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Formulazioni MINLP Per Problemi Di

Hydro Unit Committment

MINLP formulations for the Hydro Unit Commitment problems

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ABSTRACT

Many real-world optimization problems involve continuous and nonlinear decisions. Each nonlinear component of these problems can be modeled linearly, with or without taking into consideration additional integer variables. In this study, different modeling alternatives are proposed for a real-world nonlinear optimization problem, in particular the hydroelectric unit commitment problem (1-HUC). The 1-HUCs non-linearities comes from the energy produced in each period. It is defined as a two-dimensional non-convex and non-concave function of decisions variable related to water. Flow and head, being themselves variable decisions are nonlinear, convex and are one-dimensional functions of the turbine volume. A common simplification is also considered, assuming that the hydraulic head is fixed and thus defining the power output as a one-dimensional, non-convex function of water flow. Several linear and nonlinear models are described for the 1-HUC and fixed-head 1-HUC. These models cover several families of modeling alternatives, including models common in the literature as well as new models with less common features. Different sets of instances are generated to assess performance sensitivity against key 1-HUC features. Several available solvers are used for each nonlinear model and the best virtual solver is selected to focus on model capabilities rather than solver performance. Based on the numerical experiments, counter-intuitive recommendations are given to help practitioners select the most appropriate model and solver based on instance characteristics.

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LIST OF ABBREVIATION

MINLP: Mixed integer nonlinear programing
MLIP: Non-linear mixed model with linear

Program PWL: Piecewise linear function

MLP: Non-linear mixed integer program KKT karush - kuhm - tucker.

SBB: Algorithm involved in the global

Optimization PNLM: Mixed non-linear programming

G: affine function

F: power function

PGEN: Persistent generator

CPU: Central processor unit
P: function
BB: BRANCH AND BOUND

5PL: Five Parameters Logistic

TC: Computing Time

REFERENCES

AL: scientific author

ZHUAN: scientific author

MADRIGAL scientific author

INTRODUCTION

In real life, we are regularly subjected to continuous optimization through well-defined methods. These methods are mainly the linear and nonlinear methods that are done by means of additional variables and represent a more precise physical system, faster than a linear model but requiring more computational at time, especially when the possibility of convexity doesn't apply. To implement and present the specificity of these two possibilities, firstly, we shall present in a detail manner the different illustrations that represent a real-world non-linear function. We will mainly focus on the HUC model of hydroelectricity. HUC commitment models cover a wide range with or without linearity. All 1HUC models offer two methods, one is specific to one-dimensional nonlinear convexity and the other is proper to non-concave two-dimensional nonlinear whose parameters are precision, feasibility and computation time. With the 1HUC model, we will have the ability to provide optimal and effective solutions that improve the different models. Furthermore, this model presents a set of solutions that possessed linear and non-linear properties with integer or non-integer values. This thesis aims to define and explain the 1HUC approach in the general recommendations of numerical modeling base on an experimental model and briefly summarize the usage of nonlinear optimization in literature reviews, to explain his numerical experiment that illustrates the performance of different 1HUC disjunctions, that can be used to partition the solution set or to obtain bounds on the optimal solution of the problem. Within the framework of the MINLPs, the use of disjunctions for branching has been the subject of intense research, while the practical usefulness of disjunctions as a means of generating valid linear inequalities has attracted attention only recently. Secondly, we will describe some applications of MINLPs well-known for their separation and disjunction method which has proved its effectiveness in Mixed Integer Linear Programming (MILP). As showed by the experimental results, this application has obtained encouraging results in the case of MILP, while using a simple separation method.

1 HYDRO UNIT COMMITMENT

The HUC is like a hydroelectric container with an upstream and downstream reserve. The production follows the working principle that water flows from the upstream to the reserve passing by the turbines of the unit. The horizon time is discretized in time periods of delta duration. At each period of time "T", the water flow "DT" passing by the unit, has to be in the interval (D, D). The power point produced at the time period T depends on the water flow rate DT but also the tank head. The quantity of water in the reserve depends on the water in upstream; the minimum capacity is defined as the target volume when we have additional positive or negative water entries. The reserve contains water that has an expected unit value. A higher level will lead to retain more water and produce less electricity, which is the opposite for a low level. The HUC presents in this context the problem of selecting price takers to maximize electricity revenue. It has parameters such as the water value, the external in flows and the capacity of the reservoir which is related to the water value of each reservoir at the end of the period.

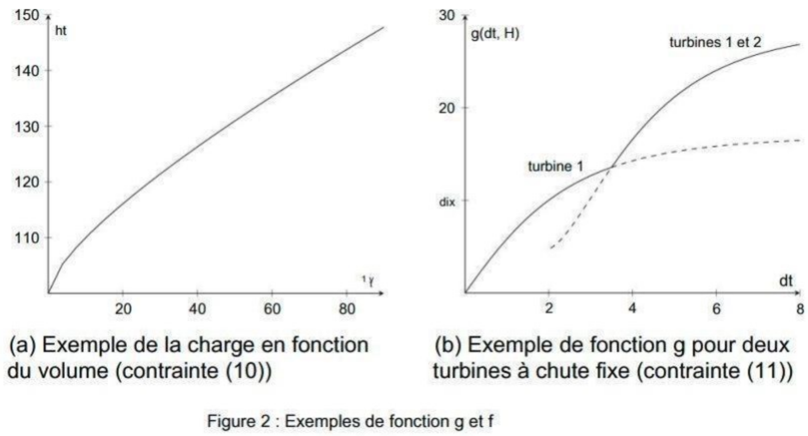
The main purpose of HUC is maximizing profit by meeting target of capacities and volumes. At each period, we have volume conservation constraints which calculate the hydraulic load. The simplification of HUC consists in assuming a constant load $\boldsymbol{ht} = \boldsymbol{H}$. This simplification is relevant for small variations.

1-1 HUC NON-LINEARITY LITERATURE

The goal of linear and nonlinear modeling is to approximate the power function F from a hydroelectric to the power function $\boldsymbol{PT} = \boldsymbol{RHO} \times \boldsymbol{G} \times \boldsymbol{HT} \times \boldsymbol{DT}$. PT are the RHO powers. The density of water **G** is the universal gravitational constant. **DT** represents the flowing water and HT the height to consider. HUC considered multiples units; a power function is derived from linearity which is a head bilinear function and the flow of water. The function also depends on the height and the downstream of the volume more than the upstream volume. **(F)** Is a bilinear function that depends on the flow and management. SPACIAL HYDRO BRANCH AND BOUND (SHBB) exists to optimally solve the HUC with cascading units and the MINLP. A bilinear function of water level and flow is obtained by comparing it again with the pgen**.** The function F is a polynomial function of degree 4. The function F is bilinear as a function of flow and height in articles. It is considered convex and concave, but in HUC the authors have introduced a family of univariate linear functions by a power of a room model depending on the water flow. Each partially linear function represents a volume VS. Water flow, using exactly four parts VS. NLP, which has become a univariate linear function by the introductory function. Moreover, it approaches constraints by a univariate flow family that involves a polynomial function. He also has included an improvement of 1-HUC which is the rectangle method. However, several actors will be ignored and considered people of no economic value for optimization.

1-2 NONLINEAR FUNCTION FOR 1-HUC

From the Pgen model, we want to specify the functions F and G. The function f is used to calculate the head and it shows the evolution of the head as a function of the volume, for an instance where Υ_i are instance-dependent parameters and Υ_4 belong to a certain interval. Meaning that the function is necessarily convex. According to the shape of the tank, the function can be quasi-linear or presented in a very sensitive non-linearity. For each turbine, the function g is almost linear when the turbine starts and after it curves more and more until the next turbine starts up. When a turbine starts, we notice a break in the form of the function.



<u>Figure 1</u>: example of g and f functions

Following the evolution of the problem, we were interested to the power function f because this function is neither convex nor concave. It doesn't necessarily correspond to our data because it has been a generic hydroelectric function of the literature and described by its equation not being a fixed head. The following functions correspond more to the data that will be considered as a model instead of the equation with g non-convex and non-concave

$\boldsymbol{PT} = \boldsymbol{RHO} \times \boldsymbol{G} \times \boldsymbol{ht} \times \boldsymbol{g}$ (*dt.ht*). With a fixed head, the power is the product of the function and the constants but unfortunately remains non-convex and non-concave. We can say with certainty that the power regime of each turbine is convex and makes the unit to have N turbine which starts in the prescribed order. The addition of several turbines accentuates the concavity of the functions to be much more refined. The function is almost linear at the start of the turbine and becomes more linear at the start of the next turbine. The tea binary variable sea is equal to the << **i** >> if the first turbine rotates and uses the power accumulated in the first turbines. In general cases, the function g also depends of the head (figure3.). It illustrates the evolution of the function for the minimum and maximum load of instance BT-1 in the case of 1HUC. This function is as close as possible to physics. The tea Pgen model is considered as the original model that cannot be used to solve the 1-HUC for the following reasons:

- In terms of calculation, when we consider the function **G** as described, it is nonlinear with mixed variable;
- Preliminary calculations show that this model involves more calculations times than all the other models.

A possible solution would be to derive more treatable models than the original one, to capture the nonlinearity in the power function. This power function presents the following characteristics (useful) non-convex, non-concave, locally linear. When you start a turbine, it's convex with respect to the water flow

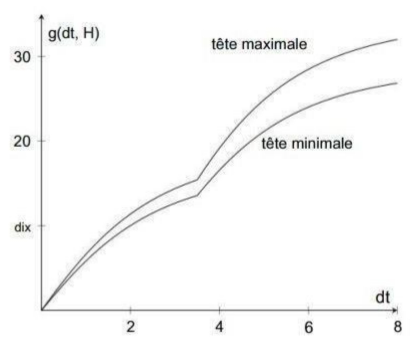


Figure 3 : Exemple de fonction g pour la charge minimale et maximale (contrainte (4))

Figure 2: example of g function for minimum and maximum load.

2 MODEL FOR 1-HUC

It's has as purpose to represent the non-linearity of the 1-HUC, resulting from the energy production function and from its characteristics. The models are also described for the 1-HUC with a fixed head.

2-1 (MI) NLP MODELING ELEMENTARY NONLINEAR FUNCTIONS

Here, the models are representations of each turbine to realize. It is enough to consider each turbine separately and to have a representation close to physics. The problem remains at the level of the auxiliary variables of the turbines because they are necessary.**3** models are presented:

- polynomial function (formula family of functions)
- 5pl with the Max () function
- 5PL function family without the Max () function

The MILP and MINLP convex problem contain non variables constraints of the first type only. When these constraints are relaxed, we obtain a relaxing suite which gives a lower bound on the optimal solution value and a vector solution **x** that satisfies convex constraints but can violate the requirements of entirety. We made deepest research on the way to use this type of disjunction, how to derive disjunctive cuts from it and how to add effectively such inequalities while using the couple BB method to a generator procedure of cutting. Duration (7, 8, 49). Many generalizations can be introduce at the MILP level. A well-known example is the split disjunction $\boldsymbol{\pi x} \leq \boldsymbol{\pi 0}\ \boldsymbol{V\pi x} \geq \boldsymbol{\pi 0} + \mathbf{1}$ where

- (π, π0) Є Z
- P + 1 and x Є Z
- P is a vector of p integer variables (10, 17, 22, 36).

The formulation of a MINLP problem is submitted to the two types of no convex constraints. The optimal solution to the relaxation LP, x can be unrealizable for entirety constraints or for one or many reformulation constraints, xk = Ɵk(x). In this work, we are principally going to consider disjunctions issue from the constraints of the second type, and concentrate on the problem of finding valid disjunctions which is violated by a solution to the relaxation LP of P0.

In a simple manner, let's consider a no convex constraints xk = 0k (xi) with 0k univariate and xi constant (disjunctions can be derived with a similar procedure when 0k is multivariate and/or xi is integer). Let's suppose that this constraint is violated by a solution x of the relaxation LP, which means that the spatial disjunction, although valid for any ß € ii of the user interface, it is not violated by x. It is a starting point with the MILP, for which disjunctions on integer variables are valid and obviously violated by fractional solutions

2-2 MILP MODELING A PIECEWISE LINEAR FUNCTION

In the modeling of linear problems, piecewise linear approximations have a fundamental use, especially for obtaining MILPs. The purpose of comparing a piecewise linear model to a non-linear model is to compare the accuracy of the values of the solutions. This is the standard linear formulation that we have considered more precisely, the convex formulation. There are several methods to obtain a piecewise linear function in two dimensions, but they are much more focused on the computation time than on the values of the solution. The one-dimensional method allows to obtain a piecewise linear function in two dimensions. It represents a generalization of the convex combination.

The UVLS is an important means to prevent the acceleration of the tension. This article uses LM as a criterion of tension stability. LM designs the maximal quantities of supplementary charge at a point of a given functioning by the UVLS organogram approach. When an eventuality is detected by the surveillance system online after a certain period, the relay command is no more furnishing energy to the following charge and the UVLS strategy is detected. In normal circumstances, the system actualizes periodically the UVLS strategy according to the state of the system online. The set of contingence is defined by operators whose can include the N 2 unexpected with relatively high probabilities.

For each contingence, the LM post-contingence can be obtained by the method based on the immediate optimization of the power flow obtain by the surveillance system online. If is inferior to the exigency LM demanded, the UVLS optimization is solved to generate and refresh the UVLS strategy of contingence. The refreshment rate is limited by the resolution speed of optimization. Parallel calculation techniques can accelerate that process. The UVLS optimization is a problem based on the MILP and is presented in section III-B. The UVLS strategy meaning the optimal solution, determines which start has to be abandoned. When a certain contingence produces, the last correspondent strategy is adopted by the relay command to assure that the LM satisfied the exigencies.

The objective function is to minimize the total cost of the effacement. Binary variables Xcz designs the jettison strategy. Xcz is equal to 1 if all the loads at the start z is 0. Let's note that the continuously controllable load can easily be extended in this model. The constraints of the modeling DPWLPF presented in section II are included to provide an image of the states of the system post-contingence after UVLS. Nodal power balance equations are described as the active and reactive output powers of the unit that must be limited within the authorized range. Bus voltage amplitudes are restricted within the safe range. Constraints representing the flow constraints of branches apparent quadratic linear per piece.

2-3 INTEGER LINEAR PROGRAMMING

Dillon and Al. developed an integer programming method for the practical size. In what concerns problems based on the extension and modification of the branch and bound method, the HUC problem can be divided into a nonlinear economic dispatch problem and linear pure integer one. HUC problem can also be based on the Benders approach while the mixed integer programming approach solves the HUC problem by reducing the solution search space and rejecting infeasible subsets. A linear programming problem can be solved either by decomposing the entire problem into sub problems using the Danzig Wolf

Decomposition principle, and then solving each sub-problem using linear programming or by solving the problem directly using the revised simplex technique. To solve logic contradictions, the Hitting set tree (HST) of REITER is often used to satisfy the principle of minimal changes. The given work applies the HST algorithm to the minimal incoherent subset to solve the incoherence. The work of [8] proposes various strategies to calculate axiom weights then modifying the HST algorithm to find a solution with a minimum weight sum. That algorithm is more efficient because of the normal optimality criterion of minimum path length has been replaced by the minimum path rank where could exist a number of path that could be interrupted sooner. This algorithm corresponds to HST Swoop in our experimentation. The work of provides a graph based algorithm for calculating conflicts and apply the HST algorithm to find solutions. To ameliorate the efficacy of algorithms based on HST, some works provide algorithms to reduce the search space and satisfy other types of minimum change definitions. Such algorithm first extract a subset of each conflict then apply the HST algorithm to its subsets. A typical work is given in the lines that follow, there we show two algorithms to treat the problem of revision of the ontology. An algorithm uses the notation function to choose the axioms with highest score of each contradiction. The other exploits weights to select the axioms with the lowest weight. The two algorithms correspond to HST Score and HST Weight. However, the effectiveness of these algorithms remains a problem if the extracted subsets are not quite small.

To further improve efficiency, researchers offer heuristic strategies to find an approximately minimal solution and adopt a new semantics to avoid building a HST. The previous algorithm removed axiom with the highest score of MIPS and has chosen another in the left MIPS and this process is repeated until there is no more MIPS. Obviously, the solution consisting of all deleted axioms could not be minimal. The work of furnishes equally a notation function to select axioms and revise an ontology. There are several work that deal with the incoherence of ontology learning tasks and the ontology versions . They don't calculate conflicts but stop an axiom to be added to a coherent ontology because that axiom can cause a potential incoherence. The article furnishes an effective algorithm by replacing the semantics standard in the characterization of inference tasks Lite by an alternative semantic call semantics of type. In such cases, the conflicts calculation and their minimal set of hit can be avoiding.

2-4 BRANCH AND BOUND

Cohen and Al. presented a new approach to solve the HUC problem based on the branch and bound method, which incorporates all temporal constraints and does not require unit priority ordering. Huang and Al proposed constraint logic programming and branch-bound technic to provide an efficient and flexible approach to the HUC problem. The branch and bound procedure consists of the repetitive application of the following steps. Firstly, the decision variables (i.e.: the set of decision variables considered) through which the optimal solution is obtained is known to be partitioning in subset. Secondly, if all the elements of a subset violate the constraints of the minimization problem, then this subset will be eliminated for further consideration.

2-5 LAGRANGIAN RELAXATION

Based on the Lagrangian relaxation approach, the HUC problem can be written in terms of 1) a cost function that is the sum of terms implicating each single unit; 2) a set of constraints involving single unit; 3) a set of coupling constraints (the generation and reserve constraints). In the first part, we have a sub-division of the basic solution. Officially, the HUC problem can be written as a pumped storage hydroelectric power station. Zhuang and Al proposed three new phases of Lagrangian relaxation algorithm for HUC. In the first phase, the Lagrangian dual of the UC is maximized with the standard sub gradient technique; the second phase finds a dual feasible solution in reserve and is followed by a third economic dispatch phase. Wang and Al presented a rigorous mathematical method to deal with the bounds on the rampart UC and the rotor fatigue effect.

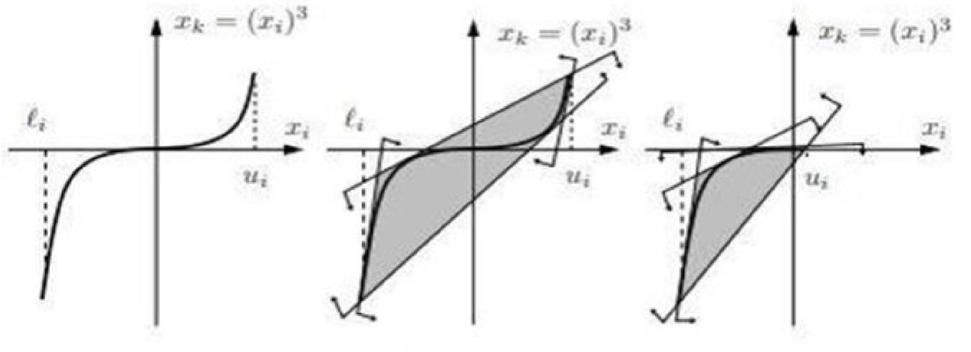
2-6 OPTIMIZATION OF INTERIOR POINTS

Interior point methods have been used not only to successfully solve very large linear and nonlinear programming problems, but also to solve combinatorial and a non-differentiable problem. The interior point method has now been applied to solve scheduling problems in electric power systems. Madrigal and Al applied the interior point method to solve the HUC problem. Based on his observation, he noticed that it possesses advantages and meets no problem with the use of parameters.

3 DISJUNCTIONS IN A MINLP

For the MINLP, in order to apply the disjunctive programming paradigm to the non-convex, MINLP disjunctive programming paradigm need a description of valid disjunctions set that will be violated by an optimal solution x. Convex problems of MILP and MINLP contain only convex constraints of the first kind. When these constraints are relaxed, we obtain a continuous relaxation which produces a lower bound on the optimal solution value and a solution vector. A standard procedure for selecting disjunctions is to sort them in ascending order of infeasibility. During the selection of a disjunction for branching, the one with the maximum infeasibility is chosen. If a set of disjunctive cuts is desired, the first p will choose the first p disjunctions from the sorted list and p disjunctive cuts are generated. As we can see, this measure of infeasibility may not be the best way to classify disjunctions. More sophisticated techniques have been proposed for disjunction selection in branching process. For example, strong branching, pseudo-cost branching, reliability branching and violation transfer. A generalization of the reliability in the case of MINLP has recently been presented disjunctions in special cases of the MINLP

DISJUNCTIVE CUTS FOR NONCONVEX MINLP 7



(a) Reformulated constraint (b) A linearization (c) A *refined* linearization

FIG. 1. *Linearization of constraint $x_k = (x_i)^3$: the bold line in (a) represents the nonconvex set $\{(x_i, x_k) : \ell_i \leq x_i \leq u_i, x_k = (x_i)^3\}$, while the polyhedra in (b) and (c) are its linearizations for different bounds on x_i.*

<u>Figure 3:</u> linearization of constraint.

3-1 MINLP WITH A FAMILY OF POLY FUNCTIONS

The polynomial function has a parabolic form and can be represented by a polynomial function of degree 2, where each function represents a power generated by a turbine plus the contribution of the previous turbines, according to their starting order (figure 5.2) as shows in the example with 03 turbines

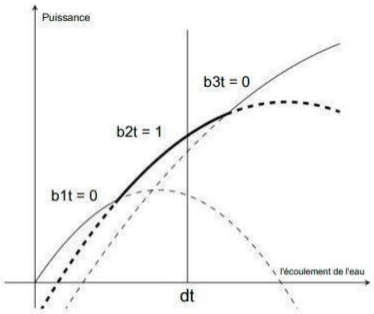


Figure 4 : Fonctions polynomiales représentant la puissance pour chaque combinaison de turbines

<u>Figure 4:</u> polynomial function representing the power for each combination.

3-2 NLP WITH 5PL FUNCTIONS USING MAX FUNCTION

The function **g** can be represented as a sum of 5PLs and by the power of a turbine, where each of the 5PLs represents the power of a turbine and, if the sum is correctly built; it can be a correct approximation of the physical sums. The figure below shows well an approximation of the sum of

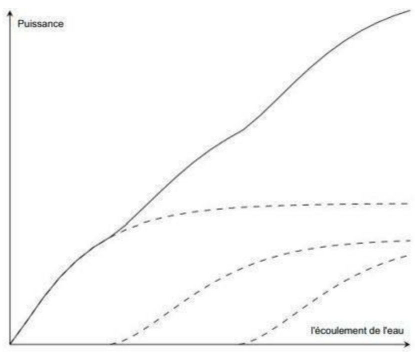


Figure 5 : Somme des fonctions 5P L représentant la puissance pour un volume fixe

The functions 5PL as continuous lines and **035PL** as a dotted line

<u>Figure 5:</u> sum of 5PL functions representing the power for a fixed volume

This model is a variation of (P5P L-max), where the max () is linearized by adding linear inequalities, auxiliary variables and binary variables. The inequality set ensuring the unit= max (0, dt – y1it). The model (P5P L – bin) contains inequalities and contrary to the model NLP (P5P L – max), the model (P5P L – bin) is a MINLP, because it needs an auxiliary binary variable. The model (P5P L – bin) can be solved by many MINLP solvers because the function max () has been linearized. The non-linearity is the same for (P5P L – max) and (P5P L – bin); the two models take into account the characteristics C1 and C2. Let's note that the model (Pgen) and the piecewise nonlinear function with 5PL are also a MINLP. The difference between the two is that the binary variables are not the same as those of the (P5P L – bin). Indeed, the binary variables of (P5P L – bin) act only to linearize the function while the (Pgen) are decision variables to fit the model (P5P L – bin) to the fixed head 1-HUC.

3-3 NLP WITH A BILINEAR FUNCTION

A model used in the literature for solving the HUC as a MINLP is a bilinear model. The water flow represents the linear power in relation to the head. Therefore, it must be done while respecting the flow of water. The following notations illustrate the non-constant phenomenon such that the power depends linearly on water$\boldsymbol{PT} = \boldsymbol{RHOGHT} + \boldsymbol{y} + (\boldsymbol{t})$. The inequalities calculate the power which depends linearly on the height of the stream ht, the pbilin contains inequalities for the MINLP functions. It is easy to have an under estimator and an over estimator because a bilinear function is convex. The Pbilin model contains MINLP inequalities which are effective tools on this type of nonlinearity because a binary function is convex and it is easy to build an over estimator and an under estimator. (PHD – poly), (P5P L – max) or (P5P L – bin), (Pbilin) do not require any additional variables. This makes this model a potential candidate for quickly solving problem. Indeed, the binary function does not present any of the nonlinear characteristics; even MILP models such as piecewise linear models might have better accuracy. When we adopt the (Pbilin) model to a 1-HUC fixed-head, the model becomes a linear model where the power is a linear function of water flow.

The problem with this model is the risk of approximation when fitting a pbilin model to the 1HUC which has fixed head and a height h. The linear model and the power become a linear function. The water flow for this purpose adapts to the power $\boldsymbol{pt} = \boldsymbol{RHOgh}\ (\boldsymbol{u} + \boldsymbol{vdt})$ which contains inequalities. The pop model is a necessary model for finishing the workflow. Its advantage remains a considerable solution but has the disadvantage that its target volumes may be inaccessible. The combination of 1-HUC model and the pop model give us a MINLP model. The pop model adapted to the fixed head 1-HUC contains very few variables.

3-4 SUMMARY OF NONLINEAR MODELS AND FUNCTIONS

All of these models generally have the same constraints. In table 1, we have a summary of all the models. Table 2 shows us the characteristics and the type of program for each model.

The convexity and the linearity does not take into consideration the integer variables. Theoretically, no model presented fits perfectly the power function of the original model. None of the nonlinear models present the 4 nonlinear characteristics of the power functions. However, it will be demonstrated in the numerical experiments that these models admit very small approximation errors, while the shortest times are obtained in other models.

Model	General cases inequalities	Fixed-head cases inequalities
$(P_{2D-poly})$	S1,(12)-(15)	S2,(13)-(14), (16)
$(P_{5PL-max})$	S1,(17)-(18)	S2,(19)
$(P_{5PL-bin})$	S1,(17),(20)-(23)	S2,(24)-(27)
$(P_{HD-poly})$	S1,(28)-(29)	S2,(30)
(P_{bilin})	S1,(31)	S2,(32)
(P_{op})	(1), (6),(10), (33)-(36)	(1), (6), (33)-(35), (37)
(P_{pwl})	S2,(38)-(48)	S2,(38)-(41), (42)-(44), (49)

Table 1: Summary of the proposed models

	1-HUC		Fixed-head 1-HUC		Characteristics			
Model	Type	Convexity	Type	Convexity	C1	C2	C3	C4
$(P_{2D-poly})$	MINLP	non-convex	MINLP	convex	✓	✗	✓	✓
$(P_{5PL-max})$	NLP	non-convex	NLP	non-convex	✓	✓	✗	✗
$(P_{5PL-bin})$	MINLP	non-convex	MINLP	non-convex	✓	✓	✗	✗
$(P_{HD-poly})$	NLP	non-convex	NLP	non-convex	✓	✗	✓	✗
(P_{bilin})	NLP	non-convex	LP	linear	✗	✗	✗	✗
(P_{op})	MINLP	non-convex	MILP	linear	✗	✗	✗	✗
(P_{pwl})	MILP	linear	MILP	linear	✗	✗	✗	✗

Table 2: Comparison of the models non-linear characteristics

Figure 6: : comparison of the models non-linear characteristic

4 SOLUTION APPROACHES FOR NON-LINEAR OPTIMIZATION PROBLEM

T

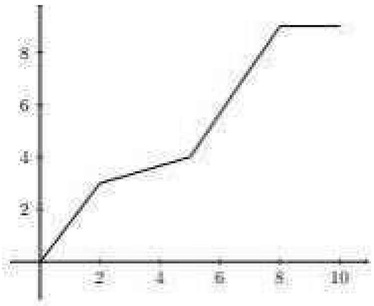
The solution to approaches nonlinear optimization goes through the linear and non-linear system. A nonlinear function can be approximated in 4 possible ways: approximations with a linear function, approximations with a family of elementary functions, approximations with a piecewise linear function, approximations with a discrete set of decisions. These models lead to the (MLIP) for the set of mixed nonlinear models, the non-linear program (NLP) or the non-linear mixed integer program. If the function is convex or concave, and the set of constraints is convex, there are specialized methods, called convex optimization methods that can be used. In other word, there are several solutions. For example: the use of principle separation and evaluation to split and treat several parameters separately. The algorithm can also be stopped before it succeeds, if it can be proven that no subsequent solution will be better within a certain tolerance. The Karush-Kuhn-Tucker (KKT) conditions guarantee that a solution obtained in this way is optimal. These algorithms essentially use the principle of separation at run time, means the presence of a lower and upper bound in each subspace. The first minimization solution is derived from all solutions of the feasible subspace. This lower bound is obtained by solving a relaxation of the problem for a MILP. In the case of the relaxed problem, we obtain the integrity constraints through the convex sub estimator. Since our study is limited to the exact methods, we selected solvers who will make us visualize the optimal solution of our system.

4-1 MODELING

In the framework of modeling of a nonlinear problem, the choices of modeling can have an impact on the processing speed. In this case, the convex problem of the system admits a single local optimum which is also a global optimum based on the convex estimators NINLP and NLP. The computation of lower bounds also means a very fast convergence towards the global optimum. The choice of the solution can modify the feasibility of the problem, particularly in the case of polynom of the second degree. Of course, we can underline that each solver is unique but the modeling will impose its impact on the calculation time, if the MLIP is in it.

It is usually motivated by the shorter computation time and the structure of the function which influences its model. The PWL is a piecewise linear function and affine collection on intervals.

They are described through disjunctive solutions that make it possible to describe a PWL model. They are also useful in the case of linear non-continuity of a univariate function.



(d) Approximation with a piece-wise linear function

An example of figure (Figure12.1).

Figure 7: approximation with a piece-wise linear function.

4-2 ALGORITHMS

Solving a MLP or MINLP models consists in looking an underestimation of the convex function; there is a study by BMF16 which describes a list of underestimations for several linear functions. Many sub- estimators have been created in order to obtain the best one or the use of symbolic reformulation will consist in adding auxiliary variables in order to maintain simplistic nonlinearities. Another method consists in reducing the domain of variables as above. These two methods are based on feasibility and optimality. Another approach is to find a linear polyhedron that includes any other nonlinear function on an interval. In a way that a polyhedron will be both a sub-estimator and an over-estimator of the linear function. We can obtain spaces by splitting; here, we will obtain what we call disjunctive inequalities. To solve a MILP, the algorithm of branch and bound (BB) and its derivatives, in particular the branch and cut. Branch, cut and price are widely used. The strategy divides the search space, only if the optimal solution of the linear and the relaxation are not integer solution. A search space can be ignored if its lower bound value is superior than the global upper bound of the best integer holder solution. To solve a NLP or a MINLP when the nonlinear functions are non-convex, we need a local optimum which is not necessarily the global optimum. Global optimization refers to all the techniques that seek the global optimum. The main algorithm involved in most global optimization tools is the spatial and bound branch which can be applied to solve a MINLP. The main characteristic that differs the SBB and the BB is the lower bound which is obtained by a convex underestimation instead of linear relaxation. The strategy to construct a search tree is the same as a classic branch. There are many variants of the SBB leading to the implementation of different tools. These tools can solve both a MINLP and a NLP or just the NLP. If the modeling doesn't require any binary variable, the model remains continuous and results in a PNL. A SBB variant for PNL is the Branch and Bound using a parameter α to calculate the under estimator functions. The reduced space Branch and Bound is an amelioration of SBB for NLP with a branching process performed only on a subset of variables. The branch and the contract permit to reduce the domains of variables in order to obtain a better under estimator convex. The difference between the BB, the SBB and the lower bound is obtained by a under estimator convex while the linear relaxation of this modeling remains continuous.

5- DISJUNCTIVE CUTS FOR NONCONVEX MINLP

This selection of instances shows that, in some cases, the benefit of disjunctive cuts is worth than the CPU time spent on generation. This is especially true for box QP instances which although a large amount of time is spent on disjunctive cuts, it translates in a better lower bound or a lower CPU time. Again, the fact that the current separation algorithm is rather simple, suggest that a more efficient implementation would permit to obtain the same benefit advantage in a shorter time.

We have also schematized the performance of the four variants using performance profiles. This performance profile only considers instances that could be solved in less than two hours by at least one of the variants. Therefore, it also compares the quality of a variant in terms of the number of solved instances. The numbers of instance (plotted on the y-axis) for which a deviation is less than the corresponding value on the abscissa axis. We can observe once again that, in some cases, we considered the use of reliability branches and disjunctive cuts as something more profitable for MINLL instances. Reliability branches is solved in a short time while the disjunctive cuts will obtain a better lower bound but both remain expensive.

5-1 MOTIVATION FOR NON-CONVEX MINLP

Mixed Nonlinear Programming is a powerful modeling tool for problems that are generally defined in optimization. There are many applications of NLP in the field of chemical engineering and computational biology. Among them, there are special subclasses of MINLP such as mixed integer linear programming (MINLP).

When f is linear and all GI are affine, the convex MINLPs (i.e., MINLPs whose continuous relaxation is a convex nonlinear program), admit special solvers (that are more efficient). So the only reason to use a general purpose non convex MINLP solver is that the problem cannot be classified into any of these special cases. Efficient algorithms for non-convex optimization aim to find a relaxation and obtain a good lower bound on the problem and on the value of the optimal solution. In the case of MILPs, a lower bound can be found by solving the LP relaxation obtained by relaxing the set of all variables. In the case of convex MINLPs, the relaxing integrality produces a convex nonlinear problem and thus a lower bound. In a general case, it can be difficult to find a relaxation and a lower bound on the global optimum of P0 since that the relaxing integrality produces a nonlinear and non-convex problem. When relaxation doesn't yield a strong lower bound, one approach to strengthening relaxation is to use logical methods and disjunctions that are satisfied by all solutions of P0. In their most general form, disjunctions are logical operators that return true whenever one or more of their operands are true. In this work, we consider disjunctions that are involve in linear inequality, although there are more general disjunctions

6-NUMERICAL EXPERIMENTS

The tests are carried out via Neos Server [CMM98] using the following five MINLP solvers: ANTIGONE, BARON, COUENNE, LINDO Global, SCIP, as well as the MILP solver CPLEX. For MINLP solvers, the GAMS format is used for input files, while that for the MILP solver use the LP format.

All experiences are performed on the Neos Server prod-exec-7 machine (a 2x Intel Xeon Gold 5218 @ 2.3 GHz processor with 384 GB of RAM), using a single thread. The calculation time limit is set to 10800 seconds.

6-1 MODELING CHOICE

The parameters presented in the different models are obtained by adjusting the power of the original model. Indeed, this adjustment is made by the nonlinear method of least squares. Remember that our goal is to study and analyze the different approximations of the power function whose parameters are head functions. The PPWL is a model on which we can compare the impact of the number of linear PPWL. All these models include variables that are subject to equality constraints; which have the particularity of not having a sufficiently explicit bound constraint.

6-2 MODEL COMPARISON

As mentioned above, the 1-HUC and its simplified version 1-HUC fixed have configurations not resolvable by all the solvers. The following diagrams show the proportions of the different configurations solved with their VBS while AE are the follower

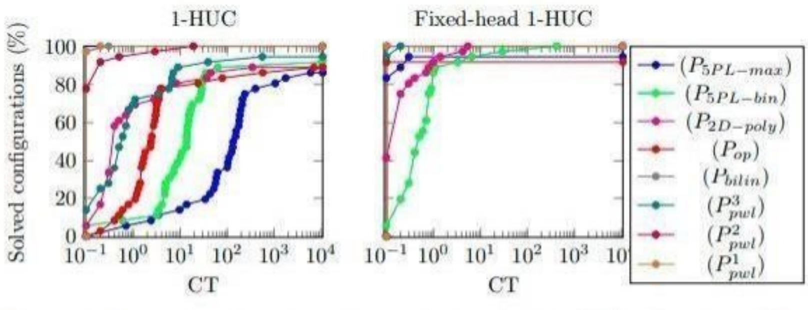


Figure 7: Proportion of configurations solved with their VBS where the CT is under a CT threshold

<u>Figure 8:</u> proportion of configuration solved with their VBS where the CT is under a CT threshold

6-3 COMPARISON OF SOLVERS

Note that some configurations cannot be solved with all solvers. Indeed, the model (P5PL - max) is only supported by LINDO Global and SCIP. Moreover, none of the configuration with (PLD - poly) returns a feasible solution. It follows that the results related to the (PLD - poly) model are not included. All figures and tables for the results are with VBS, except Tables 4 and 5 which show the results for each solver. Figure 7 shows on the ordinate, the proportion of configurations solved with their VBS, under a given CT on the abscissa. Similarly, figure 8shows on the ordinate, the proportion of solved configurations with their VBS, but this time under a AE given in abscissa. Figure 9 shows on the ordinate the proportion of patterns resolved with their VBS, under a DB given in abscissa. For these three figures, the configurations are color coded according to the configuration model.

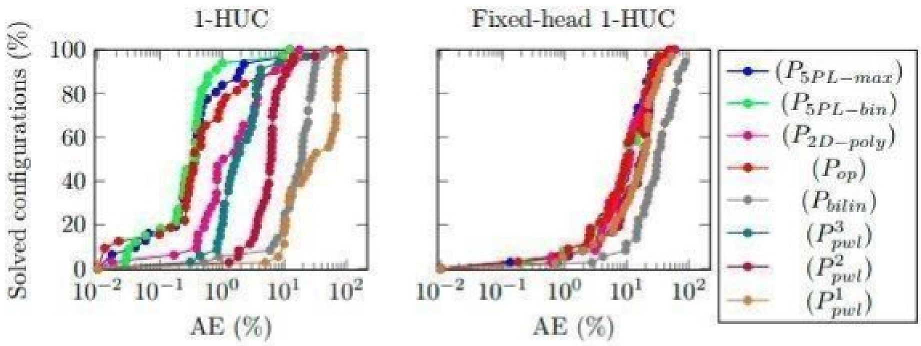


Figure 8: Proportion of configurations solved with their VBS where the AE is under an AE threshold

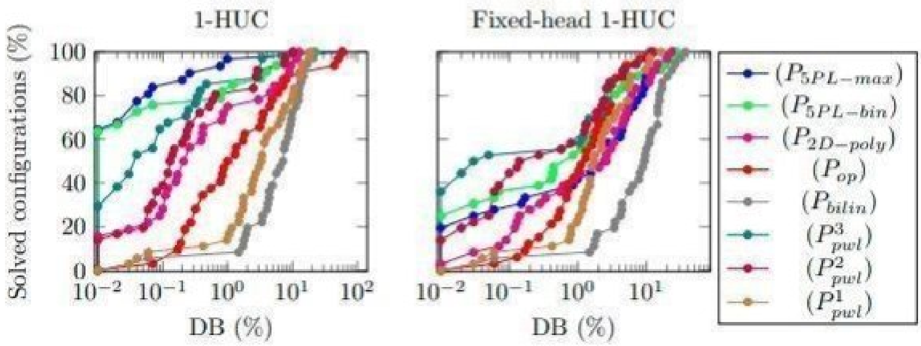


Figure 9: Proportion of configurations solved with their VBS where the DB is under a DB threshold

<u>Figure 9:</u> proportion of configurations solved with their VBS

To finish, the disjunctive cuts are as effective in the MINLP solvers as in MILP. Although, they are generated from an LP relaxation of a non-convex MINLP. They can considerably improve the lower bound that will permit to perform the performance of a branch and bound method. A disadvantage of the CGLP procedure is to solve a large LP in order to obtain a single cut. Itis reflected in a single cut that is carried over to the MINLP case. Figure 4(c) is a comparison between the remaining gap and the reports of all instances for which none of the variants could obtain an optimal solution in two hours or less. This graph shows for each algorithm, the number of instances (plotted on the y-axis) with a deviation remaining below the corresponding entry on the x - axis.

Some algorithms have been developed for the MILP to overcome this problem. Unfortunately, as shown, their extension to the MINLP case is not so simple.

6-4 NUMERICAL PROBLEMS

Numerical problems represent an interesting compromise in respect to the accuracy versus speed of resolution while some work has been realized in a rigorous global optimization that formally verifies nonlinear functions, including semi-algebraic and transcendental functions (Domes, 2009; Domes and Neumaier, 2014). The most commonly used solutions oft ware generates relaxations and cutting planes via a floating point arithmetic, and then uses LP and NLP solvers based on floating point to find under estimators and heuristic solutions. Numerical instability can be at least partially mitigated by using validated interval arithmetic (Brönnimann, Melquiond, Pion, 2003, 2006; de Moura Pass more, 2013) for FBBT. Especially for ill-dimensioned optimization problems, the combination of divergent solution strategies can induce numerical problems due to the variation in tolerances between solvers (including conflicts) and the different sub-solvers of the same meta-solver)

7. GLOBAL SEARCH METHODS

7-1 DIRECT SEARCH METHODS

Theoretical developments focuses on positive coverage sets and insurance descent mechanisms. Convergence to the optimal solutions of first and second order is guaranteed under certain assumptions of smoothness and differentiation of the objective function, and the constraints within the direct local search region based on simplex model methods. Convergence has been extensively studied in the literature, where a set of feasible region update rules guarantees a convergence to local optima when the size of the feasible region is sufficiently small. All CDFO local direct search methods include a set of non-stationarity rules that is the principal mechanism followed by derived algorithms. Direct search methods require that the function should be sampled at locations defined by positive covering sets in order to move in the directions of the best value of the objective function. In addition, the mechanism method of CFD must ensure that strict design geometry criteria are meet, in order to theoretically guarantee true stationarity, geometry measurements like the cosine measurement of positive covering sets. Finally, local CDFO algorithms converge when the mesh size, simplex diameter or line search parameter are small enough and which have been theoretically related to convergence towards stationary points. A diagram of the local search is showed in Figure3, where initially a random point is selected, followed by a set of expansions and contractions until the step size is reduced to zero. It should be noted that the purely direct search methods do not follow exactly what is showed in figure 3, because in the case of a direct search, the new top must be located on a pre-defined network. Figure 3 is expanded as a general representation of the forward search and feasible region that will be described. Following the disadvantages of the CDFO, the local direct searches are strongly dependent on the initial point, the trap in the nearest local optimum and the large number of functions required to guarantee convergence. In order to increase the probability of convergence towards the global optimum, multi-start approaches can be used. However, these are not effective in cases where the model of interest is computationally expensive, when a good starting point is available and when sampling does not require a large computational cost.

7-2 STOCHASTIC METHODS

Stochastic CDFOs constraints originates from the evolutionary literature and it is based on random sampling strategies or on strategies based on probabilistic criteria. Based on probabilistic criteria, the random search was initiated as a concept in the 1950s, proving asymptotic convergence results. The first algorithm to use random samples from a centroid model was

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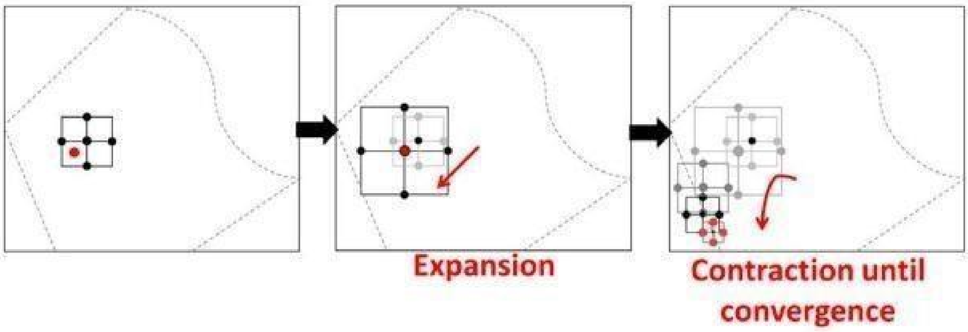

Fig. 3. Schéma d'un algorithme CDFO de recherche de région de confiance locale.

Figure 10: : search for local trust area

the COMPLEX method (Box, 1965), which proceeded by replacing the worst possible points identified. Controlled random search algorithms were then developed on the basis of which new solutions are generated for a sequence of probability distributions. Other developments that have long been in this category are genetic algorithms category, particle swarm, memetic algorithms and Tabu search (Das Suganthan, 2011; ONG, Nair, Keane, 2003; Pal, Csendes, Markot, Neumaier, 2012; Sun, Garibaldi, Krasnogor, Zhang, 2013). There are several developments that extend the stochastic methods to mixed integer optimization. Cases in which the space is necessarily large, complex, or poorly understood and a more sophisticated mathematical analysis is not applicable.

7-3 HYBRID METHODS

Several algorithmic developments combine two or more techniques in order to exploit the techniques and advantages of different methodologies in terms of convergence, sampling requirements and efficiency. The hybrid methodology has significantly better performance compared to pure evolutionary algorithms (Egea, Martí, Banga, 2010; Egea, RodriguesFernandez, Banga, Marti, 2007a; Egea, Vries, Alonso, Banga, 2007b). Griffin and Kolda (2010) integrate the DIRECT algorithm with a stochastic generator search approach to solve constrained box problems, while Hemker and Werner (2011) combine DIRECT with a local search based on a substitute of general problems of CDFO constraints. Recently, Liuzzi, Lucidi and Piccialli (2015a) have proposed modifications to the algorithm DIRECT in order to speed up convergence by using local searches and transformations of the feasible domain. Vaz and Vicente (2007), (2009) integrate Particle Swarm Optimization (PSO) with model search components to solve box constrained and linearly constrained CFD problems. Martelli and Amaldi (2014) combine three different techniques, constrained particle swarm, pattern search and the COMPLEX to solve non smooth problems with multiple nonlinear constraints. In addition, there are several methodologies that uses arrogate models to approximate a part of the models to an original MINLP model. This is treated as a black-box and develop an iterative approach collecting additional samples in order to improve the convergence to a global solution (DavisIerapet,2007,2008,2009; HenaoMaravelias,2011). Finally, Garcia-Palomares, Gonzalez, Castano and Burguillo-Rial (2006) propose to combine the global search with a final local search; a recent development combines a global and direct local search and optimization in a box constrained DFO algorithm (GLODS) which aims at identifying several local solutions without using multi-start random methods (Custódio Madeira, 2015). In addition, decomposition methods have been developed to solve complex problems, such as the optimization of steam networks (Colmenares Seider 1989), where steam pressures and temperatures are optimized at the lower level, using a global deterministic optimization, while water and mass flow are optimized upwards by using the stochastic level and the stochastic COMPLEX method. Similarly, Gassner and Marechal (2009) develop a decomposition algorithm for total site optimization integrated with heat exchangers and utility networks.

7-4 SEARCH TABU

Tabu search is a powerful optimization procedure that has been successfully applied to a number of combinatorial optimization problems. It has the ability to avoid trap in local by employing a flexible memory system. Morietal presented an algorithm, incorporating the list of priority into the Tabu search for unitary commitment. Rajan and Al. solved the CU problem using the neural-based Tabu search method. Lin and Al developed an improved Tabu search algorithm to solve economic allocation problems. Mantawy and Al. presented CPU solutions using Tabu research and also solved long-term hydro power planning problems very effectively using a new Tabu search algorithm.

CONCLUSION

Different linear and nonlinear models are compared to solve nonlinearity problems in terms of resolution, feasibility, approximation error, distance to best recalculated value and computation time. The nonlinear problem considered is 1-HUC, comprising two non-linearity: a one-dimensional convex and a two-dimensional non-convex function. A common simplification of 1-HUC and the fixed head 1-HUC is also considered, with only one nonlinearity: a non-convex and non-concave dimensional function. A first model is defined for the 1-HUC and for the fixed-head 1-HUC. However, this model contains too many non-linearity which are difficult to solve in a reasonable time, even for small instances. Several simpler models are proposed, the objective being to represent the non-linearity of the 1-HUC and the 1-HUC with fixed head. These models cover a wide range of modeling families, including models for 1-HUC from the literature, but also new models with nonlinear functions. Multiple instances sets with 1-HUC and fixed-head 1-HUC features are solved with each of the specific solver choices. The three solvers that give the lowest TC and solve the most instances are CPLEX for linear models and BARON for nonlinear models. There are linear and nonlinear models which are not supported by BARON. This result is valid for both 1-HUC and 1-HUC fixed head. Configurations with a large number of break points (number of turbines) and models with a family of elementary functions, namely (P2D-poly), (P5PL max) and (P5P L-bin), introduce more variables, making their TCs larger. Likewise, instances with more breakpoints have less nonlinearities and create trade-offs between computation time, approximation error and some are not even recommended for any instance class. Moreover, the computation time of a nonlinear model depends on the available global solver. Thus, no model is perfect, the choice of model depends on the characteristics of the instance and the solver used. Recommendations are given to guide the selection of the mathematical model and an appropriate solver. Future work could be devoted to describe models with alternative functions to derive an efficient reformulation of nonlinear models. Models can be tested on variants of Hydropower Unit Commitments, including more constrained from the literature, either with a multi-unit topology or with a pumping systems. Generally, the same type of study can be done for any nonlinear problem and could complement the results presented. The approach envisaged is to use optimal algorithms with approximations of the original model. A complementary approach would be needed to solve the original model with heuristics and compare the results of each approach.

RIASSUNTO THESI

In questo progetto, abbiamo dovuto sviluppare dal punto di vista theorica il modello e la tecnica per risolvere il problema dell'impegno dell'unità idroelettrica, tenendo pienamente conto dei vincoli dinamici dell'unità idroelettrica per ottenere un'economia complessiva del funzionamento del sistema elettrico. Il problema dell'ingaggio di unità idrotermali combinate è risolto mediante un approccio di scomposizione e coordinamento. L'impegno dell'unità termica viene risolto utilizzando una tecnica di rilassamento lagrangiana convenzionale. Il sistema idroelettrico è suddiviso in bacini idrografici, a loro volta suddivisi in invasi. I bacini idrografici sono ottimizzati dalla programmazione del flusso di rete (NFP). La programmazione dinamica basata su un elenco di priorità viene utilizzata per risolvere il problema dell'impegno dell'unità idroelettrica (HUC) nel serbatoio. Viene utilizzato un metodo di approssimazioni successive per aggiornare i valori marginali dell'acqua (moltiplicatori di Lagrange) al fine di migliorare la convergenza dell'innesto dell'unità idraulica, a causa delle grandi dimensioni e dei molteplici accoppiamenti dei vincoli di conservazione dell'acqua. L'integrazione dell'impegno dell'unità idroelettrica nel pacchetto esistente di ottimizzazione idrotermica (HTO) migliora significativamente la qualità della sua soluzione nel sistema.

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