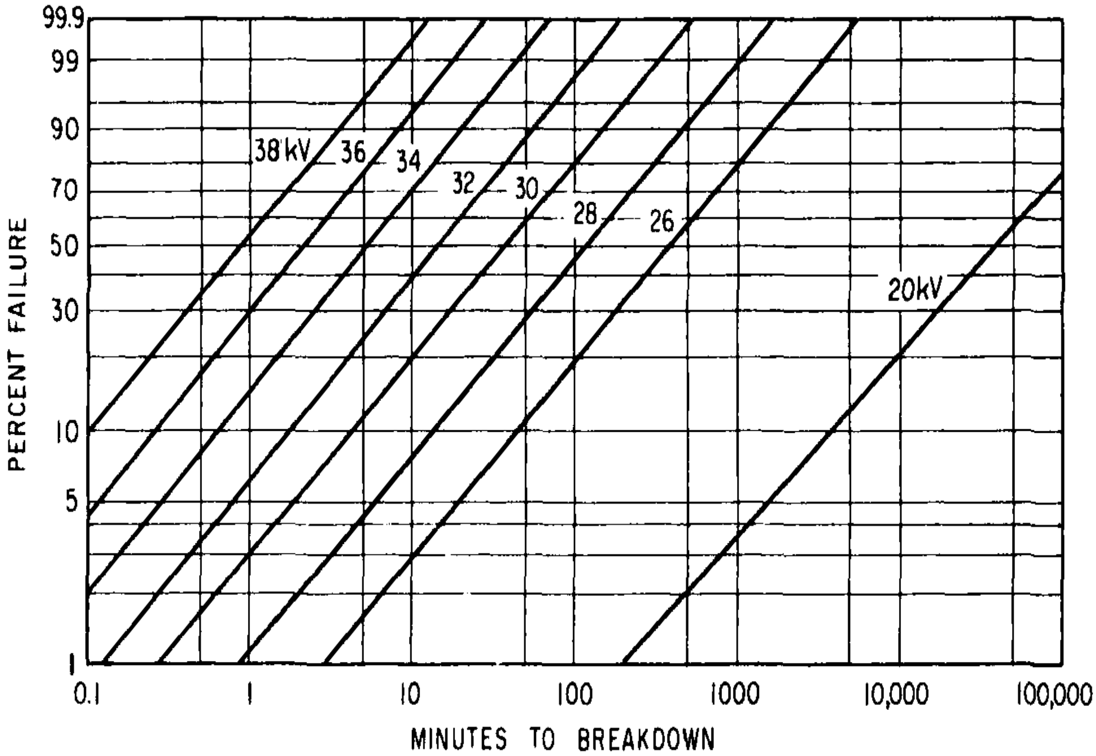

Figure 10.2. **Cumulative distributions on Weibull probability paper – power-Weibull model.**

Percentiles. At stress level V, the $100P_{th}$ percentile (P fractile) is

$$\tau_p(V) = \alpha(V)[-\ln(1-P)]^{1/\beta}$$

This equation shows that any $\tau_p(V)$ plotted against V on log-log paper is a straight line, as in Figure 10.1. The spacing of such parallel percentile lines depends on the Weibull distribution through the factor $[-\ln(1-P)]^{1/\beta}$.

The corresponding percentile of ln life is

$$\eta_p(x) = \ln[\tau_p(V)] = \xi(x) + \mu_p\delta$$

where $\delta = \frac{1}{\beta}$.

Design stress level

Suppose a desired life is specified as a percentile value τ_p^*. For the power-Weibull model, the stress level that yields this life is $V^* = \left\{e^{\gamma_0}[-\ln(1-P)]^{1/\beta}/\tau_p^*\right\}^{1/\gamma_1}$.

10.4. Power-Exponential Model

The life of semiconductor and solid states devices and other electronic components is often (incorrectly) represented with an exponential distribution. Generally, the power-Weibull model provides a much better representation of the life of electronic components. The power exponential model is a special case of the power-Weibull model with $\beta = 1$.

Assumptions. The assumptions of the power-exponential model are:

- At any stress level V, life has an exponential distribution;
- The mean life θ is an inverse power function of V:
-

$$\theta(V) = e^{\gamma_0}/V^{\gamma_1}$$

Model parameters γ_0 and γ_1 are characteristic of the product and test method. $\theta(V)$ plots as a straight line on log-log paper. Equivalently,

- The failure rate $\lambda = 1/\theta$ is a power function of V:

$$\lambda(V) = e^{-\gamma_0}V^{\gamma_1}$$

Also, $\lambda(V)$ plots as a straight line on log-log paper.

It follows that the natural log of life has an extreme value distribution with scale parameter $\delta = 1$ and location parameter

$$\xi(x) = \ln[\theta(x)] = \gamma_0 + \gamma_1 x$$

Fraction failed.
At stress level V, the cumulative distribution function (population fraction failed by age t) is

$$F(t;V) = 1 - exp\left[\frac{-t}{\theta(V)}\right] = 1 - exp[-te^{-\gamma_0}V^{\gamma_1}]$$

For a specific V, this fraction failed plots as a straight line versus t on Weibull probability paper. Such distribution lines are parallel as in Figure 10.2.

Percentiles. At stress level V, the $100P_{th}$ percentile (P fractile) is

$$\tau_p(V) = \theta(V)[-\ln(1-P)] = \left[\frac{e^{\gamma_0}}{V^{\gamma_1}}\right][-\ln(1-P)]$$

For fixed P, $\tau_p(V)$ plots against V on log-log paper as a straight line, as in Figure 10.1. Such percentile lines for different P are parallel.

Design stress level.

Suppose that a desired life is specified as a mean time to failure θ^*. For the power-exponential model, the stress level that yields this life is

$$V^* = (e^{\gamma_0}/\theta^*)^{1/\gamma_1}$$

In terms of a specified failure rate λ^*, $V^* = (e^{\gamma_0}\lambda^*)^{1/\gamma_1}$.

11. ENDURANCE (OR FATIGUE) LIMIT RELATIONSHIPS AND DISTRIBUTIONS

Fatigue data on certain steels suggest that specimens tested below a cer- tain stress run virtually indefinitely without failure. That stress is called the *fatigue limit.* Graham (1968) and Bolotin (1969) discuss this phenomenon, and ASTM STP 74 gives a number of proposed life-stress rela- tionships with a fatigue limit for steels. For many components, such low design stress is uneconomical, and such components are designed for finite life and removal before failure.

Similarly, voltage endurance data on certain dielectrics and insulations suggest that specimens tested below a certain voltage stress (electric field strength in volts per mil) run virtually indefinitely without failure. That volt- age stress is called the *endurance limit.* While the existence of such a limit stress may be in doubt, it would allow designers to put critical components under a low enough stress to prevent failures during design life.

The simple relationship below may be useful even if there is no physical endurance limit. It has three coefficients and may be a better fit to non-linear data over the range of interest than the quadratic relationship, which also has three coefficients.

11.1. Power-Type Relationship

A commonly used relationship for "nominal" time τ to failure with an en-durance (or fatigue) limit $V_0 > 0$ is

$$\tau = \begin{cases} \dfrac{\gamma_0}{(V - V_0)^{\gamma_1}}, & V > V_0 \\ \infty & V \leq V_0 \end{cases}$$

Here V is the positive stress and γ_0 and γ_1 are product parameters. V_0, γ_0, and γ_1, are estimated from data. The previous equation reduces to the inverse power law if $V_0 = 0$

11.2. Straight Line

Another simple relationship with endurance (or fatigue) limit $V_0 = 0$ is

$$\log(\tau) = \begin{cases} \gamma_0 + \gamma_1 \log(V), & V > V_0 \\ \infty, & V \leq V_0 \end{cases}$$

Here V is the stress, and γ_0 and γ_1 are product parameters. Previous system of equation is a straight line on log-log paper for $V > V_0$. It is used to represent fatigue data on certain steels as described in ASTM STP 744. It is the inverse power law if $V_0 = 0$.

11.3. Fatigue Limit (or Strength) Distribution

The preceding simple relationships involve a sharp fatigue limit. Stressed below that limit, all specimens last forever. Stressed above that limit, all specimens have finite life. A more plausible model assumes that each specimen has its own distinct fatigue limit. Thus there is a distribution of fatigue limits of such specimens. This distribution is also called the strength dlstri- bution; it is depicted as a vertical distribution in Figure 11.1. Of course, fatigue specimens cannot be tested an infinite time.

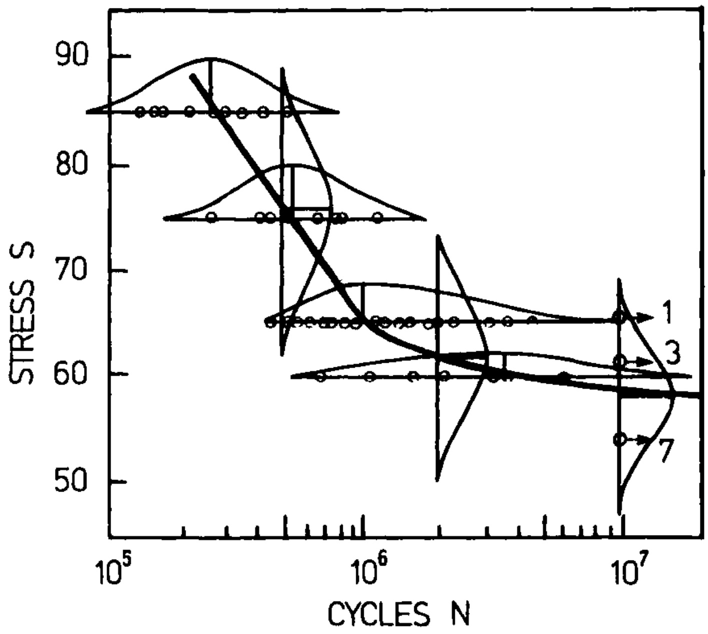

Figure 11.1. **Fatigue life distributions and strength distributions (○→ is a runout).**

Thus, such fatigue tests of steel typically run for 10^7 cycles. For most applications, 10^7 cycles well exceed design life. A designer then chooses a design stress such that (almost) all product units survive 10^7 cycles. The desired design stress is typically the 0.001 fractile (0.1 percentile) or 10^{-6} fractile (1 in 10^6 fails) of the fatigue limit distribution. In some applications, design life is short, and the design stress is well above the fatigue limit. Then one uses the strength distribution at the design life. The following paragraphs survey such fatigue limit (strength) distributions, the form of the test data, and data analyses.

Fatigue limit (or strength) distributions have been represented with the normal, lognormal, Weibull, extreme value, logistic, and other distributions. The normal distribution provides a simple and concrete example. Suppose the entire population runs at stress level *S.* Then the population fraction that fails before *N* cycles is

$$F(S; N) = \Phi[(S - \mu_N)/\sigma_N]$$

Here $\Phi()$ is the standard normal cumulative distribution function, and μ_N and σ_N are the mean and standard deviation of *strength* at N cycles. The vertical strength distributions in Figure 11.1 is normal distributions. For a design life of N^* cycles with a small percentage 100P failing, the design stress from the previous equation is

$$S^* = \mu_{N^*} + z_p\sigma_{N^*}$$

here z_p is the standard normal $100Pth$ percentile. In practice, one estimates μ_{N^*} *a*nd σ_{N^*} from data.

Fatigue data used to estimate a strength distribution generally have the form depicted in Figure 11.1. Several specimens are run at each of a small number of test stresses, Testing is stopped at, say, 10^7 cycles. The runouts (nonfailures) are depicted with arrows in Figure 11.1. For a design life (say, 10^7 cycles), the data at each test stress consist of the number of specimens that failed before that life and the number that reached that life without failure. The exact failure ages are ignored. Such binary data (failed before or survived the design life) are called quantal-response data. Such data are sometimes obtained from up-down testing. Such testing involves a few stress levels and running one specimen at a time. If a specimen fails before (after) the design life, the next specimen is run at the next lower (higher) stress level. Thus, most specimens are run near the middle of the strength distribution.

Methods and tables for fitting a strength distribution to such quantal-response data are given, for example, by Little (1981) and Little and Jebe (1975). Also, many computer programs fit distributions to such quantal-response data. Typically, such methods and up-down testing, which were developed for biological applications, yield efficient estimates and confidence limits for the median of the strength distribution. In fatigue design, estimates of low percentiles of a strength distribution are usually more important. Efficient tests plan for estimating low percentiles have not been used in practice. Meeker and Hahn (1977) present some optimum plans for a logistic strength distribution. Use of quantal-response data and the strength distribution at a design life ignores the relationship between fatigue life and stress. This simplification has a drawback; namely, run-outs below the design life cannot be included in a quantal-response analysis. A more complex model (including the fatigue curve and life distribution) is required to properly include such early runouts in an analysis.

12. OTHER SINGLE STRESS RELATIONSHIPS

The Arrhenius and inverse power relationships are the most commonly used life-stress relationships. People, of course, use a great variety of other relationships. This section briefly surveys several such relationships with a single stress variable.

This section first presents simple relationships with just two coefficients. Then it proceeds to relationships with three or more coefficients. It also presents special purpose relationships. The relationships are typically a function of temperature, voltage, or current.

12.1. Exponential Relationship

The *exponential relationship* for "life" τ as a function of stress *V* is

$$\tau = \exp\left(\gamma_0 - \gamma_1 V\right)$$

This has been used, for example, for the life of dielectrics, according to Simoni. Moreover

$$\ln(\tau) = \gamma_0 - \gamma_1 V$$

12.2. Exponential-Power Relationship

The exponential-power relationship for "nominal" life τ as a function of (possibly transformed) stress x is

$$\tau = \exp\left(\gamma_0 - \gamma_1 x^{\gamma_2}\right)$$

This relationship has three parameters $\gamma_0, \gamma_1 \; and \; \gamma_2$: thus, it is not linear on any plotting paper.

12.3. Quadratic and Polynomial Relationships

The quadratic relationship for the log of nominal life τ as a function of (possibly transformed) stress x is

$$\log(\tau) = \gamma_0 + \gamma_1 x + \gamma_2 x^2$$

This relationship is sometimes used when a linear relationship ($\gamma_0 + \gamma_1 x$) does not adequately fit the data. For example, the linearized form of the Arrhenius relationship or of the power law may be inadequate. It is sometimes used, and its curve on the corresponding plotting paper is a quadratic. For example, Nelson (1984) applies it to metal fatigue data. A quadratic relationship is often adequate over the range of the test data, but it can err much if extrapolated much outside of that range. It is best to regard it as a curve fitted to data rather than as a physical relationship based on theory.

A polynomial relationship for the log of "nominal" life τ as a function of (possible transformed) stress x is

$$\log(\tau) = \gamma_0 + \gamma_1 x + \gamma_2 x^2 + \cdots + \gamma_k x^k$$

Such relationships are used for metal fatigue data over the stress range of the data. Such a polynomial for $K \geq 3$ is virtually worthless for extrapolation, even short extrapolation.

12.4. Elastic-Plastic Relationship for Metal Fatigue

Used for metal fatigue over a wide range of stress, the *elastic-plastic relationship* between "life" *N* (number of cycles) and constant strain (or pseudo-stress) amplitude *S* is

$$S = AN^{-a} + BN^{-b}$$

This relationship *(S-N* curve) has four parameters *A,* a, *B,* and *b,* which must be estimated from data. Here *N* cannot be written as an explicit function of *S.* Consequently, some data analysts incorrectly treat *S* as the dependent (and random) variable in least-squares fitting of this and other fatigue relationships to data. Hahn (1979) discusses the consequences of incorrectly treating stress as the dependent (and random) variable. Least-squares theory, of course, assumes that the random variable (life) is the dependent variable.

12.5. Eyring Relationship for Temperature Acceleration

An alternative to the Arrhenius relationship for temperature acceleration is the Eyring relationship. Based on quantum mechanics, it is presented as a reaction rate equation for chemical degradation by Glasstone, Laidler, and Eyring (1941).

The Eyring relationship for "nominal" life τ as a function of absolute temperature T is

$$\tau = (A/T)exp[B/(kT)]$$

*H*ere A and B are constants characteristic of the product and test method, and k is Boltzmann's constant. For the small range of absolute temperature in most applications, (A/T) is essentially constant, and it is close to the Arrhenius relationship. For most applications, both fit the data well.

The methods for fitting the Eyring model to data are the same as those for fitting the Arrhenius model. One just analyzes transformed times $t' = t\,T$ as if they come from an Arrhenius model.

In the Eyring-lognormal model, the last is the relationship for median life. Also, the standard deviation σ of log life t is assumed to be a constant. Then the standard deviation of the log of transformed life is the constant value σ.

13. MULTIVARIABLE RELATIONSHIPS

The life-stress relationships involve a single accelerating stress. Such relationships are appropriate for many accelerated tests. However, some accelerated tests involve more than one accelerating stress or an accelerating stress and other engineering variables. For example, an accelerated life test of capacitors employed both high temperature and high voltage - two accelerating variables. The effect of both stresses on life was sought. Also, for example, in an accelerated life test of tape insulation, the accelerating variable was voltage stress, but the dissipation factor of specimens was included in the relationship. The factor was related to life and could be used to determine which insulation batches to install at higher voltage. The curves are functions of temperature, voltage, current, vibration, and other variables.

It is useful to divide nonaccelerating variables into two groups. One group consists of variables that are experimentally varied. That is, the value of such an *experimental variable* is chosen for each specimen. For example, a taping experiment involved insulating tape wound on a conductor. The amount of overlap of successive layers of tape was varied/different specimens with different amounts of overlap. The other group of variables are uncontrolled, and such *uncontrolled variables* are only measured. For example, the same tape specimens were each measured for dissipation factor, a property of the specimen, because it is related to life. In experimental design books such uncontrolled variables which are observed and included in a relationship are called *covariates.*

13.1. Log-Linear Relationship

General relationship. A general, simple relationship for “nominal” life τ (says, a percentile) is the *log-linear relationship*

$$\ln(\tau) = \gamma_0 + \gamma_1 x_1 + \cdots + \gamma_j x_j$$

Here $\gamma_0, \gamma_1\ and\ \gamma_j$, are coefficients characteristic of the product and test method; they are usually estimated from data. $x_1, \ldots\ x_j$ are (possibly transformed) variables. Any x_j may be a function (transformation) of one or any number of basic engineering (predictor or independent) variables. It is used in parametric analyses with an assumed form of the life distribution. It is also used in nonparametric analyses without an assumed form of the life distribution.

Last equation is a linear function of each of the coefficients $\gamma_0, \gamma_1 \ldots\ \gamma_2$. It is not necessarily linear in the original variables used to calculate $x_1 \ldots\ x_j$. Relationships that are linear in the coefficients are used mostly because they are mathematically convenient and physically adequate rather than “correct.” The log-linear relationship includes a number of special cases below. Also, the relationship can be used to represent the spread in life (lognormal σ or Weibull β) as a function of a number of variables.

Taping experiment. An experiment with insulating tape sought to evaluate the effect on life of the amount w that tape overlaps itself when wound on a conductor

$$x_1 = sin\left(\frac{2\pi w}{W}\right),\ \ x_2 = \cos\left(2\pi w/W\right)$$

where W is the tape width. Also, the life test was voltage accelerated; so, the effect of voltage stress was modeled with $x_3 = -\ln(V)$, that is, with the inverse power law.

13.2. Generalized Eyrlng Relationship

*Relationship***.** The *generalized Eyring relationship* has been used to describe accelerated life tests with temperature and one other variable. Glasstone, Laidler, and Eyring (1941) present it as a reaction rate equation. Rewritten to express "nominal" product life τ as a function of absolute temperature *T* and a (possibly transformed) variable P, it is

$$\tau = (A/T)exp[B/(kT)] \times exp\{V[C + (D/kT)]\}$$

Here *A, B, C,* and *D* are coefficients to be estimated from data, and *k* is Boltzmann's constant.

Capacitors. It was assumed that the *interaction term* between temperature and voltage was zero (D=0). If $D \neq 0$, then in an Arrhenius plot, the straight line for τ has a different slope for each voltage level. Because V is the ln of voltage, there was an assumed inverse power relationship between life and voltage. Figure 13.1 depicts this relationship as a plane in three dimensions. In the figure, temperature is on a reciprocal absolute scale, and life and voltage are on log scales. Figure 13.2 depicts contours of constant

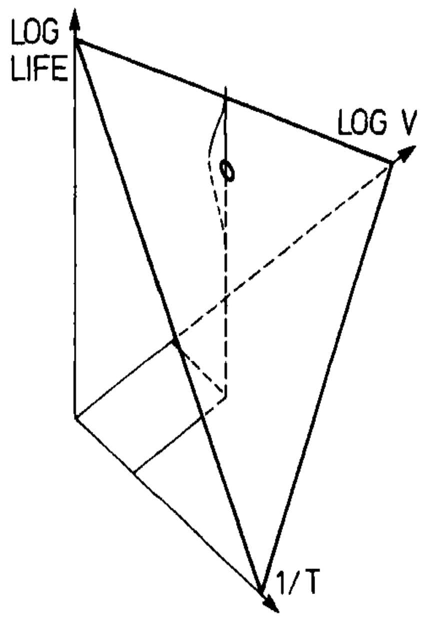

Figure 13.1. Generalized Eyring relationship with a power relationship in *V*.

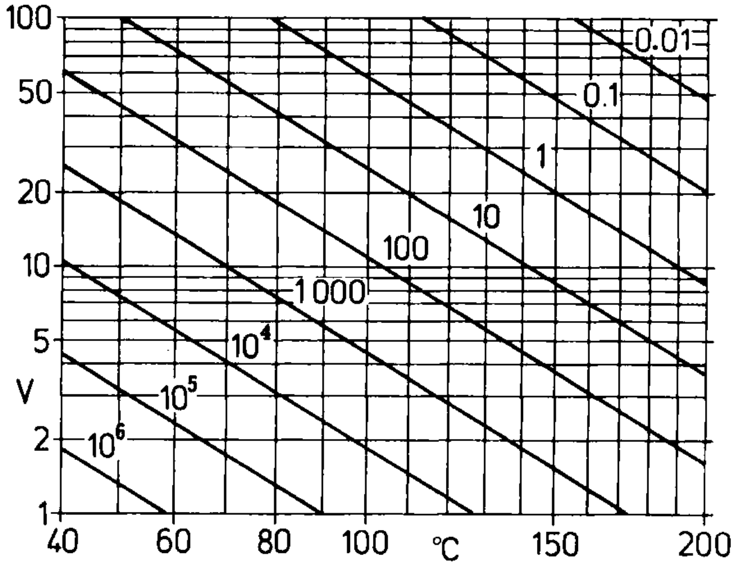

Figure 13.2. **Contours of constant life τ for a generalized Eyring relationship with no interaction ($D = 0$).**

life (height on the relationship plane) projected onto the temperature-voltage plane of Figure 13.1. That plane is scaled like Arrhenius paper in Figure 13.2. The contours are straight lines since *D=0*. Montanari and Cacciari (1984) extend the model by assuming that the Weibull shape parameter is a linear function of temperature. They fit their model to accelerated life test data on low density polyethylene.

*Temperature-humidity tests***.** Many accelerated life tests of epoxy pack- aging for electronics employ high temperature and humidity. For example, 85°C and 85% relative humidity (RH) is a common test condition. Peck (1986) surveys such testing and proposes an Eyring relationship for life

$$\tau = A(RH)^{-n} \exp\left(\frac{E}{kT}\right)$$

called *Peck's relationship.* Data he uses *to* support it yield estimates $n = 2.7$ and $E = 0.79eV$.

Figures 13.1 and 13.2 depict this relationship if relative humidity RH replaces voltage V and its axis is linear instead of logarithmic. Intel (1988) uses another Eyring relationship

$$\tau = A \exp(-B \times RH) \exp\left(\frac{E}{kT}\right)$$

Intel notes that this differs little from Peck's relationship relative to uncertainties in such data.

Rupture of solids. Zhurkov's (1965) relationship for "time" τ to rupture of solids at absolute temperature P and tensile stress S is

$$\tau = A\, exp\left[\left(\frac{B}{kT}\right) - D(S/kT)\right]$$

This is the Eyring relationship with $C = 0$ and a minus sign for *D.* Also, it is a form of the Larsen-Miller relationship (9.1.7). Zhurkov motivates this relationship with chemical kinetic theory, and he presents data on many materials to support it. He interprets *B* as the energy to rupture molecular bonds and *D* as a measure of the disorientation of the molecular structure. Ballado Perez proposes this relationship for life of bonded wood composites; he extends it to include indicator variables (below) for type of wood, adhesive, and other factors.

Indicator Variables

Category variables. Most variables in relationships are numerical, and they can mathematically take on any value in an interval. For example, absolute temperature can mathematically have any value from 0 to infinity. On the other hand, some variables can take on only a finite number of discrete values or categories. Examples include

1) insulation made on three *shifts,*

2) insulation coated on two different conductor *metals,* and

3) material from two *vendors.* Relationships for such categorical variables are expressed as follows in terms of indicator variables.

Shift example. For concreteness, suppose that insulation is made on three production shifts, denoted by 0, 1, and 2. Also, suppose that insulation life in a voltage-endurance test is modeled with the power-Weibull model. Also, suppose that the insulations from the shifts have the same power in the power law but different constant coefficients (intercepts). Then the characteristic life α_j for shift j is

$$\ln[\alpha_0(V)] = \gamma_0 + \gamma_3 \ln(V)$$

$$\ln[\alpha_1(V)] = \gamma_1 + \gamma_3 \ln(V)$$

$$\ln[\alpha_2(V)] = \gamma_2 + \gamma_3 \ln(V)$$

here V is the voltage stress, γ_3 is the power coefficient and is negative, and γ_j is the intercept coefficient for shift $j = 0, 1, 2$. The straight lines for the three shifts are parallel on log-log paper. The power γ_3 is assumed to be the same for all three shifts, as it is regarded as a physical property of the insulating material. The intercept is assumed to depend on shift, as skills of the workers differ. Of course, both assumptions were assessed using data.

Definition. Define the *indicator variable* $z_j = 1$ if the corresponding test specimen is from shift j; otherwise, $z_j = 0$ if the specimen is from another shift. For example, a specimen made on shift 1 has values $z_0 = 0, z_1 = 1, z_2 = 0$ for the three indicator variables, also called *dummy variables.* An indicator variable takes on only the values 0 and 1. For that reason it is also called *0-1 variable.* Let $z_3 = \ln(V)$. Previous relationships can then be written in a single equation

$$\ln[\alpha(V)] = \gamma_0 z_0 + \gamma_1 z_1 + \gamma_2 z_2 + \gamma_3 z_3$$

This equation has four variables (z_1, z_2, z_3, z_4) and four coefficients $(\gamma_1, \gamma_2, \gamma_3 \gamma_4)$; it has no intercept coefficient, that is, a coefficient without a variable. Most computer programs that fit linear relationships to data require that the relationship have an intercept coefficient. This can be achieved by awkwardly rewriting as

$$\ln[\alpha(V)] = \delta_0 + \delta_1 z_1 + \delta_2 z_2 + \gamma_3 z_3$$

here $\delta_0 = \gamma_0$ is the intercept, $\delta_0 + \delta_1 = \gamma_1$, and $\delta_0 + \delta_2 = \gamma_2$. Equivalently, $\delta_1 = \gamma_1 - \gamma_0$ and $\delta_2 = \gamma_2 - \gamma_0$. The δ_j coefficients are not as natural or simple a way of representing the relationship. Yet this representation is better suited to most computer programs, which require an intercept term.

Some relationships have more than one categorical variable. For example, the three shifts each make two types of insulation. Shift is a categorical variable requiring two indicator variables (z_1 and z_2), insulation is another requiring one (z_3). Then

$$\ln[\alpha(V)] = \delta_0 + \delta_1 z_1 + \delta_2 z_2 + \delta_3 z_3 + \gamma_3 \ln(V)$$

A relationship that is just a linear function of the indicator variables for two or more categorical variables is call a *main-effects* relationship. More complex relationships involve *interaction terms,* they appear in many books on analysis of variance. Zelen (1959) presents an application with interaction terms for life of glass capacitors over a range of temperature and of voltage.

logistic Regression Relationship

The logistic regression relationship is widely used in biomedical applications where the dependent variable is binary; that is, it is in one of two mutually exclusive categories, for example, dead or alive. The logistic relationship for the proportion p in a category (says, "failed") as a function of J independent variables $x_1, \dots, x_j$ is

$$\ln[(1-p)/p] = \gamma_0 + \gamma_1 x_1 + \cdots + \gamma_j x_j$$

here γ_0, γ_1 *and* γ_2 are unknown coefficients to be estimated from data.

In accelerated testing, might be used when the life data are quantal-response data; that is, each specimen is inspected once to determine whether is has failed by its inspection age. Then p is fraction failed, and one of the independent variables is (log) age at inspection.

Nonlinear Relationships

The log-linear relationship is linear in the unknown coefficients. Engineering theory may suggest relationships that are nonlinear in the coefficients. However, most computer packages fit only linear relationships. Nonlinear relationships usually must be programmed into certain packages.

Nelson and Hendrickson give an example of such a non-linear relationship. A test involved time to breakdown of an insulating fluid between parallel disk electrodes. The voltage across the electrodes was increased linearly with time at different rates R (volts per second). Electrodes of various areas A were employed. The assumed distribution for time to breakdown is Weibull with parameters

$$\alpha(R; A) = \{\gamma_1 R/[A \exp{(\gamma_0)}]\}^{1/\gamma_1} \quad \beta = \gamma_1$$

The $\ln(\alpha)$ relationship is nonlinear in γ_1.

Cox (Proportional Hazards) Model

Used in biomedical applications, the *Cox* (or *proportional hazards) model* can be used as an accelerated life testing model. It does not assume a form for the distribution - possibly an attractive feature. It can be used to extrapolate in stress but not in time, because it is distribution free ("non- parametric"). It cannot be used to extrapolate the distribution to early time in the lower tail or later time in the upper tail outside the range of the life data. So, it is useful only for estimating the observed range of the life distribution at actual use conditions. Such extrapolation in stress is desired in many applications. A brief description of the model follows.

Let $x_1, \ldots, x_j$ denote the (possibly transformed and centered) variables, and let $h_0(t)$ denote the hazard function of the unknown life distribution at $x_1 = x_2 = x_3 = \cdots = x_j = 0$. The Cox Model for the hazard function of the distributions at variables values $x_1, \ldots, x_j$ is

$$h(t, x_1, \ldots, x_j) = h_0(t) \exp{(\gamma_1 x_1 + \cdots + \gamma_j x_j)}$$

The base hazard function $h_0(t)$ and the coefficients $\gamma_1, \ldots, \gamma_j$ are estimated from data. The corresponding reliability function are

$$R(t; x_1, \ldots, x_j) = [R_0(t)]^{\exp{(\gamma_1 x_1, \ldots, \gamma_j x_j)}}$$

The reliability function at $x_1 = \cdots = x_j = 0$ is

$$R_0(t) = exp\left[-\int_0^t h_0(t) dt\right]$$

The life distribution is complex. Its typical life depends on $x_1, \ldots, x_j$ in a complex way. Moreover, the spread and shape of the distribution of log life generally depends $x_1, \ldots, x_j$ in a complex way. Previous parametric models, such as the Arrhenius-lognormal and linear-Weibull, have simpler form.

14. SPREAD IN LOG LIFE DEPENDS ON STRESS

In many accelerated life test models, the spread in log life is assumed to be the same at all stress levels of interest. For example, the standard deviation σ of log life in the Arrhenius-lognormal model is assumed to be a constant. Similarly, the shape parameter β in the power-Weibull model is assumed to be a constant. A constant spread is assumed for two reasons. First, the data or experience with such data suggests that a constant spread adequately models log life. Second, the analyst uses a model with constant spread because it is traditional or easy to use. For example, almost all model fitting programs (especially least-squares programs) assume that the data spread is constant.

On the other hand, experience with certain products indicates that the spread of log life is a function of stress. For example, for metal fatigue and roller bearing life, the spread is greater at lower stress. For some electrical insulations, the spread is smaller at lower stress. The following paragraphs present some simple “heteroscedastic” relationships for spread as a function of stress. For concreteness, the standard deviation σ of log life with a lognormal distribution is used. Equivalently, one could use the shape parameter β of a Weibull distribution or any measure of spread of a life distribution.

Log-Linear Relationship for Spread

The simplest relationship for a spread parameter σ is the log-linear relationship

$$\ln[\sigma(x)] = \delta_0 + \delta_1 x$$

Here δ_0 and δ_1 are parameters characteristic of the product; they are estimated from data. x is the (possibly transformed) stress. Equivalently

$$\sigma(x) = exp[\delta_0 + \delta_1 x]$$

Figure 14.lA depicts previous equation where $\mu(x)$ is a linear function of $x = \log(stress)$. Nelson gives an example of fitting to metal fatigue data with runouts. Of course, σ must be positive. The logarithmic form of $\sigma(x)$ assures that is so. Glaser assumes σ is simply a linear function (no logarithm) of x. His function yields an incorrect negative σ for extreme values of x. Thus, it is satisfactory over only a limited range of x.

Other mathematically plausible relationships could be used, for example, the log-quadratic relationship

$$\ln[\sigma(x)] = \delta_0 + \delta_1 x + \delta_2 x^2$$

A model with a lognormal (or Weibull) distribution where $\sigma(x)$ is a function of stress has the following drawback. Figure 14.1B depicts on lognormal (or Weibull) paper a life distribution for low stress and another for high stress. If the straight lines are extended into the lower tail, the distributions cross there. This is physically implausible and undesirable if the lower tail is important. Then such a model with $\sigma(x)$ is inaccurate low enough in the tails. This drawback is not apparent in Figure 14.lA. There is a need for more sophisticated models that do not have distributions that cross.

The spread may depend on several accelerating and other variables. This dependence can be represented with the log-linear and other multivariable relationships.

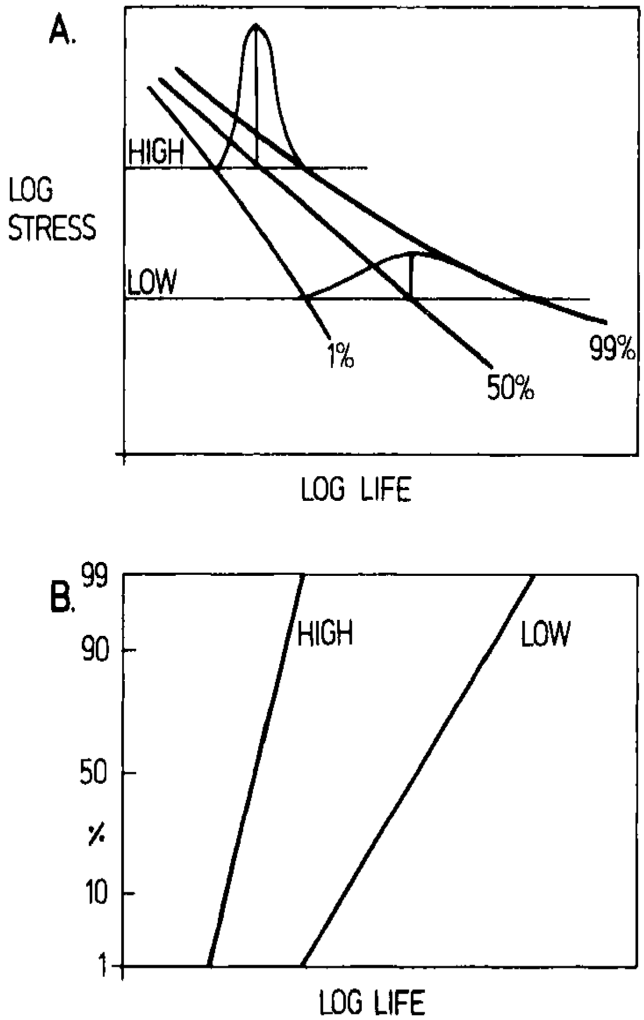

Figure 14.1. (A) Log spread as a function of stress, (B) Probability plot of crossing distributions.

Components of Variance

Metals and many other products are made in batches. For metals, the fatigue life distribution may differ appreciably from batch to batch. Thus, the life distribution of the entire product population is a mixture of the distributions of the batches. Such a situation with many batches is modeled with a components-of-variance model. Such models for a single stress level appear in most books on analysis of variance. Some recent research is extending such models to regression situations. However, such extensions do not yet apply to censored data or to standard deviations that depend on the accelerating variable. Such models need to be developed for metal fatigue and other applications.

Graphical Data Analysis

1. INTRODUCTION

Purpose. This basic chapter presents simple data plots for analysis of accelerated life test data. These plots provide desired estimates of a product life distribution at design stress and of model parameters. Also, the data plots are used to assess the validity of the model and data. In addition, the plots help one understand the complex numerical methods of later chapters.

Advantages. Graphical methods for data analysis have advantages and disadvantages relative to analytic methods. Graphical methods are multipurpose, and their advantages include:

- They are simple, quick to make and easy to interpret. Moreover, they do not require special computer programs, as do analytic methods. However, computer programs readily make such plots. Moreover, well-made computer plots are convincing and authoritative.
- They provide estimates of the product life distribution (percentiles, percentage failed) at any stress and estimates of model parameters.
- They allow one to assess how well a model fits the data and how valid the data are. Such assessments are also needed before using analytic methods.
- Most important they help convince others of conclusions based on plots or analytic results, which others accept more readily after seeing such plots.

Disadvantages. Analytic methods have certain advantages over graphical methods. These include:

- The statistical uncertainty of analytic estimates can be given objectively by means of confidence intervals. This is important because inexperienced
- data analysts tend to think estimates are more accurate than they really are. On the other hand, it is often apparent that graphical estimates are (or are not) accurate enough for practical purposes.
- Comparisons can be made objectively with a statistical confidence interval or hypothesis test. Such a test indicates whether an observed difference is statistically significant, that is, convincing. Of course, graphical comparisons can be convincing, namely, when observed differences are large compared to the scatter in the data. Subjective judgments of what is convincing can differ. Viewing the same data plots, three data analysts can have six different opinions on what they see. Thus, use subjective judgement cautiously, aided by objective analytic methods.
- Appropriate sample sizes can be determined, as well as, optimum or good test plans, based on analytic methods.

For most work, it is essential to use both graphical and analytic methods. Each provides certain information not provided by the other. A proper analysis of data always requires many different analyses. Examples repeatedly appear in different chapters to show the wide variety of analyses that should be applied to a data set.

Method. The graphical method employs the simple model and involves two data plots. The first employs probability paper for the assumed life distribution (for example, lognormal, Weibull, and exponential). The second employs paper which linearizes the assumed relationship (for example, the Arrhenius and inverse power laws). The simple model is usually suitable only for data with a single cause of failure.

2. COMPLETE DATA AND ARRHENIUS-LOGNORMAL MODEL

Introduction. This section presents simple graphical methods for analyzing complete data. The methods are illustrated with the lognormal distribution and the Arrhenius relationship. The methods yield estimates of model parameters and the product life distribution at any stress. The plots are also used to assess the validity of the model and data.

Moreover, they provide information that numerical methods do not, including checks on the validity of the data and the model. On the other hand, graphical estimates have unknown accuracy whereas numerical confidence intervals indicate the accuracy of estimates. It is best to use a combination of graphical and numerical methods.

2.1. Lognormal Probability Plot

The life data for each test stress level are plotted as follows on probability paper. The plot provides estimates of the model parameters and life distribution percentiles. On such probability paper, the theoretical cumulative distribution function of life is a straight line.

Plotting Positions. Order the *n* failure times at a test stress from smallest to largest. Give the earliest failure rank 1, the second earliest failure rank 2, etc. Calculate the probability plotting position for each failure from its rank *i* as

$$F_i = \frac{100(i - 0.5)}{n}$$

These midpoint plotting positions approximate the percentage of the population below the *i*th failure. People also use the expected plotting position

$$F_i' = 100i\frac{1}{(n + 1)}$$

People also use the median plotting position approximated as

$$F_i'' = \frac{100(i - 0.3)}{(n + 0.4)}$$

Probability plot. Use probability paper for the model distribution. Choose probability and data scales with the smallest range that encloses the data and any distributions and percentiles of interest. This spreads out the plotted data and reveals details. Two or more plotting papers can be joined to obtain more log cycles on the data scale. Label the data scale to span the data and any distribution of interest, say, at the design temperature. Plot each failure time against its plotting position on the probability scale. Figure 2.1A is a plot of the Class-H data on lognormal probability paper.

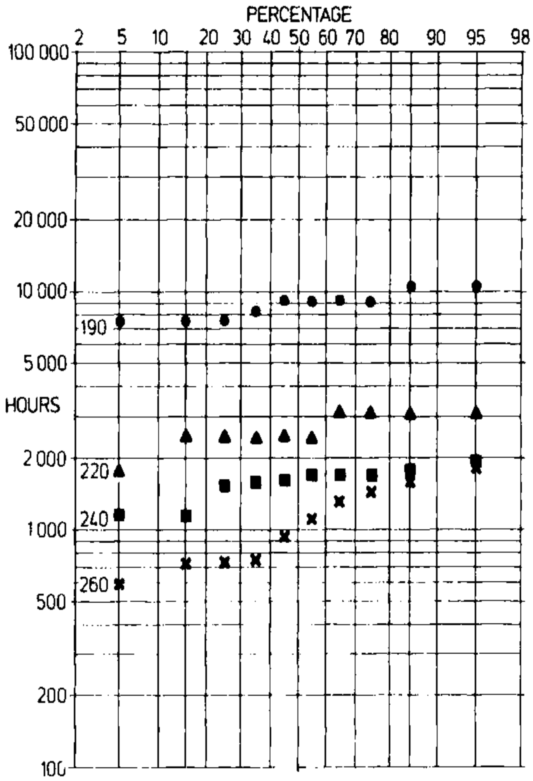

Figure 2.1A. Lognormal plot of Class-H data.

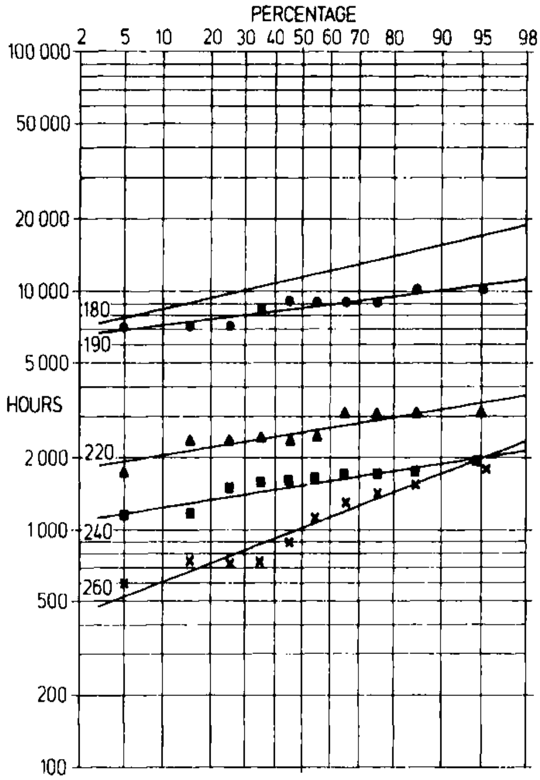

Figure 2.1B. Separate fits to Class-H data.

Assess distribution. If the plotted points for a stress level tend to follow a straight line, then the distribution appears to describe the data adequately. Sample sizes are typically small (below 20 specimens) at each stress level in accelerated life tests, and the plots may appear erratic.

Distribution lines. Draw a separate straight distribution line through the data for each test stress as in Figure 2.1B. The vertical (time) deviations between the line and the plotted points should be as small as possible. Such fitted lines are not necessarily parallel due to random variation in the data or unequal true slopes. The slope of a line for a lognormal distribution corresponds to the log standard deviation σ. The 260° data have a slope different from the others. Note that the fitted lines distract the eye from the data and can distort the data. Thus, it is best to examine plots both with and without fitted lines.

Parallel lines. If parallel lines are appropriate, a more refined method fits parallel lines to the data as in Figure 2.2. Guided by the separately fitted lines, fit parallel lines with a compromise common slope to the data for each stress. The numbers of specimens may differ from stress to stress. Then try to choose the common slope by weighting the slope at a stress proportional to its number of specimens. Estimates are obtained from the parallel lines as described below.

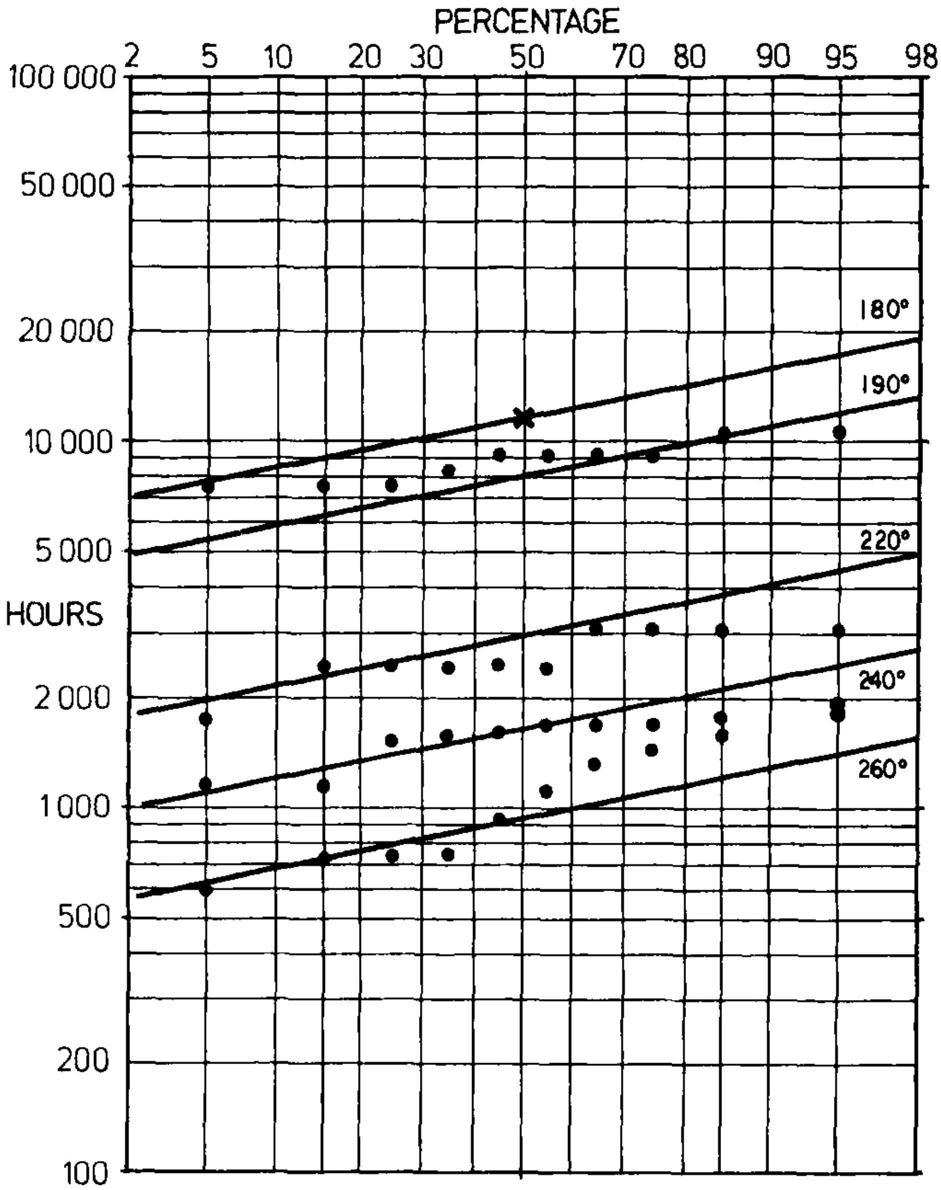

Figure 2.2. Parallel fit to Class-H data.

Percent failed at a test stress. Either type of fitted distribution line provides an estimate of the percentage of units failing by a given age. Enter the probability plot at that age on the time scale, go sideways to the fitted line for that stress, and then go up to the probability scale to read the percentage. For example, for 190˚C in Figure 2.1B, the estimate of the percentage failing by 7,000 hours is 7 percent. Similarly, from Figure 2.2, the estimate is 27%. Estimates from the parallel distribution lines tend to be more accurate, provided the model is correct.

Percentile estimate. The following provides an estimate of a percentile at a test stress. Enter the probability plot at the desired percentage on the probability scale, go vertically to the fitted distribution line for the test stress, and go horizontally to the time scale to read the estimate. For example, this estimate of the median life at 190˚C is 8,700 hours from Figure 2.1B. These estimates of the medians are x's in Figure 2.3 on Arrhenius paper.

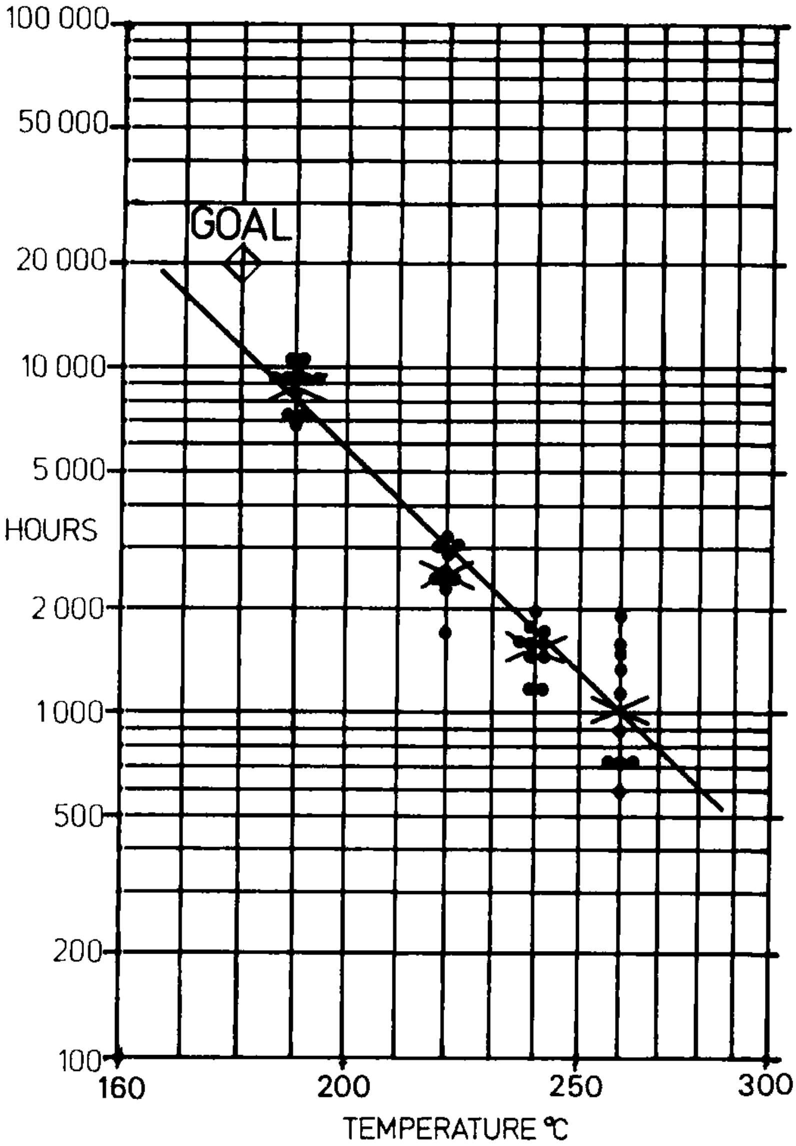

Figure 2.3. Arrhenius plot of Class-H data (× = median).

2.2. Relationship Plot (Arrhenius)

Data plot. The plot graphically estimates the relationship between life and stress. The method employs plotting paper on which the relationship is a straight line. For example, Arrhenius paper (Figure 2.3) has a logarithmic time scale and a reciprocal scale for absolute temperature. Plot each failure time against its temperature as in Figure 2.3, which shows the Class-H data of Table 2.1. Draw a line through the data to estimate "life" as a function of temperature. Note that Figure 2.3 also shows that the 260° data has greater scatter than data from other test temperatures. This was noted earlier from the lognormal plot, Figure 2.2, which shows this more clearly. Probability plots often reveal more. Figures 2.2 and 2.3 are side by side and have identical time scales. A failure time is plotted at the same height on both figures. This pair of figures depicts that the probability and relationship plots are two views of the same model and data.

Life estimate at any stress. The "life" line is used to estimate life at any stress level. Enter the plot at the stress level, go up to the fitted “life” line, and then go horizontally to the time scale to read the estimate. For the Class-H insulation, the estimate of life at the design temperature of 180˚C is 11,500 hours from Figure 2.3. This is well below the desired 20,000 hours. "Life" as used here is a vague "typical" life and is common engineering usage. The uncertainty of a graphical estimate can be gauged subjectively. Wiggle the fitted line (clear straight edge) to change the estimate while still passing the line through the data well. The largest and smallest such estimates for which the line fits well enough indicate the uncertainty. For example, a line through 20,000 hours at 180˚C does not pass through the data well enough. This is convincing that the insulation does not meet the 20.000 hours requirement.

Percentile lines. The "life" line has more specific meaning if fitted as follows. On the relationship paper, plot the estimate of a chosen percentile at each stress. Such percentile estimates from the distribution line on a probability plot are described above. The sample percentile and the geometric mean are other useful estimates. Fit a line to the percentile estimates. Try to weight each estimate proportional to the number of test units at its stress. This line graphically estimates the relationship between the percentile and stress. Such a line is fitted to medians $x's$ in Figure 2.3. Plot all points to display the data and to help spot peculiar data.

Sample percentile. The sample percentile is another useful percentile estimate. Suppose that the chosen percentage is a plotting position, say, 25%. Then the sample percentile is the corresponding observation. For example, the sample 25th percentile at 240˚C is 1521 hours from Table 2.1. Suppose the chosen percentage is not a plotting position, say, 50%. On the relationship plot, one graphically interpolates appropriately between the observations with plotting positions just above and below the chosen percentage. For the Class-H data, the sample 50th percentile is graphically midway between the 5th and 6th largest observations, which have plotting positions of 45 and 55%. Mark such sample percentiles with $x's$ in relationship plots. The sample percentile is robust. That is, it is a valid estimate of a distribution percentile even when the assumed distribution is not valid over the entire actual distribution. This is so because the estimate does not use the distribution line on the probability plot. If the assumed distribution is valid, then the sample percentile is not as accurate as the previous estimate. This is so because the sample percentile uses one or two observations, and the previous estimate uses the entire sample at that stress. Usually the percentile is chosen in the middle or lower tail of the distribution, near the percentiles of interest at the design stress.

Geometric mean. Another useful estimate of a median is the geometric mean. For a stress, take the log10 of each time, sum those logs, and divide the sum by the number of observations in the sum. The antilog of this result is the geometric mean. For example, for the Class-H data at 190˚C, the geometric mean is 8.701 hours. This estimate is valid only for the lognormal distribution and when there are no peculiar data. Then it is the most accurate estimate of the median. If the lognormal distribution is in doubt, the previous two estimates are likely more accurate.

2.3. *Graphical Estimates*

Graphical estimates of the life distribution at any stress (such as a design stress), of the model parameters, and of a design stress are given below.

Distribution for a stress. The distribution line for any stress, such as a design stress, is fitted as follows. First, use the percentile line on the relationship plot to estimate the percentile at the chosen stress level. For example, the estimate of median life at the design temperature of 180˚C is 11,500 hours from Figure 2.3. Plot that estimate on the probability plot. This point is shown as an x on Figure 2.2. Draw a distribution line through that point parallel to the distribution lines for the test stresses. Such a 180˚ line with a compromise ("average") slope appears in Figure 2.2. This line provides estimates of percentiles and fraction failed for the 180° life distribution.

If the plot or other knowledge suggests that the distribution lines are not parallel, fit a line with an appropriate slope, for example, with the slope of the data at the nearest test stress. In Figure 2.1B, one would use the slope of the 190° data, not the compromise 180° line in the plot. Information given later suggests that the 260° data should be ignored in drawing the 180° line.

Percent failed at a (design) stress. As follows, estimate the percent failed by a given age at a specified stress, such as a design stress. Use the preceding distribution line for that stress. Enter the probability plot on the time scale at the given age, go horizontally to the distribution line, and then go vertically to the probability scale to read the percentage failed. For the Class-H insulation, the estimate of the percentage failed by 10,000 hours at 180°C is 27% from Figure 2.2.

Percentile at a (design) stress. A percentile at any specified stress (such as design stress) is estimated from the fitted distribution line as follows. Enter the probability plot at that percentage, go to the fitted line, and then go to the time scale to read the estimate. For example, in Figure 2.2, the estimate of median life at the design temperature of 180°C is 11,500 hours. This is much below the desired 20,000 hours median. Also, the estimate of the 1st percentile (off scale) at 180°C is 00 hours.

Other percentile lines. Other percentile lines are drawn on the relationship plot as follows. As described in a previous paragraph, obtain a percentile estimate at the design stress. For example, the estimate of the 1st percentile at 180°C is 6600 hours. Plot this estimate on the relationship plot. This added line estimates the desired percentile line. It is used as described next to estimate the design stress that yields a specified life. Such parallel lines for several percentiles may be added.

Design stress. One may need to estimate a design stress that yields a specified life. To do this, enter the relationship plot at the specified life on the time scale, go to the fitted line for the desired percentile, and go to the stress scale to read the estimate. For example, a median life of 20,000 hours results from an estimated temperature of 165°C from Figure 2.3.

Relationship parameters (activation energy). Sometimes one wishes to estimate the coefficients γ_0 and γ_1 of the simple linear relationship. For the Arrhenius relationship, γ_1 is related to the activation energy E of the chemical degradation that produces failure; thus γ_1 has a physical interpretation. Choose two widely spaced temperatures T < T'. Obtain graphical estimates of the median lives $t^*_{.50}$ and $t^{*\prime}_{.50}$ from the relationship plot. Then the estimates of γ_1, γ_0 and the activation energy E are

$$\gamma_1^* = [TT'/(T'-T)]\log(t^*_{.50}/t^{*\prime}_{.50})$$

$$\gamma_0^* = \log\left(t^{*\prime}_{.50}\right) - (\gamma_1^*/T')$$

$$E^* = 2.303k\gamma_1^*$$

here k = 0.00008617 is Boltzmann's constant in electron volts per degree Kelvin. Peck and Trapp's (1978) Arrhenius paper has a special scale that estimates activation energy directly. Other papers listed above lack this scale.

For example, from Figure 2.3, the graphical estimates of the medians at

T = 453.2°K (180°C);
T' = 533.2°K (260°C);

$t^*_{.50}$ = 11,500 and

$t^{*\prime}_{.50}$= 950 hours

Thus,

$$\gamma_1^* = 3271$$
$$\gamma_0^* = -3{,}17$$
$$E^* = 0{,}65\ eV$$

Log standard deviation. Estimate from the slope of a fitted line in the lognormal probability plot as follows. Enter the plot on the probability scale at the 50% point, go down to one of the parallel fitted lines, and then go sideways to the time scale to read the median estimate $t^*_{.50}$. Similarly, obtain the estimate $t^*_{.16}$ of the 16th percentile from the same line. The estimate of σ a is

σ* = log ($t^*_{.50}/t^*_{.16}$)

For example, for the 180° line in Figure 2.1B, $t^*_{.50}$ = 11,500 hours and $t^*_{.16}$ = 9,000 hours. Then σ* = log (11,500/9,000) = 0.11. This small estimate indicates that such insulation has a failure rate that increases over most of the distribution. The line for a test temperature could also be used to estimate a, say, the 190° line.

2.4. Assess the Model and Data

The validity of the graphical analyses and estimates above depend on how valid the assumptions of the model are. The methods below check the assumptions of the simple (Arrhenius-lognormal) model and the validity of the data. In particular, the methods check that the assumed distribution fits the data, that the distribution spread is the same for all stresses, and that the (transformed) life-stress relationship is linear. Also, the methods check the data for outliers and other peculiarities due to blunders or faulty testing. However, the methods apply to data from any simple model.

Lognormal distribution. A probability plot allows one to assess how well the theoretical distribution fits the data. Relatively straight lognormal plots suggest the distribution adequately fits the data. To judge straightness sensitively, hold the plot at eye level and sight along the line of plotted points. For example, the four data plots in Figure 2.1A are relatively straight and indicate that the lognormal distribution adequately fits at the four temperatures.

Curved distribution plots. Plots curved the same way suggest that another distribution, such as the Weibull distribution, may fit the data better. Another analysis of curved plots involves drawing smooth curves through the points. Such curves (nonparametric fits) can be used as distribution lines. Estimates of percentiles and fraction failed are read from them as explained above. Such percentile estimates can be plotted on a relationship plot and percentile lines fitted to them. Sometimes the lower tails of curved plots are straight. If interested only in the lower tail, one can treat the data above some point in each lower tail as censored. Then the fitted model and estimates describe only the lower tail of the fitted distribution. Hahn, Morgan, and Nelson (1985) present methods for such censoring and analytically fitting to just the early failure data.

Outliers. Individual points out of line with the rest may indicate unusual or mishandled specimens, rather than poor fit of the distribution. Such "outliers" usually are failures that are too early relative to others at the same stress. Peculiar data are discussed in detail below.

Overinterpretation. Those inexperienced in analyzing data tend to overinterpret plots. They assume that any noticeable gap, flat spot, curvature, etc., of a data plot has physical meaning and is a property of the population. But only pronounced features of a plot should be assumed to be properties of the population. Hahn and Shapiro (1967) and Daniel and Wood (1971) display probability plots of Monte Carlo data from a true normal distribution. Their plots are sobering. Their samples of 20 and even 50 observations appear erratic, having peculiarities such as curvature, gaps, and outlying points. Most people's subjective notions of randomness are stringent and orderly; they incorrectly expect points in a probability plot to fall on a straight line.

Constant standard deviation. The Arrhenius-lognormal model assumes that the standard deviation a of log life is a constant. If does depend on stress, then the estimates for percentiles at a stress may be inaccurate. A a that depends on stress may be a property of the product, may result from a faulty test, or may be due to competing failure modes. The following method assesses whether the spread a is independent of stress. If the spread is constant, probability plots of the data should yield parallel lines. Most samples at each test stress are small (less than 20 specimens). So, the slopes of the distribution lines may randomly vary a lot when the true population spreads are the same. Thus, only extreme differences in the slopes are convincing. The slope changing systematically with stress may indicate that the spread depends on stress or there are competing failure modes. If the slope for one test stress differs greatly from the others, the data or the test at that stress may be faulty.

Linear (Arrhenius) relationship. The Arrhenius relationship is a linear relationship between transformed life and temperature. This assumed linearity is important when one extrapolates the fitted relationship to estimate life at low stress. The relationship plot may be nonlinear for various reasons. The life test may not have been carried out properly. The data may contain several competing failure modes, each with a different linear relationship. Also, the (transformed) true relationship just may not be linear. Subjective assessment of the linearity comes from examining the relationship plot of the data (or better the percentile estimates). The data and the medians of the Class-H insulation in Figure 2.3 are close to a straight line compared to the scatter in the data. This suggests that the Arrhenius relationship adequately fits the data.

Nonlinear relationship plot. If nonlinearity is convincing, examine the relationship plot of the percentile estimates to determine how the relationship departs from linearity. For competing failure modes, the resulting relationship is concave downwards in Figure 2.3. If a sample percentile is out of line with the others, the data for that stress may be in error. Look for erroneous data and determine if those data should be used or not. After examining the relationship plot, one may do any of the following:

- If the plot shows that a smooth curve describes the relationship, fit a curve to the data. Be sure that the apparent curvature is not a result of erroneous data. Extrapolating such a curve to the design stress may be difficult to justify and is likely to be inaccurate.
- Analyze the data with a linear fit but subjectively consider the nonlinearity in interpreting the data and coming to conclusions. The linear fit may ignore certain data or weight some data.

Valid data. The probability and relationship plots may reveal peculiarities of the data. In a sense, the data are always valid, and our assumptions are often invalid if the data differ from the model. Sometimes peculiar data arise from blunders in recording or transcribing the data. More often such data arise from specimens that are mismade or mistested or an inaccurate model. Methods above check the adequacy of the model, namely, the distribution, a common standard deviation of log life, and a linear relationship.

Peculiar data. If the model does not fit the data well, it is important to determine the reason. Some people speak backwards and say the data do not fit the model. The data are almost always right and understanding the reason for peculiar data is often more important than the good data. For example, the greater scatter in the 260° Class-H data led to yearly savings of $1,000,000. Thus it is essential to determine the cause of peculiar data. It is less important to decide whether to include or exclude peculiar data from an analysis. Usually it is best to do two or more analyses, excluding some or all of the peculiar data. Often the practical conclusions are the same for such analyses. When the conclusions differ, then one must choose an analysis, say, the most conservative or most realistic analysis.

Outliers. Sometimes there is one or a few data points that stand out in a plot. Such points are usually failures that are too early relative to the rest of the data. As noted above, it is usually most informative to determine the cause of such "outliers." Otherwise, one can do analyses with and without such outliers.

3. COMPLETE DATA AND POWER-WEIBULL MODEL

Introduction. This section presents simple graphical methods for complete life test data, using Weibull and exponential life distributions and the (inverse) power relationship. To use the methods, one needs background on the Weibull and exponential distributions and the inverse power law in Chapter 2. While the example employs the inverse power law, the methods apply to other relationships between life and stress. Thus, this section aims only to acquaint readers with Weibull probability paper and the inverse power law, since both are widely used for accelerated life test data.

3.1. Data (Insulating Fluid)

The data in Table 3.1 illustrate the graphical methods for complete data, using the power-Weibull model. The data are the times to oil breakdown under high test voltages. High voltages quickly yield breakdown data. At design voltages, time to breakdown runs thousands of years. The tests employed two parallel plate electrodes of a certain area and gap. The electrical stress is given as a voltage, since the electrode geometry was constant.

The main purpose was to estimate the relationship between time to breakdown and voltage. This involves fitting the model to the data. The model is used to estimate the probability of product failure during a factory test at 20 kV. Another purpose was to assess whether the distribution of time to breakdown is exponential.

Table 3.1. Times to Breakdown of an Insulating Fluid

26 kV	
Minutes	Plotting Position
5.79	16.3
1579.52	50.0
2323.70	83.3

28 kV	
Minutes	Plotting Position
68.85	10.0
108.29	30.0
110.29	50.0
426.07	70.0
1067.60	90.0

30 kV	
Minutes	Plotting Position
7.74	4.5
17.05	13.6
20.46	22.7
21.02	31.8
22.66	40.9
43.40	50.0
47.30	59.1
139.07	68.2
144.12	77.3
175.88	86.4
194.90	95.5

32 kV	
Minutes	Plotting Position
0.27	3.3
0.40	10.0
0.69	16.7
0.79	23.3
2.75	30.0
3.91	36.7
9.88	43.3
13.95	50.0
15.93	56.7
27.80	63.3
53.24	70.0
82.85	76.7
89.29	83.3
100.58	90.0
215.10	96.7

34 kV	
Minutes	Plotting Position
0.19	2.6
0.78	7.9
0.96	13.2
1.31	18.4
2.78	23.7
3.16	28.9
4.15	34.2
4.67	39.5
4.85	44.7
6.50	50.0
7.35	55.3
8.01	60.5
8.27	65.8
12.06	71.1
31.75	76.3
32.52	81.6
33.91	86.8
36.71	92.1
72.89	97.4

36 kV	
Minutes	Plotting Position
0.35	3.3
0.59	10.0
0.96	16.7
0.99	23.3
1.69	30.0
1.97	36.7
2.07	43.3
2.58	50.0
2.71	56.7
2.90	63.3
3.67	70.0
3.99	76.7
5.35	83.3
13.77	90.0
25.50	96.7

38 kV	
Minutes	Plotting Position
0.09	6.2
0.39	18.7
0.47	31.2
0.73	43.7
0.74	56.2
1.13	68.7
1.40	81.2
2.38	93.7

3.2. Weibull Probability Plot

Weibull plot. However, Weibull probability paper is used (Figure 3.1). Such paper has a log scale for data (time) and a Weibull scale for probability. Data from all of the stresses are plotted on in Figure 3.1 and clutter the plot. There is a separate straight line through the data for each test stress. The

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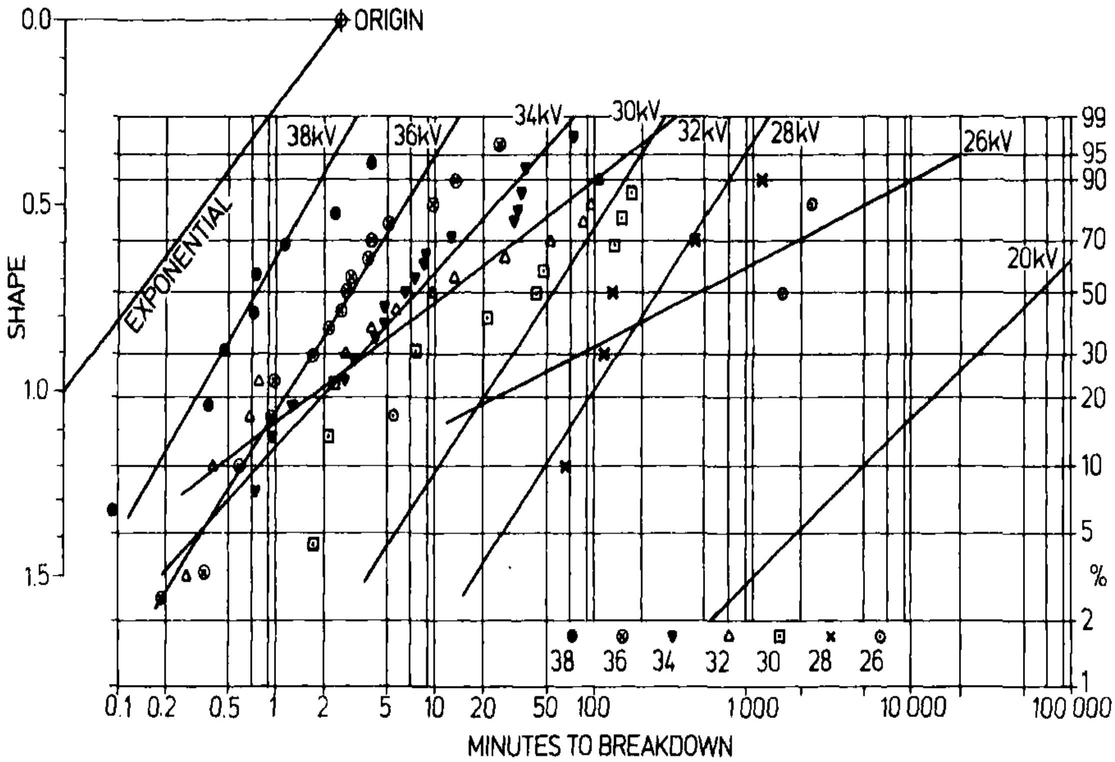

Figure 3.1. Weibull plot of insulating fluid data.

lines estimate the cumulative distribution functions at those stresses, that is, the percentage failed versus time.

For the model for the example, the Weibull life distribution has the same shape parameter value at any stress. This means that the distribution lines are all parallel. Such parallel lines are fitted in Figure 3.2; there data from half of the stresses are plotted to avoid chiller.

Shape parameter estimate. The Weibull shape parameter indicates the behavior of the failure rate with time. It is estimated from the Weibull probability plot (Figure 3.2). Through the point labeled "origin" draw a line parallel to the common slope of the fitted lines. The line intersects the shape parameter scale at a value which is the graphical estimate. Figure 3.2 gives an estimate of 0.81. The value less than 1 indicates that the failure rate decreases with time. Also, the life distribution is close to exponential. It is not clear that this estimate convincingly differs from 1. Also, uncontrolled test conditions (voltage etc.) may have created greater scatter in the data and thereby lowered the observed shape value below 1.

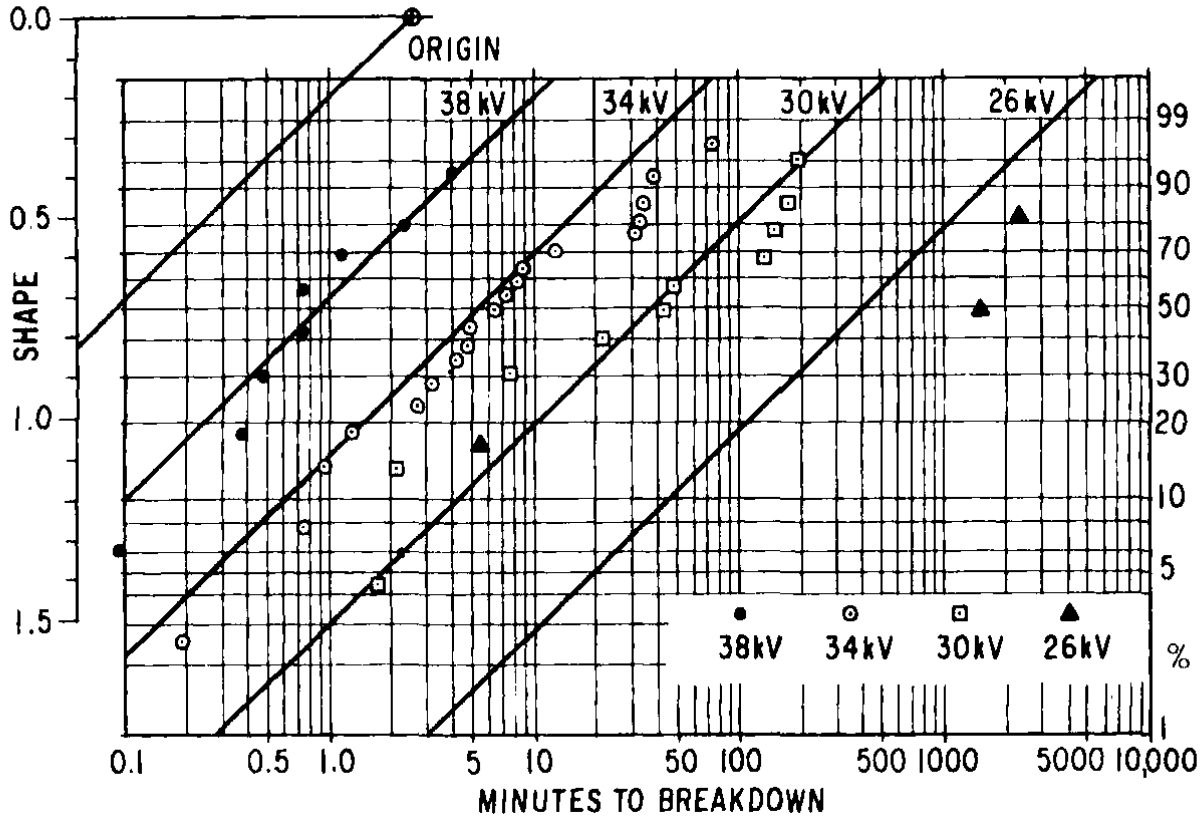

Figure 3.2. Parallel fits to insulating fluid data.

Specified Weibull shape. Sometimes the value of the Weibull shape parameter is specified. In the example, the life distribution is assumed to be exponential — a shape parameter of 1. One can fit distributions with that shape parameter value to the data as follows. On the Weibull paper, draw a line from the point labeled "origin" through the shape parameter scale at the specified value. Such a line in Figure 3.1 passes through the value 1 (an exponential distribution). Through the data for a stress draw a line parallel to the shape parameter line. That fitted distribution has the specified Weibull shape parameter value.

3.3. Relationship Plot (Inverse Power)

Relationship line. For the inverse power law, plot the data on log-log paper (Figure 3.3). Then fit a straight line by eye to pass through the data. This is best done by fitting the line to an estimate of a percentile at each stress. Such estimates of the characteristic life (63.2 percentile) are shown as x's in Figure 3.3. The fitted line in Figure 3.3 graphically estimates the inverse power law relationship between the characteristic life and stress. Here the relationship plot (Figure 3.3) and the corresponding probability plot (Figure 3.1 or 3.2) do not appear side by side with identical time scales, as do Figures 2.2 and 2.3, and one is turned 90° from the other. Log-log and Weibull papers with identical time scales show the correspondence between the two data plots. However, such pairs of papers have not been developed.

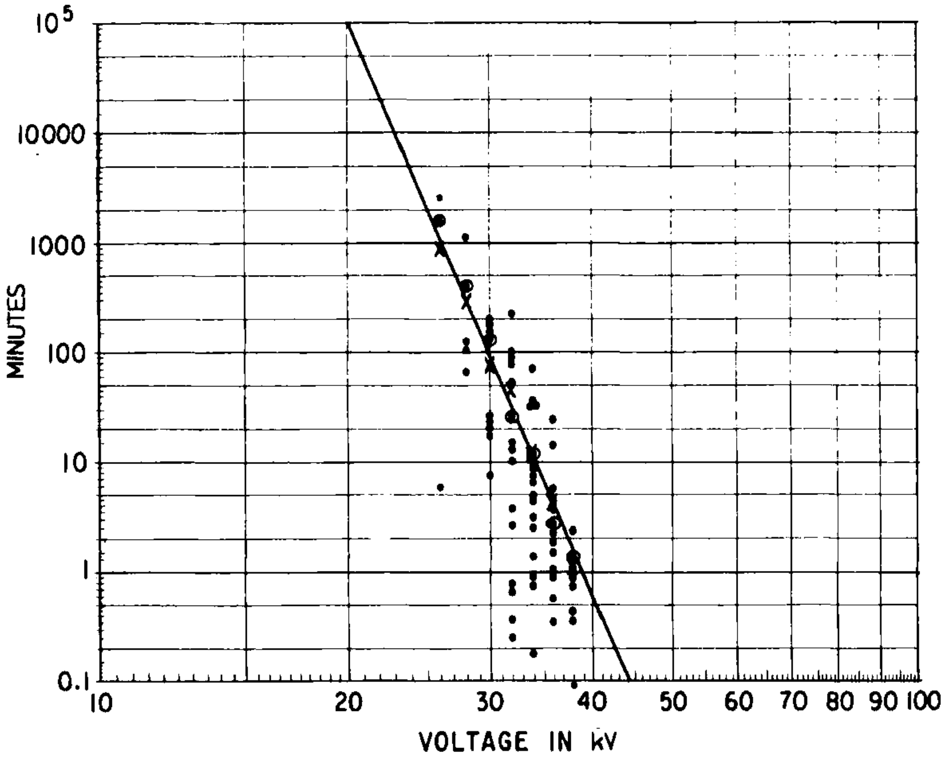

Figure 3.3. Log-log plot of insulating fluid data (× is a graphical estimate of α and O is the sample 63rd percentile).

Percentile estimates. Two estimates of a percentile at a stress can be used. Such estimates appear as x's in Figure 3.3. The other is the sample percentile of Section 2.3. These observations (63.2 percentiles) are circles in Figure 3.3. In practice, only one such estimate is marked at each test stress. The two percentile estimates differ with respect to simplicity and statistical accuracy. The sample percentile is easier to use and less accurate. For most purposes, the graphical estimate is recommended.

Log-log papers. Most commercial log-log paper has cycles of the same length on both axes. Such paper is often not suitable for accelerated test data. Better paper (Figure 3.3) has one or two large cycles for stress and many small cycles for life; some engineering departments have developed such papers. Semi-log paper with one or two log cycles can be used. Mark the linear scale with powers of 10 and treat it as a log scale.

3.4. Graphical Estimates

Graphical estimates are obtained as described previously. Examples of such estimates follow and include the Weibull characteristic life, percentiles, relationship parameters, and the design stress that yields a specified life.

Characteristic life. The estimate of the characteristic life at any stress can be read directly from the fitted line in the relationship plot. Enter the plot at that stress on the horizontal scale. Go up to the fitted line, and then go sideways to the time scale to read the estimate. For example, in Figure 3.3, the estimate of the characteristic life at 20 kV is 105,000 min. The estimate may be in error if the relationship is nonlinear on log-log paper.

Relationship parameters (power). The power $\gamma_1$in the inverse power law $\alpha = e^{\gamma_0}/V^{\gamma_1}$ is often of interest. The larger its (absolute) value, the more sharply life decreases as stress increases. For two stresses V < V', graphically estimate their characteristic lives $\alpha*$ and $\alpha'*$. Then the graphical estimates of γ_1 and γ_0 are

$$\gamma_1^* = \ln(\alpha^*/_{\alpha'^*}) / \ln(\frac{V'}{V})$$

$$\gamma_0^* = \ln(\alpha^* V^{\gamma_1^*})$$

For example, in Figure 3.3 $\alpha*$ = 105,000 min at V = 20kV and $\alpha'*$ = 0.60 min at V = 40 W. Thus

$$\gamma_1^* = 17{,}4$$

$$\gamma_0^* = 63{,}7$$

A power of 17.4 is unusually large, even for insulation life.

Choice of design stress. Sometimes one needs to estimate a design stress with a desired characteristic life. Enter the log-log plot at that life on the time scale, go sideways to the fitted line, and go down to the stress scale to read the estimate. For example, in Figure 3.3, a characteristic life of 1.000.000 min is provided by an estimated voltage of 17.6 kV. This method can also be used with percentile lines to estimate a design stress with a specified low percentile.

Distribution line at any (design) stress. The distribution line at any stress is estimated on the probability paper as described in the second chapter. Such a line for the insulating fluid at the factory test voltage of 20 kV appears in Figure 3.1. Percentiles and percentage failing are estimated from the line as described in Section 2.4. For example, the estimate of the 1st percentile at 20 kV is 210 min. from Figure 3.1. Also, the estimate of the percentage failing by 10 min. at 36 kV is 95%.

3.5. Assess the Model and Data

Graphical methods of Section 2.5 are used to assess the (Weibull) distribution, the (power law) relationship, and the data. Use both graphical and analytic methods to get the most information from the data.

The straight Weibull plots (Figure 3.1) indicate that the Weibull distribution fits adequately. The relatively parallel Weibull plots (Figure 3.1) suggest a common shape parameter is reasonable. The straight relationship plot (Figure 3.3) indicates that the power law fits over the range of the data. In both plots, a low outlier at 26 kV is evident. No reason for it was found. Eliminating it has little effect on estimates, since the sample is relatively large (76 times to breakdown) and the Weibull distribution has a long lower tail.

4. SINGLY CENSORED DATA

Introduction. Often life data are not complete. When the data are analyzed, some units may still be running. Such data are singly censored when the failure times of unfailed units are known only to be beyond their current common (single) running time. A censored life clearly cannot be discarded or treated as a failure as this ignores and arbitrarily changes such data.

This section presents simple graphical methods for estimating from singly censored data the model and the life distribution at any stress. The methods include

- a probability plot of singly censored data from each test stress;
- a relationship plot of life against stress.

Also, a distribution may adequately fit only data in the lower tail at each stress. When the upper tail is not of interest, then the influence of data in the upper tail can be removed. One treats all upper tail data as censored at some time in the lower tail. Hahn, Morgan, and Nelson (1985) present such artificial censoring.

4.1. Data (Class-B Insulation)

Table 4.1 displays censored data from a temperature-accelerated life test of a Class-B insulation for electric motors (Crawford 1970). Ten motorettes were tested at each of four temperatures (150˚C, 170˚C, 190˚C, 220˚C). The test purpose was to estimate the life distribution (in particular, its median and 10% point) at the design temperature of 130˚C. At the time of the analysis, 7 motorettes at 170˚C had failed, five each at 190˚C and 220˚C had failed, and none at 150˚C had failed. The + in Table 4.1 indicates a running motorette at that number of hours. The motorettes were periodically checked for failure, and a failure time in Table 4.1 is the upper endpoint of the period in which the failure occurred. It is better to use the midpoint.

Table 4.1. Class-B Insulation Life Data and Plotting Positions

150 °C		170°C		190°C		220°C	
Hours	F_i	Hours	F_i	Hours	F_i	Hours	F_i
8064+	-	1764	5%	408	5%	408	5%
8064+	-	2772	15	408	15	408	15
8064+	-	3444	25	1344	25	504	25
8064+	-	3542	35	1344	35	504	35
8064+	-	3780	45	1440	45	504	45
8064+	-	4860	55	1680+	-	528+	-
8064+	-	5196	65	1680+	-	528+	-
8064+	-	5448+	-	1680+	-	528+	-
8064+	-	5448+	-	1680+	-	528+	-
8064+	-	5448+	-	1680+	-	528+	-

The lognormal distribution and Arrhenius relationship are used to analyze these data. The methods also apply to other simple models, using the Weibull distribution and the inverse power law and other relationships.

4.2. Probability Plot (Lognormal)

A probability plot of censored data at each test stress is made as follows. Use probability plotting paper for the model distribution (lognormal here). Suppose, at a test stress, a sample of *n* units has *r* failure times. Order the times from smallest to largest and assign rank *i* to the *i*th ordered failure time. As before, the plotting position of the *i*th failure is

$$F_i = 100(i - 0.5)\frac{1}{n} \qquad i = 1, 2, \dots, r$$

These plotting positions are in Table 4.1. Nonfailure times are not assigned plotting positions.

On the probability paper, plot each failure time against its plotting position. Nonfailure times are not plotted. Figure 4.1 shows a lognormal probability plot of the Class-B data. By eye fit a straight line (or, if necessary, a curve) to the plotted points as shown in Figure 4.1. Fitted lines can be parallel, since the model has parallel lines. As before, the fitted lines obscure the data. So, it is best to also have a plot without lines.

The probability plot of singly censored data yields the same information and is interpreted the same as a plot of complete data. Such information includes estimates of distribution percentiles and fraction failing by a given age. For example, Figure 4.1 yields median estimates of 4300 hours at 170°, 1650 hours at 190°, and 510 hours at 220°. For 190 or 220°, estimating the median involves extrapolating from data in the lower tail of the distribution to the middle of the distribution. The lognormal (cumulative) distribution is used to extrapolate over time. This is similar in spirit to using the Arrhenius relationship to extrapolate over temperature.

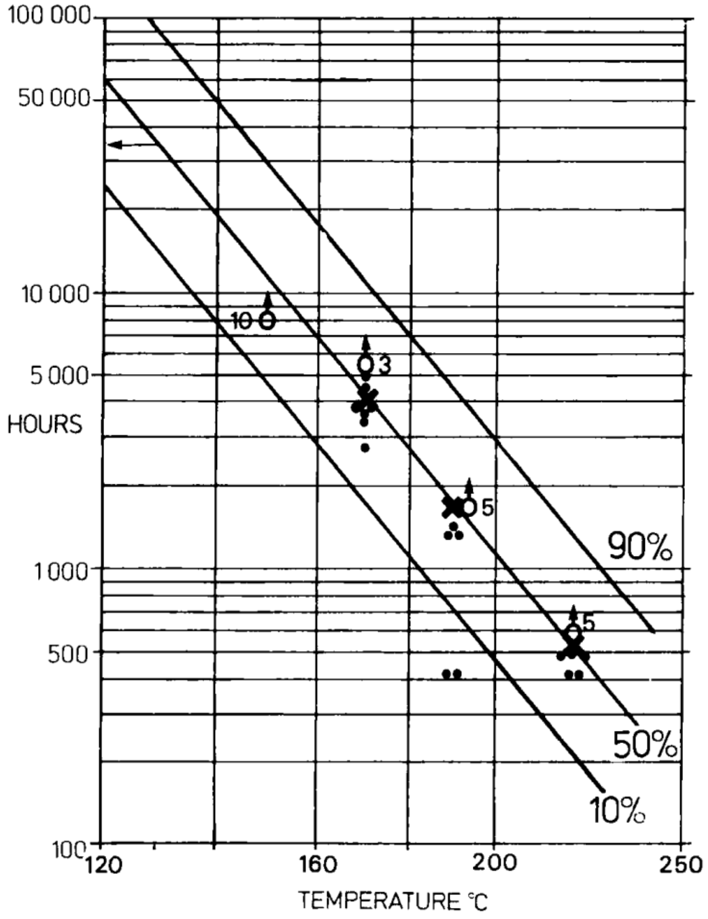

Figure 4.2. Arrhenius plot of Class-B data (• observed failure, × estimate of median, Ô# unfailed).

4.3. Relationship Plot (Arrhenius)

A relationship plot of life against stress is made as described in Section 2.3. In particular, estimate a specific percentile at each test stress from the probability plot. Then plot each estimate against stress on paper where the relationship between "life" and stress is linear. Finally draw a line through the plotted estimates to estimate the relationship.

For the Class-B insulation, each median estimate is plotted with a cross against its test temperature on Arrhenius paper in Figure 4.2. A straight line is fitted by eye to the crosses in Figure 4.2. The failure and nonfailure times are also plotted to display the data. Nonfailures make such a plot more difficult to grasp. This is one reason the line is fitted to percentile estimates rather than directly to the data.

The fitted relationship is used as described to estimate model parameters and life at a given stress. For example, the estimate of median life at the design temperature of 130˚C is 35,000 hours. This key estimate indicates that insulation life is satisfactory.

4.4. Graphical Estimates

Example. To estimate the life distribution line at 130˚C, mark the median life of 35,000 hours on the lognormal probability paper in Figure 4.1. Then draw a straight line through this median. Choose the slope of this 130˚ line as the visual average of the slopes for the test temperatures. The slope (log standard deviation) is assumed to be the same for all temperatures. This line estimates the 130˚ life distribution. For example, the estimate of the 10th percentile at 130˚C is 17,300 hours.

4.5. Assess the Model and Data

Graphical methods of Sections 2.5 and 3.5 are used to assess the (lognormal) distribution, the (Arrhenius) relationship, and the data. Censored data require greater care in interpretation; for example, the nonfailures in a relationship plot must be visually assessed differently from the failures. Thus, it is best to use estimates of percentiles in such a plot to assess linearity.

The lognormal plots (Figure 4.1) of the 170 and 220˚ data are relatively straight. However, their slopes differ some. When informed of this, the insulation engineer revealed that postmortem of failures had identified different dominant failure modes for these two temperatures.

At 190˚C, two failures are much too early. Review of the data and test method did not reveal their cause. The data were reanalyzed without them. The estimate of median life at 130˚ changed little. Of course, the estimate of the log standard deviation decreased, and the estimate of the 10% point at 130˚C increased.

Examination of the plot of the medians suggests that over the range 170˚C, 190˚C, and 220˚C the linear relationship is adequate.

5. MULTIPLY CENSORED DATA

Introduction. In some accelerated life tests, data at a stress level are multiply censored. Such data contain running and failure times that are intermixed. Such data result from

- analysis of the data while specimens are still running;
- removal of specimens from test at various times;
- starting specimens on test at various times and
- loss of specimens through failure modes not of interest or through extraneous causes such as test equipment failure.

This section describes graphical estimates of the accelerated test model and of the product life distribution at any stress. Graphical analysis involves

- a hazard plot of the multiply censored data at each test stress (like a probability plot);
- a relationship plot of life versus stress.

The example employs the lognormal distribution and Arrhenius relationship. Of course, the methods apply to other distributions and other (transformed) linear relationships.

5.1. Data (Turn Failures)

Data in Table 5.1 illustrate the graphical methods here. The data are hours to Turn failure of a new Class-H insulation system tested in motorettes at high temperatures of 190, 220, 240, and 260^{0}C. A purpose was to estimate the median time to Turn failure at the design temperature of 180^{0}C. A median life over 20,000 hours was desired.

Ten motorettes were run at each temperature and periodically inspected for failure. The time in Table 5.1 is midway between the time when the failure was found and the time of the previous inspection. The times between checks are short, and using the midpoint has little effect on the plots. The times between checks (called cycle lengths) were nominally 7, 4, 2, and 2 days for 190, 220, 240, and 260˚C, respectively.

The data on Turn failures are not complete, since some motorettes were removed from test before having a Turn failure. Each running time is marked with a + in Table 5.1. Failure times are unmarked. Such multiply (or progressively) censored data and must be analyzed with special methods like those below.

Table 5.1. Turn Failure Data in Hours

190°C	220°C	240°C	260°C
7228	1764	1175	1128
7228	2436	1521	1464
7228	2436	1569	1512
8448	2436+	1617	1608
9167	2436	1665	1632+
9167	2436	1665	1632+
9167	3108	1713	1632+
9167	3108	1761	1632+
10511	3108	1881+	1632+
10511	3108	1953	1896

5.2. Hazard Plot (Lognormal)

A hazard plot of the multiply censored data from each test stress is made to estimate each life distribution. A hazard plot is a probability plot and is used and interpreted like one. Other methods for plotting multiply censored data are given by Kaplan and Meier (1958), Herd (1960), Johnson (1964). They employ probability paper. Hazard plotting also applies to singly censored and complete data; for such data, probability plotting is usually used, as it is better known and understood by others. The hazard plotting method is explained with the 220° data in Table 5.2.

Hazard calculations. For a test stress, suppose there are *n* tests units (*n* = 10 for the 220° data). Order the *n* times from smallest to largest as shown in Table 5.2 (ignore whether they are running or failure times). Then label the times with reverse ranks *k*; that is, label the first with *n*, the second with $n - 1$ and the *n*th with 1 as in Table 5.2.

Calculate a hazard value for each failure time as 100/*k*, where *k* is its reverse rank. The hazard values for the Turn failures are shown in Table 5.2. For example, the failure at 2436 hours with reverse rank 8 has a hazard value of 100/8 = 12.5%. Hazard values are not calculated for running times.

Calculate the cumulative hazard value for each failure as the sum of its hazard value and the cumulative hazard value of the preceding failure. For example, for the same failure at 2436 hours, the cumulative hazard value is 33.6 = 12.5+21.1. The cumulative hazard values of the Turn failures are shown in Table 5.2. Cumulative hazard values have no physical meaning and can be larger than 100%. They are just proper plotting positions.

Table 5.2. Hazard Calculations for 220° Turn Data

220°C Hours	Reverse Rank k	(100/k)% Hazard	% Cum. Hazard	Modified Cum. Haz.
1764	10	10.0	10.0	5.0
2436	9	11.1	21.1	15.6
2436	8	12.5	33.6	27.4
2436+	7			
2436	6	16.7	50.3	42.0
2436	5	20.0	70.3	60.3
3108	4	25.0	95.3	82.8
3108	3	33.3	128.6	112.0
3108	2	50.0	178.6	153.6
3108	1	100.0	278.6	228.6

+ censoring time

Modified values. Modified cumulative hazard values may be better for plotting small samples. The modified value for a failure is the average of its cumulative hazard value and that of the preceding failure. The modified cumulative hazard value of the first failure is half of its cumulative hazard value. Such modified values appear in Table 5.2.

Hazard paper. Choose the hazard paper of a theoretical distribution. There are hazard papers for the exponential, Weibull, extreme value, normal, and lognormal distributions. Lognormal hazard paper is used for the insulation life data (Figure 5.1). Suitably label the vertical (data) scale.

Hazard plot. On the hazard paper, plot each failure time vertically against its cumulative hazard value on the horizontal axis. Nonfailure times are not plotted. Such a plot is made with the data for each stress as shown in Figure 5.1. Fit parallel straight lines to the data at each stress if desired. Each line is a graphical estimate of the cumulative distribution at that stress.

How to use a hazard plot. The probability (percentage) scale on a hazard paper for a distribution is exactly the same as that on the corresponding probability paper. Thus, a hazard plot is used the same way as a probability plot. The hazard scale is only a convenience for plotting multiply censored data.

Percentile estimates. An estimate of a distribution percentile comes from a hazard plot in the same way as from a probability plot. Enter the hazard plot on the probability scale at the desired percentage. Go down to the fitted line for the stress, and then go sideways to the time scale to read the percen-

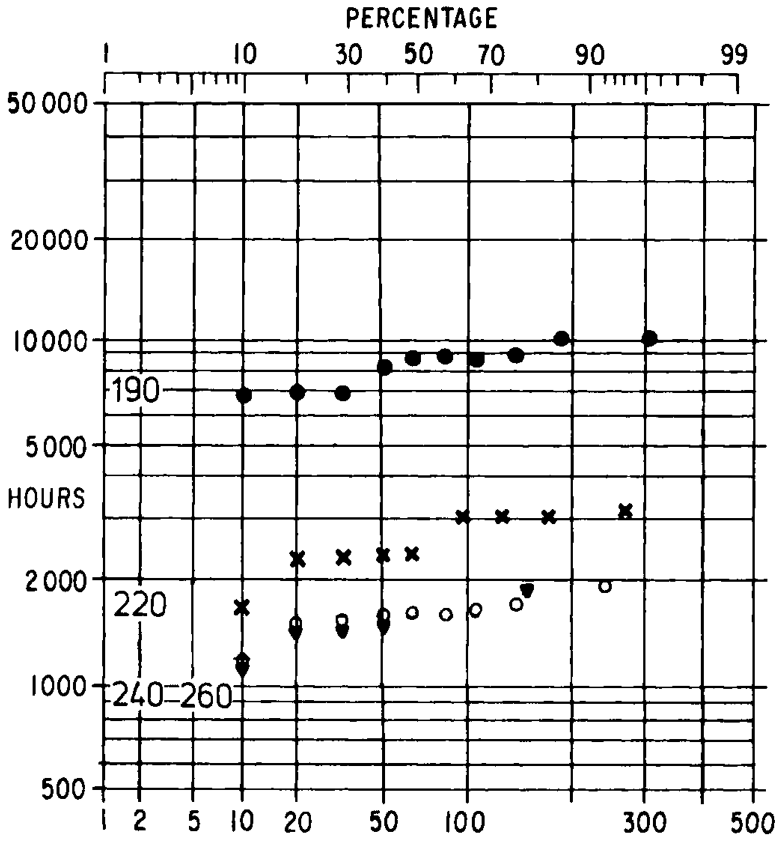

Figure 5.1. **Lognormal hazard plot of Turn data.**

tile estimate. For example, the estimate of the median time (50th percentile) to Turn failure at 220˚C is 2.900 hours from Figure 5.1. Estimates of the medians at the test temperatures are plotted as crosses in Figure 5.2.

Estimate of a percentage failing. A hazard plot is used like a probability plot to estimate the percentage of units that fail by a given age at a stress. Enter the hazard plot on the time scale at that age. Go sideways to the fitted line for the stress, and then go up to the probability scale to read the percentage. For example, in Figure 5.1, the estimate of the percentage that fail by 3.000 hours at 220˚ is 55%.

5.3. Relationship Plot (Arrhenius)

The relationship between life (some distribution percentile) and stress is estimated with the same method described in Sections 2.3 and 3.3. Namely, use paper where the relationship is a straight line, and plot the estimate of the chosen percentile for each test stress against the stress. The Turn failure medians are plotted on Arrhenius paper in Figure 5.2. The Turn failure times and running times could be plotted to display the data. However, the plot of failure and running times is cluttered and difficult to interpret.

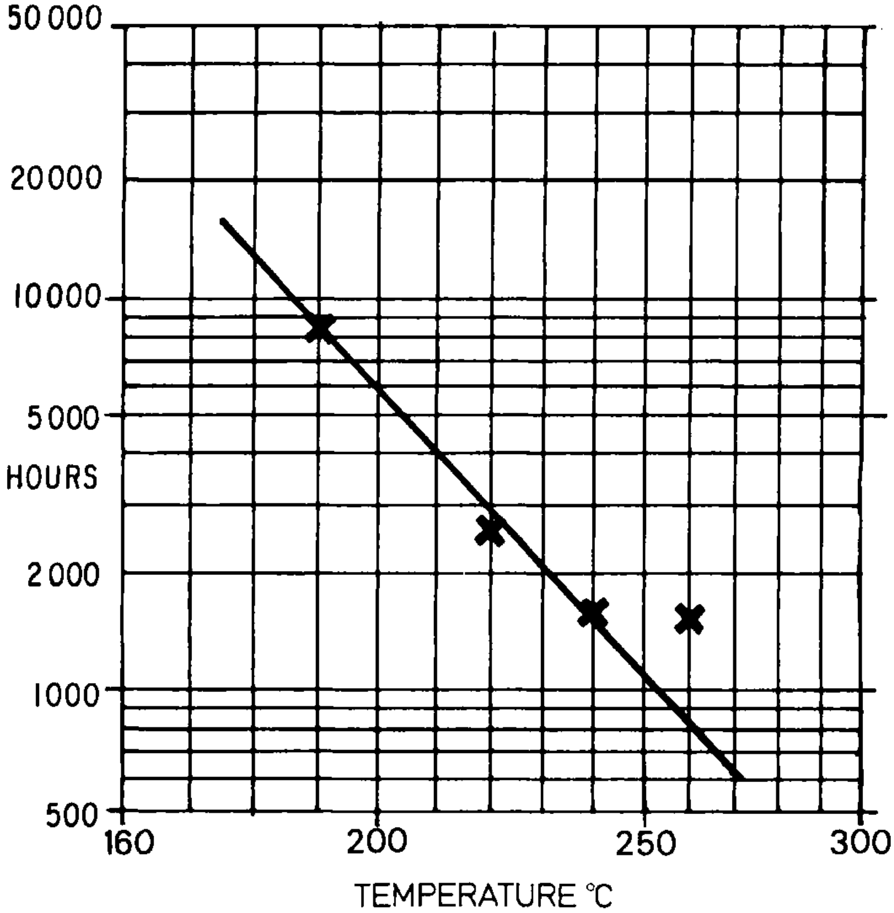

Figure 5.2. Arrhenius plot of Turn medians ×.

Finally fit a straight line by eye to the percentile estimates. This line graphically estimates the relationship between life (the percentile) and stress. For reasons given later, the 260° data were not used to estimate the line in Figure 5.2. The median life at any temperature is estimated from this line. In particular, the estimate of median life of Turn insulation at the design temperature of 180°C is 12.300 hours. This is well below the desired 20.000 hours.

5.4. Graphical Estimates

Estimates of μ and σ. The hazard plot yields estimate of the mean log life μ and the log standard deviation σ. As before, the estimate of the mean log life at a stress is just the log of the median there. For example, the graphical estimate of the median at 220°C is 2900 hours from Figure 5.1. The corresponding mean log life is $\log(2900) = 3.462$. The estimate of the log standard deviation a is the difference between the logs of the 50th and 16th percentiles at a stress. For 220°C, the estimate of the 16th percentile is 2400 hours. The estimate of the log standard deviation is $\log(2900) - \log(2400) = 0.08$. This small value indicates an increasing failure rate.

Median at design temperature. The relationship plot yields an estimate of the median at any temperature. For example, the estimate for 180°C, the design temperature, is 12.300 hours.

5.5. Assess the Model and Data

The graphical methods of Section 2.5 are used to assess the distribution (lognormal here), the relationship (Arrhenius here), and the data. Censored observations in a relationship plot are difficult to interpret. Thus, one uses estimates of percentiles in such a plot. Some remarks on the plots follow.

Common σ. Figure 5.1 shows noteworthy features. The plots at the four temperatures are parallel. This indicates that a has the same value at all test temperatures. This is consistent with the Arrhenius-lognormal model. In contrast, Figure 2.1A does not have parallel plots because the Class-H data there contain a mix of failure modes. One expects the model to fit data on a single failure mode better than it fits data with a mix of failure modes.

Relationship not Arrhenius. The 260° data coincide with the 240° data in Figure 5.1. Consequently the 260° median is the same as the 240° median in Figure 5.2. However, insulation life should be less at 260° than at 240°. One possible reason for the peculiar 260° data is that the 260° motorettes were not made at the same time as the others. So, they may differ with respect to materials or fabrication and consequently life.

$1,000,000 insight. Another possible reason for the peculiar 260° data is that the test method may be misleading. The Standard recommends how long motorettes be held at temperature in an oven between inspections. Following the Standard, the test used 7 days between inspections at 190°, 4 days at 220°, and 2 days at 240°, but did not use 1 day at 260°. Instead the test used 2 days at 260°, the same as at 240°. The inspection involves removing the motorettes from the oven, cooling them to room temperature, and applying the design voltage to see if the insulation withstands it. Unfailed motorettes are put back into the oven and heated to the test temperature. Thus, the insulation is thermally cycled, and the resulting mechanical stressing may degrade life. According to this theory, if the 260° motorettes had been cycled every day, instead of every two days, they would have failed sooner, and the data would have looked "right." A subsequent designed experiment involved combinations of temperature and cycle length. The experiment showed that thermal cycling has an important effect on the insulation life. The insulation engineer knew that such a motor is used one of two ways: continuously or frequently on and off. The engineer saw that continuously running motors were not thermally cycled, and a cheaper insulation would suffice for them. This insight annually saves $1,000,000 at 1989 prices.

6. INTERVAL (READ-OUT) DATA

Introduction. This section describes how to graphically analyze read-out (interval) data. It shows how to estimate the product life distribution under (accelerated) test conditions and under design conditions. Topics include:

- Interval (read-out) data and Microprocessor example;
- Probability plot and confidence limits;
- Relationship plot and acceleration factors.

Tobias and Trindade (1986) present some of these topics with electronics applications.

6.1. Read-Out (Inspection) Data and Microprocessor Example

Description. Some life tests yield read-out (interval) data on time to failure of specimens. In such tests, sample specimens start on test together (at test time 0), and they are inspected periodically for failure. Finding a specimen failed on inspection i at read-out time t_i, one knows only that it failed between the previous read-out time t_{i-1} and t_i ($t_0 = 0$). The exact failure time is not observed, because it is difficult or costly to instrument each specimen to observe failure.

Purpose. A purpose of the analyses below is to estimate the life distribution of such devices at the test condition (125˚C and 7.0 V) and at the design

Table 6.1. Microprocessor Read-Out Data

Interval i:	1	2	3	4	5	6	7	8
Hours t_i:	6	12	24	48	168	500	1000	2000
f_i/n_i:	6/1423	2/1417	0/1415	2/1414	1/573	1/422	2/272	1/123

Removals and censoring. In such testing, some unfailed devices may be censored after any inspection time. For example, Table 6.1 shows that 1414 devices entered interval 4 at 24 hours and 2 failed. Of the 1414 - 2 = 1412 that survived to 48 hours, 573 devices entered interval 5 at 48 hours. Thus 1412 - 573 = 839 unfailed devices were censored at 48 hours. Such censoring arises various ways. Some unfailed devices may be removed from test at various read-outs. Such removals free the test equipment for other tests and reduce test cost. Of course, such removals result in less accurate estimates of the life distribution at later inspection times. However, it is often best to run more devices through early inspection times to accurately estimate the lower tail of the life distribution. Below it is assumed that removals occur only at inspection times. Also, censoring may result from loss of devices, say, from failure of the test equipment or other extraneous causes, for example, a failure mode not of interest. Also, censoring results from having several tests in progress, each having run a different length of time when the data are analyzed.

6.2. Probability Plot and Confidence Limits

Estimate. For read-out data, the life distribution can be nonparametrically estimated only at the inspection times. The following method yields a nonparametric estimate F_i and plot of the population fraction failed at each inspection time t_i. The random quantities in the read-out data are the numbers f_i of devices failing in each inspection period. The actual random failure times are not observed. Thus, the usual probability plotting methods of previous sections for observed failure times do not apply.

Table 6.2. Calculation of Microprocessor Reliability Estimates

(1)	(2)	(3)	(4)		(5)	(6)	(7)
			$R_i' =$		$R_i =$	$F_i =$	95%
i	t_i	f_i/n_i	$1-(f_i/n_i)$		$R_1'R_2'\cdots R_i'$	$1-R_i$	conf.
1	6	6/1423	$.99578_{35}$	→=→	$.99578_{35}$	0.42%	±.34%
2	12	2/1417	$.99858_{85}$	→=→	$.99437_{80}$	0.56%	±.39%
3	24	0/1415	1.00000_{00}	→=→	$.99437_{80}$	0.56%	±.39%
4	48	2/1414	$.99858_{55}$	→=→	$.99297_{16}$	0.70%	±.44%
5	168	1/573	$.99825_{48}$	→=→	$.99123_{86}$	0.88%	±.56%
6	500	1/422	$.99763_{03}$	→=→	$.98888_{97}$	1.11%	±.73%
7	1000	2/272	$.99264_{70}$	→=→	$.98161_{85}$	1.84%	±1.26%
8	2000	1/123	$.99186_{99}$	→=→	$.97363_{78}$	2.64%	±2.02%

Plot. On probability paper, plot each estimate F_i of the population fraction failed against its time t_i. In previous probability plots, each point corresponds to one specimen failure at its actual time of failure. However, in a plot of read-out data, a point may correspond to one or more failures. Moreover, the point appears at the inspection time after the actual failure time. To interpret such a plot, one must consider such differences. To help do this, one can write the actual number of failures in each interval in place of the point. Or, better, plot confidence limits (Figure 6.1) about each plotted F_i to indicate its accuracy. No failures occurred in interval 3. It is difficult to say whether to plot the estimate F_3, since the point does not correspond to any failures.

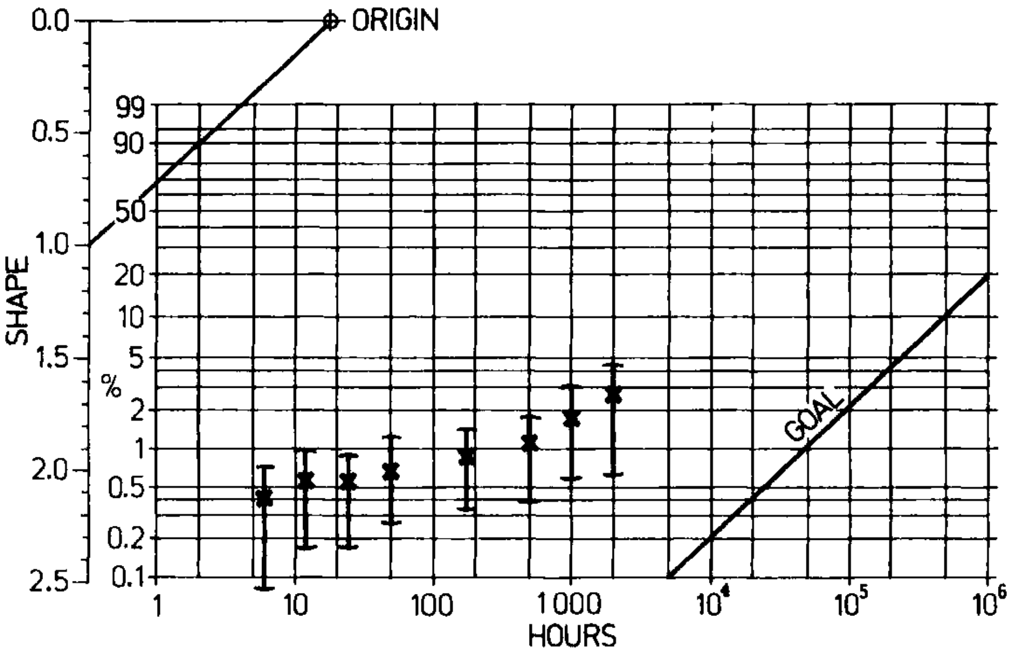

Figure 6.1. Weibull plot of Microprocessor estimates and 95% limits.

Interpretation. Interpret the plot of the Fi like other probability plots as follows. But consider the interval nature of the data.

- Distribution fit. If the plot is relatively straight, then the distribution adequately fits the data over the observed inspection times. For example, the Weibull plot of the Microprocessor data (Figure 6.1) is relatively straight. Thus, the Weibull distribution appears to adequately fit the data. To compare how well different distributions fit, plot the data on various distribution papers, and choose the distribution with the straightest plot. The confidence limits below help make this choice;
- Failure rate. The graphical estimate of the Weibull shape parameter indicates the nature of the failure rate (increasing, decreasing, or constant as the population ages). For the Microprocessor data, this estimate is 0.3. This indicates a decreasing failure rate. Thus, such devices would benefit from burn-in if the failure mode at the test condition is the dominant one at the design condition.
- Goal. A goal is a failure rate of 200 FITS. The corresponding exponential distribution appears in Figure 6.1 as a straight line with a shape parameter (slope) of 1. The distribution estimate is well below (worse than) the goal. Of course, this estimate is for an accelerated condition. Section 6.3 shows how to estimate the distribution at a design condition.

6.3. Confidence Limits

Uncensored data. Suppose n devices are tested and are uncensored before inspection i. The following simple estimate and confidence limits apply to all inspection times through t_i. For example, for the Microprocessor data in Table 6.1, the limits apply through $t_3 = 24\ hours$. Also, to a good approximation they apply through $t_4 = 48\ hours$, since only one device out of 1415 is censored at 24 hours.

Estimate. Suppose C_i is the cumulative number failed by read-out time t_i. Then the simple estimate of the population fraction failed by time t_i is the sample fraction $F_i = C_i/n$. The reliability estimate is $R_i = (C_i/n)$. The estimate above for censored data reduces to this simple one when there is no intermediate censoring. Plot these F_i on probability paper.

Exact limits. If there is no censoring before t_i, the cumulative number of failures C_i has a binomial distribution, and exact binomial confidence limits apply. Simple approximate limits follow.

Poisson approximation. For few failures (say, $C_i < 10, n > 10C_i$), use the two-sided $100P\%$ confidence limits (Poisson approximation):

$$F_i \cong \left(\frac{0.5}{n}\right) x^2 \left[\frac{1-P}{2{,}2C_i}\right] \ and\ F_i' = \left(\frac{0.5}{n}\right) x^2 \left[\frac{1-P}{2{,}2C_i+2}\right]$$

Here $x^2[P'; D]$ is the $100P'th$ percentile of the chi-square distribution with D degrees of freedom. Note that the two limits have different degrees of freedom: $2C_i$ and $2C_i + 2$. The confidence limits for reliability are $R_i = 1 - F_i$. The one-sided upper $100P\%$ confidence limit is

$$F_i = (0.5/n)x^2(P; 2C_i + 2)$$

With $100P\%$ confidence, the population fraction failing by time t_i is no worse than this F_i.

Normal approximation. For many failures (say, $10 < C_i < n - 10$), use the two-sided $100P\%$ limits (normal approximation):

$$F_i' \cong F_i - K_p \left[\frac{F_i(1-F_i)}{n}\right]^{\frac{1}{2}}, F_i'' = F_i + K_p[F_i(1-F_i)/n]^{1/2}$$

Here K_p is the standard normal $100(1 + P)/2$ percentile. For example,$K_{.95} = 1.96 \cong 2$. The one-sided upper $100P\%$ confidence limit is

$$F_i'' = F_i + z_p[F_i(1 - F_i)/n]^{1/2}$$

here z_p is the standard normal $100P_{th}$ percentile. For example,$z_{.95} = 1{,}645$. Confidence limits for reliability are $R_i = 1 - F_i$. Thomas and Grunkemeier (1975) investigate better approximations.

Censored data. For censored read-out data, the following are approximate confidence limits for the population fraction failed by read-out time t_i. Based on a normal approximation to the distribution of the estimate F_i, they are adequate when $C_i > 10$, and can be used even for $C_i > 5$. For smaller C_i, use the binomial confidence limits for uncensored data if feasible. Plot these confidence limits on the probability paper with the estimates F_i. Figure 6.1 shows such limits on the Weibull plot for the Microprocessor data.

Limits. The following limits employ an estimate $v(F_i)$ of the variance of the estimate F_i. Two-sided approximate $100P\%$ confidence limits are

$$F_i' \cong F_i - K_p[v(F_i)]^{\frac{1}{2}}, F_i'' = F_i + K_p[v(F_i)]^{\frac{1}{2}}$$

Here K_p is the standard normal $100(1 + P)/2$ percentile.

Approximate variance. For F_i small (say,$F_i < 0{,}10$), a simple approximation is

$$v(F_i) \cong R_i^2\{[F_1'/(n_1R_1')] + \cdots + [F_i'/(n_iR_i')]\}$$

The notation follows that in Table 6.2. Note that both primed and unprimed estimates appear here, and $F_i' = \frac{f_i}{n_i} = 1 - R_i'$. For example, for the Microprocessor data, $v(F_8) = 0.000100\ at\ 2000\ hours$. The approximate 95% confidence limits are

$$F_8' = 0{,}64\%$$

$$F_8'' = 4{,}64\%$$

These limits are plotted at 2000 hours in Figure 6.1.

Exact variance. The exact variance estimate entails the calculations in Table 6.3. The calculations in columns (1) through (5) of Table 6.3 are the same as those in Table 6.2. However, in column (3) of Table 6.2, use $\eta_i' = \eta_i - 1$ in place of η_i. In Table 6.3, r_i' denotes the estimate of the conditional reliability, and r_i denotes the estimate of the reliability at time t_i. Then the exact variance estimate is

$$v(F_i) = R_i(R_i - r_i)$$

Here R_i comes from Table 6.2, and r_i comes from Table 6.3. Note that $(R_i - r_i)$ is a small difference of nearly equal numbers. Thus, R_i and r_i must be accurate to seven significant figures to assure that $v(F_i)$ is accurate to two figures. The $v(F_i)$ appear in column (6) of Table 6.3. Column (7) shows $2 \times 100 \times [v(F_i)]^{1/2}$ where $2 \cong k_{.95}$ and 100 yields a percentage. This $v(F_i)$ is Greenwood's (1926) variance for the Kaplan-Meier estimate extended for read-out data.

Table 6.3. Exact Calculation of $\nu(F_i)$

(1) i	(2) t_i	(3) f_i/n_i'	(4) $r_i' = 1-[f_i/n_i']$	(5) $r_i = r_1' r_2' \cdots r_i'$	(6) $\nu(F_i) = R_i(R_i - r_i)$	(7) $K_{.95}[\nu(F_i)]^{1/2} = 200[\nu(F_i)]^{1/2}$
1	6	6/1422	$.99578_{06}$	$.99578_{06}$	$.00000_{28}$	±.34%
2	12	2/1416	$.99858_{75}$	$.99437_{41}$	$.00000_{38}$	±.39%
3	24	0/1414	1.00000_{00}	$.99437_{41}$	$.00000_{38}$	±.39%
4	48	2/1413	$.99858_{45}$	$.99296_{66}$	$.00000_{48}$	±.44%
5	168	1/572	$.99825_{17}$	$.99123_{07}$	$.00000_{78}$	±.56%
6	500	1/421	$.99762_{47}$	$.98887_{62}$	$.00001_{32}$	±.73%
7	1000	2/271	$.99261_{99}$	$.98157_{82}$	$.00003_{94}$	±1.26%
8	2000	1/122	$.99180_{32}$	$.97353_{25}$	$.00010_{24}$	±2.02%

Computer packages. The procedure LIFETEST of SAS Inst (1985) and other computer packages calculate the Kaplan-Meier estimate (1958) and such confidence limits for multiply censored data with observed failure times. Such routines can be used for censored interval data if all specimens have the same inspection schedule. Then one must consider the interval nature of the data in inputting the data and using the output. If groups of specimens have different inspection schedules, one must use the more complex confidence limits for the Peto (1973) and Turnbull (1976) estimates. STAR of Buswell and others (1984) performs the complex Peto calculations and calculates confidence limits.

6.4. Relationship Plot and Acceleration Factors

One condition. In some accelerated tests, specimens are run at just one accelerated test condition. Moreover, that condition may result from accelerating several variables, such as temperature, temperature cycling, humidity, vibration, etc. Such testing of electronics is common, and MILSTD-883 specifies standard tests. Run during development, such a test is usually intended to identify failure modes so they can be corrected. Also, they are used as demonstration tests (MIL-STD-883) to assess whether a device has satisfactory reliability. One can estimate device life at a design condition only if one knows the acceleration factor between life at the accelerated and design conditions.

Acceleration factor. Suppose that "typical life" of a failure mode is t at a design condition and is t' at an accelerated test condition. Then the acceleration factor K for those two conditions is

$$t = Kt'$$

For example, if $K = 500$, the failure mode lasts 500 times as long at the design condition as at the accelerated condition. Also, loosely speaking, one hour at the accelerated condition equals K hours at the design condition. Equivalently, a read-out time of 6 hours under acceleration corresponds to $500 \times 6 = 3000$ hours at the design condition. An acceleration factor is calculated as follows from a known life-stress relationship. Each failure mode has a separate relationship and acceleration factor.

Arrhenius factor. The Arrhenius relationship is often used to describe temperature-accelerated tests where product failure is due to chemical degradation or intermetallic diffusion. Suppose that T is the design temperature, and T' is the test temperature, both in degrees Kelvin. Kelvin = Centigrade + 273.16. Then the Arrhenius acceleration factor for a failure mode is

$$K = exp\left\{(E/k)\left[\left(\frac{1}{T}\right) - (1/T')\right]\right\}$$

Here E is the activation energy (in eV) of the failure mode, and k = 8.6171 x 10-5 is Boltzmann's constant in eV per Kelvin degree. E corresponds to the slope of an Arrhenius relationship on an Arrhenius plot. To evaluate the factor, one must know E or assume a value for it. There are such factors for other life-stress relationships.

Example. For the Microprocessor, the test temperature is T" = 125 +273.16 = 398.16$T'' = 125 + 273{,}16 = 398{,}16$, and the design temperature is $T = 55 + 273{,}16 = 328{,}16$.
For some types of failure modes, the activation energy is assumed to be$E = 1.0\ eV$. The corresponding acceleration factor is

$$K = 501$$

Thus, such a failure mode lasts about 500 times longer at the design temperature. For such microprocessors, other failure modes are also assumed to have activation energies of 0.8 and $0.3\ eV$.

Design life. The following provides an estimate of the life distribution at a design condition. For each read-out time t_i', use the acceleration factor K to calculate the equivalent time $t_i = Kt_i'$ at the design condition. Use the equivalent times to estimate the life distribution at the design condition, using the methods above. Equivalently, move the estimate of the accelerated distribution toward longer life by a factor K. For example, Figure 6.2 shows a Weibull plot of this estimate of the Microprocessor life distribution at 55˚C. The distribution at 55˚C is higher than that at 125˚C (Figure 6.1) by a factor $K = 500$. For example, the inspection at 6 hours at 125˚C corresponds to an inspection at $500 \times 6 = 3000$ hours at 55˚C. Confidence limits and parametric estimates similarly are at higher times by a factor K as in Figure 6.2.

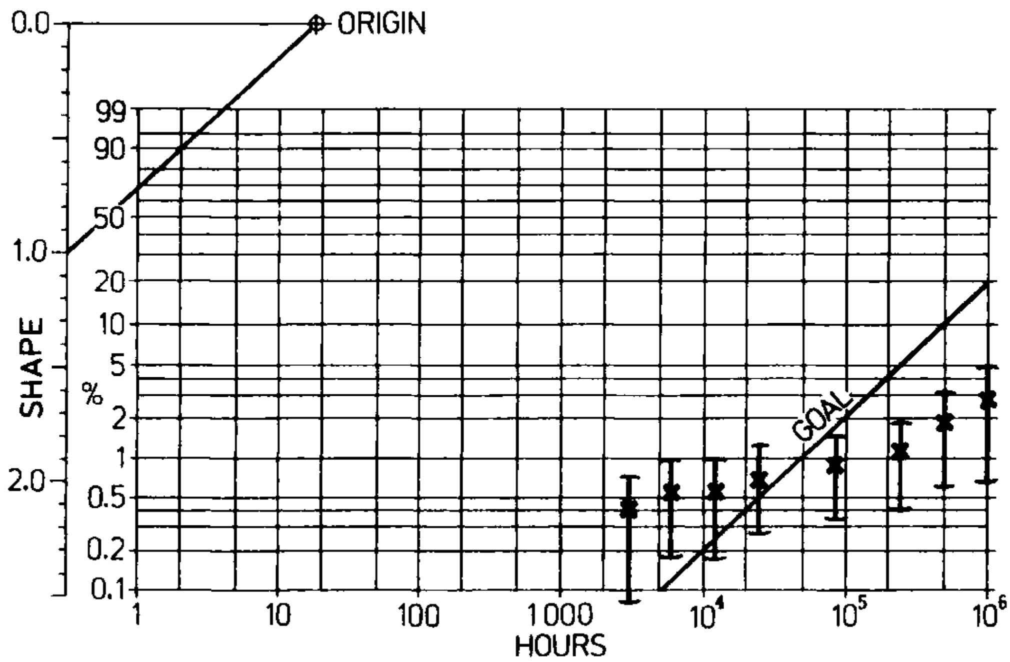

Figure 6.2. Weibull plot of 55°C estimate and 95% limits.

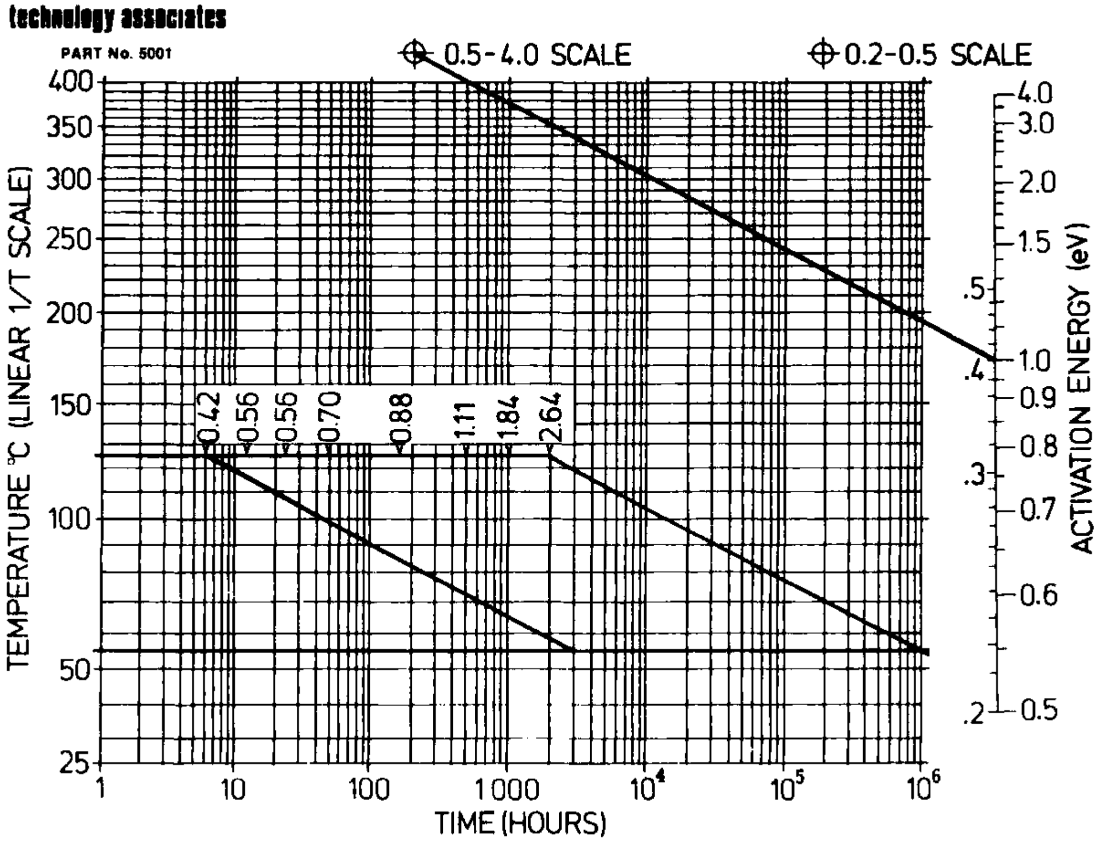

Figure 6.3. **Arrhenius plot of Microprocessor data, slope of 1.0 eV. (Paper provided by courtesy of D. Stewart Peck.)**

Multiple acceleration. There can be several accelerating variables, each with an accelerating factor. The product of those factors is the combined accelerating factor. This assumes that the variables do not interact. The generalized Eyring model represents such interactions.

Uncertainty. Note that the width of a confidence interval is the same at the accelerated and design conditions. This results from if the acceleration factor is correct. In practice, the factor is approximate. Thus, the uncertainty of the estimate of life at the design condition really exceeds the confidence limits. A better analysis would include the uncertainty in the acceleration factor.

Arrhenius plot. The acceleration factor can be graphically evaluated with Arrhenius paper as follow. The Arrhenius paper in Figure 6.3 shows each read-out time and sample cumulative percent failed at 125˚C, the accelerated temperature. In Figure 6.3, a line passes through an origin and the Activation Energy scale at 1.0 eV. Its slope corresponds to $E = 1.0\ eV$. Draw lines parallel to that line from the read-out times t_i' at 125˚C to get the corresponding times t_i at 55˚C. The estimates Fi correspond to these new t_i. The times scales in Figures 6.2 and 6.3 coincide, showing the relationship between the Weibull and Arrhenius plots.

RINGRAZIO LA MIA FAMIGLIA PER AVERMI PERMESSO DI CONTINUARE GLI STUDI, COSTRUIRMI UN FUTURO E AVERMI SOPPORTATO IN QUESTO LUNGO E FATICOSO PERCORSO.

RINGRAZIO IL PROFESSOR CIARAPICA PER LA PAZIENZA DIMOSTRATA, SOPRATTUTTO IN QUESTO PERIODO TUTT'ALTRO CHE SEMPLICE.

RINGRAZIO I MIEI AMICI PER ESSERMI SEMPRE STATI ACCANTO, AVERMI INCORAGGIATO E DATO LA FORZA DI NON MOLLARE.

RINGRAZIO MARTINA E DAVIDE PER AVER SAPUTO ASCOLTARE LE MIE CONTINUE LAGNE.

RINGRAZIO IL DOTTOR TARASCHI PER AVER CURATO LA MIA IPOCONDRIA.

RINGRAZIO LE PERSONE CHE NON CI SONO PIU', PER AVERMI RESO LA PERSONA CHE SONO.

E, INFINE, UN RINGRAZIAMENTO SPECIALE A FRANCESCO, EDOARDO, MATTEO, DENIS E MATTIA PER AVER FATTO TUTTO CIO' CHE E' SCRITTO SOPRA ED ESSERE DIVENTATI A TUTTI GLI EFFETTI PARTE DELLA MIA FAMIGLIA.